# Welfare Analysis with Discrete Choice in General Equilibrium

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# This Paper is Part of Broader Agenda

- When the economy changes, there are winners and losers.
- To summarize outcomes & guide policy, need measures of the aggregate value of changes.
- With a single agent, surplus is how much resources can be removed and still keep agent indifferent.
- With multiple agents, we define aggregate surplus to be how much resources can be removed and still keep everyone indifferent.
- Characterize aggregate surplus and its implications for policy in different settings:
  - *"Aggregate Productivity with Heterog. Agents"* (general framework, assumes continuous choice)
  - *"Misallocation due to Incomplete Markets"* (in closed & open economies)
  - *"An Approximate General Theory of Second Best"* (surplus-maximizing policy with distortions)
  - *"Aggregate Surplus-Maximizing Monetary Policy"* (in HANK environments)

# Aggregate Surplus in Discrete-Choice GE Models

- Discrete-choice GE models are standard: capture heterogeneity in tastes, choices, and outcomes.
- Most common measure of aggregate welfare in literature is called “ex-ante expected utility.”
- This talk has two parts:
  1. Show this approach depends on untestable & arbitrary assumptions mapping ordinal preferences to utils.
  2. Characterize aggregate surplus in discrete-choice economies using supply and demand, no utils.

     Characterization from our previous papers does not carry over to discrete-choice settings.

# Selection of Related Papers

- ***Aggregation using average utility***

  Redding (2016), Redding & Rossi-Hansberg (2017), Fajgelbaum et al. (2019), Dingel & Tintelnot (2020), Allen & Arkolakis (2022), Kleiner & Soltas (2023), Alam, Davis & Gregory (2026), Tsivanidis (2026).

- ***Decompositions of social welfare functions in discrete choice models***

  Donald, Fukui & Miyauchi (2023), Mongey & Waugh (2025).

- ***Cost-benefit analysis in discrete choice models in partial equilibrium***

  McFadden (1981), Small & Rosen (1981), Anderson, De Palma, & Thisse (1992).

  Dagsvik & Karlstrom (2005), Bhattacharya (2015, 2018), Kim & Vogel (2020).

- ***Aggregation using real GDP***

  Hsieh et al. (2019), Bryan and Morten (2019), Lamadon et al. (2022), Bagga et al. (2025).

- ***Aggregate surplus measures in GE, without discrete choice***

  Allais (1943), **Debreu** (1951), **Baqaee & Burstein** (2025a,b), **Baqaee et al.** (2026a,b).

# Agenda

Problems with “Ex-ante Expected Utility”

Our Measure in a Simple Economy

Our Measure in a General Economy

Comparison with Cost-Benefit Analysis

Extensions

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# Problem with "Expected Utility"

- Households make choices over locations $r \in \{1, \ldots, R\}$ with associated consumption $c_r$.
- Household $h$ has ordinal preferences $\succeq_h$ over these choices.
- Utilities $u_h(c_r, r)$ are pinned down only up to monotone transformations.
- Average utility is
$$W(c_1, \ldots, c_R) = \int_h \left( \max_r u_h(c_r, r) \right) dh.$$
- **Not** vNM expected utility, because it is an average across people not an expectation over a lottery.
- This is an SWF and so depends on how $\succeq_h$ is mapped into $u_h$.
- Note that uncertainty and dynamics do not resolve issue
(even if $u_h$ is forward-looking, $W$ still an average over people in time 0).

# Down to Earth Example

- Consider change from $(c_1^0, .., c_R^0)$ to $(c_1, .., c_R)$, let $A_W$ be consumption-equiv. change in welfare for $W$.
- If $u_h(c_r, r) = c_r \varepsilon_{hr}$, where $\varepsilon_{hr}$ is i.i.d. Frechet with tail parameter $\theta$. Then we get:

$$A_W = \left[ \sum_r (c_r)^\theta / \sum_r (c_r^0)^\theta \right]^{\frac{1}{\theta}}.$$

- But any monotone transformation of $u_h$ is a valid utility function for $\succeq_h$.
- For example, could normalize everyone's highest "taste" to one:

$$u_h(c_r, r) = c_r \frac{\varepsilon_{hr}}{\max_{r'} \varepsilon_{hr'}}.$$

- But we now obtain a different function $A_W$, with different policy implications.
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- But we now obtain a different function $A_W$, with different policy implications.
- Nothing in data says which is more "reasonable."
- Illustrate with Allen-Arkolakis (2014) model with Frechet tastes ($\theta = 3$) and no externalities.

# Welfare effects of removing the Interstate Highway System

- Study welfare loss from removing the Interstate Highway System, raising trade costs between counties.
- Report consumption-equivalent of average utility, $A_W$, for different cardinalizations.

| | Aggregate Welfare (percent) |
|---|---|
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- **Proposition:** Holding ordinal preferences fixed, by varying the cardinalization, $A_W$ can take any value between the minimum and maximum consumption-equivalent changes in the support of the cross-sectional distribution.

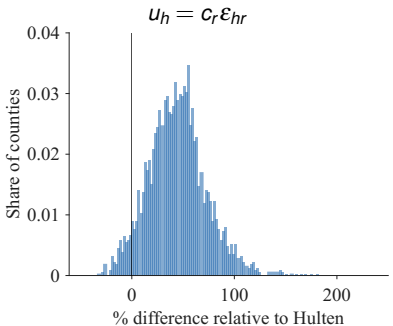
| | Aggregate Welfare (percent) |
|---|---|
| Minimum possible value | -11.5 |
| Maximum possible value | 2.8 |

- All specifications are observationally equivalent, all are “ex ante expected utility.”

# Effect of County-level Productivity Shocks

- In efficient rep agent models, value of a productivity shock is given by Hulten's theorem: sales/GDP.
- Histogram shows deviations from Hulten's theorem for productivity shocks in different counties $r$:

$$\frac{\partial \log A_W / \partial \log TFP_r - sales_r/GDP}{sales_r/GDP} \times 100$$



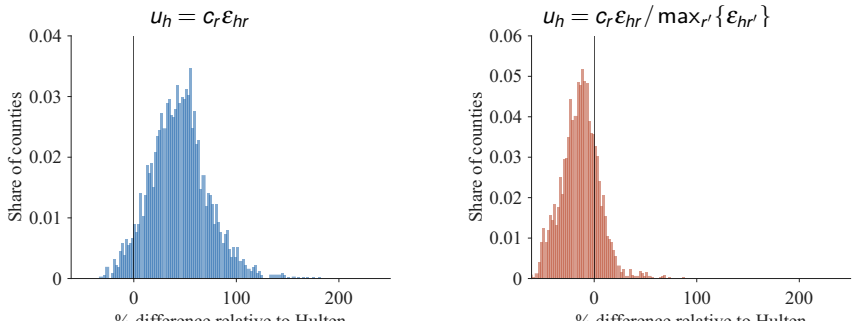

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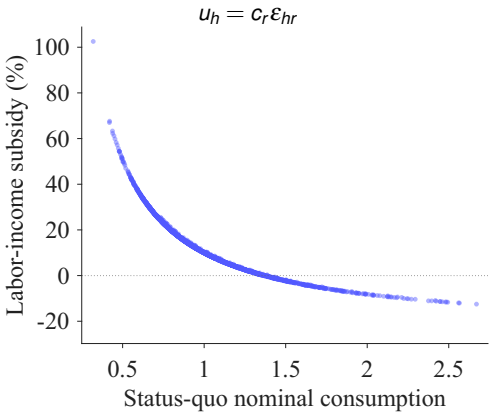



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- As pointed out by Donald et al. (2023), $A_W$ does not obey Hulten's theorem for productivity shocks.
- But deviation from Hulten depends on which cardinalization of utility is used.

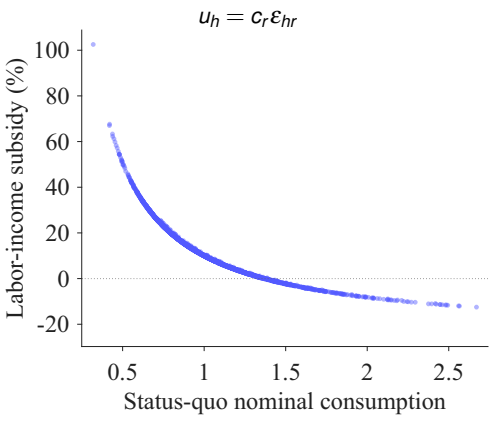
# Optimal Place Based Policies

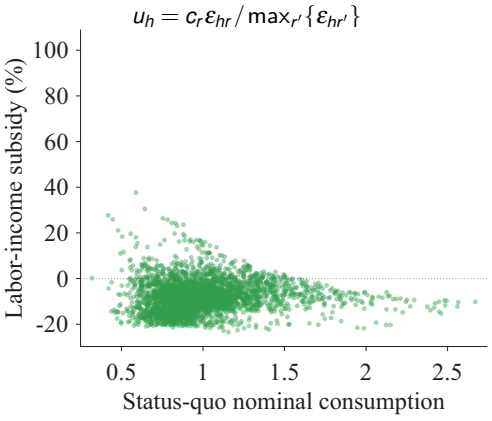
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- Suppose we give a policymaker access to place-based subsidies and have it maximize $W(\cdot)$.



# Optimal Place Based Policies

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- Suppose we give a policymaker access to place-based subsidies and have it maximize $W(\cdot)$.




- Massive intervention in markets due to arbitrary cardinalization, but equilibrium is Pareto efficient.

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# Simple Economy: Setup

- Agent $h$ has $\succeq_h$ over combinations of locations $r \in \{1, \ldots, R\}$ and consumption.
- Budget constraint implies consumption is equal to labor income plus transfer:

$$c_h = Z w_{l_h} + T_h,$$

  where $l_h$ is location choice, $w_{l_h}$ is real wage in $l_h$, and $Z$ is aggregate productivity shifter.
- The share of households that choose $r$ is $L_r = \int 1[l_h = r] dh$.
- Every location produces good linearly from labor with productivity $z_r$, so resource constraint is

$$\int c_h dh = Z \sum_r z_r L_r.$$

- Perfect competition, so equilibrium is Pareto efficient.

# TFP-Equivalent Aggregate Surplus

- Consider change in technologies from $\boldsymbol{z}^0$ to $\boldsymbol{z}$.
- Following Baqaee-Burstein (2025), define aggregate economic surplus by

  **How much endowments are left over if winners were to compensate losers.**

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  - No stance on how this surplus should be divided or transfers used.
  - Unlike average utility, it is invariant to monotone transformations of utility.
  - Applied to a single infinitesimal household, $H = \{h\}$, then $A$ is that individual's consumption-equivalent.

# First-Order Characterization

- We characterize $A$ nonlinearly. But start with a surprising first-order approximation.
- **Proposition:** Elasticity of aggregate welfare with respect to productivity of $r$ equals its GDP share:

$$\boxed{\frac{d \log A}{d \log z_r} = \frac{w_r^0 L_r^0}{\sum_k w_k^0 L_k^0}.}$$

  This holds for any collection of $\succeq_h$ (as we see later, this generalizes to much richer models too).
- To a first-order, we do not need to solve the model — spending shares suffice.
- Intuition: to a first-order, ignore relocations caused by productivity shocks. Any change in real wage from moving is offset by change in amenity value.
- Restores Hulten-logic from homog. agent spatial models (Donaldson 2025) to het. agents.

# Relationship between Aggregate Welfare and Real GDP

- Real GDP is the change in total quantity of good:

$$Y = \frac{\int_h c_h dh}{\int_h c_h^0 dh} = \frac{\sum_r z_r L_r}{\sum_r z_r^0 L_r^0}.$$

- To a first-order

$$\Delta \log Y \approx \underbrace{\sum_r \frac{w_r L_r}{\sum_{r'} w_{r'} L_{r'}} \Delta \log z_r}_{\Delta \log A} + \sum_r \frac{w_r L_r}{\sum_{r'} w_{r'} L_{r'}} \Delta \log L_r.$$

  Changing labor composition implies $\Delta \log Y \neq \Delta \log A$.

- $\Delta \log A$ does coincide with multifactor productivity (e.g. BEA procedures) because:

$$\Delta \log A \approx \underbrace{\Delta \log Y - \sum_r \frac{w_r L_r}{\sum_{r'} w_{r'} L_{r'}} \Delta \log L_r}_{\text{multifactor productivity growth}}.$$

  Equivalence breaks beyond first-order.

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- Shock occurs $\rightarrow$ agents receive compensating transfer $\rightarrow$ How much can we shrink TFP?

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- Expenditure function is minimum transfer $h$ needs, given wages, to be just indifferent to status quo:

$$T_h(\boldsymbol{w}) = \min_{c_h, l_h} \{ c_h - w_{l_h} : (c_h, l_h) \succeq_h (c_h^0, l_h^0) \}.$$

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- Note that $T_h(\boldsymbol{w})$ is a partial equilibrium object — easy to solve agent-by-agent.
- Define aggregate expenditure function to be

$$\overline{T}(\boldsymbol{w}) \equiv \int_h T_h(\boldsymbol{w}) dh.$$

- Adjust TFP until compensating transfers add up to zero

$$\overline{T}(\boldsymbol{w}/A) = 0.$$

- Equilibrium wages satisfy $\boldsymbol{w} = \boldsymbol{z}$, so $A$ solves

$$\boxed{\overline{T}(\boldsymbol{z}/A) = 0.}$$

# Constructing Aggregate Expenditure Function

- If we know aggregate expenditure function, $\overline{T}(\boldsymbol{w})$, we can easily solve for $A$ by solving for its root.
- Two ways to construct the aggregate expenditure function.
  1. Sum up household-level expenditure functions (i.e. simulate lots of households and add up).
  2. Solve for $\overline{T}(\boldsymbol{w})$ from (uncompensated) aggregate location supply function, $L_r(\boldsymbol{w})$.

     Building on Bhattacharya (2015, 2018).
- Advantage of second approach is we do not need to even specify the underlying utility functions.
- We can also compute $A$ using the derivatives of $\overline{T}(\boldsymbol{w})$ — compensated labor supply.

# Compensated Labor Supply

- The derivatives of $\overline{T}(\boldsymbol{w})$ are interpretable as aggregate compensated labor supply.
- Define $l_h^{\text{comp}}(\boldsymbol{w})$ is the location $h$ chooses once given the compensating transfer $T_h(\boldsymbol{w})$.
- Aggregate compensated labor supply in location $r$ is

$$L_r^{comp}(\boldsymbol{w}) = \int_h 1[l_h^{\text{comp}}(\boldsymbol{w}) = r]dh.$$

- Envelope theorem implies that

$$L_r^{\text{comp}}(\boldsymbol{w}) = -\frac{\partial \overline{T}(\boldsymbol{w})}{\partial w_r}.$$

  Hence, we can get $L_r^{\text{comp}}$ either by aggregating micro or differentiating aggregate expenditure function.

# Solving for Aggregate Welfare using Compensated Supply

- Totally differentiate $\overline{T}(\boldsymbol{z}/A) = 0$ and use the envelope theorem,

$$\sum_r L_r^{\text{comp}}(\boldsymbol{z}/A)\, d[z_r/A] = 0.$$

- Evaluate at status quo and rearrange to get Hulten-style formula:

$$\Delta \log A \approx \sum_r \frac{L_r^0 z_r^0}{\sum_{r'} L_{r'}^0 z_{r'}^0} \Delta \log z_r = \mathbb{E}_\lambda[\Delta \log z_r],$$

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- Differentiate twice and evaluate at status quo to get second-order approximation:

$$\Delta \log A \approx \mathbb{E}_\lambda[\Delta \log z_r] + \frac{1}{2} Cov_\lambda\left[\Delta \log\left(z_r L_r^{\text{comp}}\right), \Delta \log z_r\right].$$

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- Integrating gives the following proposition:

$$\int_{\boldsymbol{z}^0}^{\boldsymbol{z}/A} \sum_r L_r^{\text{comp}}(\boldsymbol{x})\, dx_r = 0.$$

$A$ is adjustment in real wage that makes producer surplus, area above comp. labor supply, zero.

# Example

- As mentioned, $\boldsymbol{L}^{\text{comp}}(\boldsymbol{w})$ has formula in terms of $\boldsymbol{L}(\boldsymbol{w})$, sometimes it is very simple.

- If $\succeq_h$ can be represented by $u_h(c+\varepsilon_{hr})$, then

$$L_r^{comp}(\boldsymbol{w}) = L_r(\boldsymbol{w}).$$

  Because transfers do not alter choices.

- If $\varepsilon_{hr}$ is i.i.d. type-I extreme-value distribution, then famous McFadden formula is:

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- We can integrate labor supply analytically, so $A$ solves

$$\sum_r \exp(\theta z_r/A) = \sum_r \exp(\theta z_r^0).$$

# Sufficient Statistics for Aggregate Welfare

- **First order:** GDP shares by market.
- **Second order:** elasticities of aggregate compensated labor-supply $\partial L_r^{\text{comp}} / \partial w_{r'}$.

  In paper: we show these can be recovered from elasticities of uncompensated labor supply $\partial L_r / \partial w_{r'}$.
- **Fully nonlinear:** aggregate compensated labor-supply function $L_r^{\text{comp}}(\boldsymbol{w})$.

  In paper: we show these can be recovered from uncompensated labor supply $L_r(\boldsymbol{w})$.

# Agenda

Problems with “Ex-ante Expected Utility”

Our Measure in a Simple Economy

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Extensions

# Generality

- Model so far was very simple but results generalize considerably.
- Consider a more general model allowing for arbitrary:
  - Many goods, non-traded goods, home bias, and trade costs,
  - heterogeneity in preferences and skills,
  - different “types” of primary factors (e.g. by education or landlords/labor),
  - input-output linkages,
  - commuting (agents consume & work in different places).
- Perfectly competitive benchmark (distortions and externalities at end).

# Preview of Generalized Results

- Before setting up the model, a preview of results.
- We still have the Hulten-style result: to a first-order, sales shares are sufficient statistics.
- Computing $A$ nonlinearly needs aggregate compensated **supply** like before & now **demand**.
- But demand can also be obtained from the aggregate expenditure function.

# Setting up Environment – Firms' Problem

- Many goods with arbitrary constant-returns technologies and intermediate inputs.
- Producers maximize profits taking prices as given:

$$p_i y_i - \sum_{j \in N} p_j x_{ij} - \sum_{r \in R} w_r l_{ir}, \quad s.t \quad y_i = z_i F_i \left( \{x_{ij}\}_{j \in N}, \{l_{ir}\}_{r \in R} \right).$$

# Setting up Environment – Households' Problem

- Choices $r \in \{1, ..., R\}$ index occupation, region, consumption/work location, etc.
- Agents have idiosyncratic skills across occupations, regions: $a_{hr}$.
- Consumption prices vary across locations (e.g. home bias): $p_r^c$.
- Budget constraint if $h$ chooses $l_h$ is:

$$p_{l_h}^c c_h = Z a_{h l_h} w_{l_h} + T_h.$$

- Household-side summarized by aggregate labor supply of labor type $r$

$$L_r = \int a_{hr} 1[l_h = r] dh,$$

and aggregate demand for consumption good in location $r$

$$E_r = \int p_r^c c_h 1[l_h = r] dh.$$

# Setting up Environment – Market Clearing

- Prices clear consumption goods markets:

$$E_r = p_r^c y_r^c$$

  and intermediate goods markets:

$$\sum_j p_i x_{ji} = p_i y_i,$$

  and wages clear factor markets:

$$\sum_i l_{ir} = \bar{Z} L_r.$$

- Market clearing implies that transfers add up to zero:

$$\int T_h dh = 0.$$

- Use the exact same definition of $A$ to measure aggregate surplus as before:

  **How much can we contract $Z$ while keeping everyone whole.**

# Solving for Aggregate Welfare

- Every $h$ receives a $T_h$ to keep them indifferent and factors are contracted by $A$.
- Expenditure function $T_h(\boldsymbol{w}, \boldsymbol{p}^c)$ same as before but now depends on consumption prices too.
- Once households are compensated and TFP has been contracted, we have

$$\overline{T}(\boldsymbol{w}/A, \boldsymbol{p}^c) = 0,$$

  where the wages and prices are determined in a *compensated equilibrium*.
- Solving for aggregate expenditure function $\overline{T}(\boldsymbol{w}, \boldsymbol{p}^c)$ very similar to before (it is PE problem).
- But how do we find $\boldsymbol{w}$ and $\boldsymbol{p}^c$ in the compensated equilibrium?

# Solving for Aggregate Welfare

- Wages and prices in the compensated equilibrium satisfy market clearing but using compensated supply and demand in place of the uncompensated ones.

- That is, we have

$$E_r^{\text{comp}} = p_r^c y_r^c \qquad \sum_i l_{ir} = (1/A) L_r^{\text{comp}},$$

  instead of

$$E_r = p_r^c y_r^c \qquad \sum_i l_{ir} = L_r,$$

- The extra unknown in the GE system, $A$, is still pinned down by

$$\overline{T}(\boldsymbol{w}/A, \boldsymbol{p}^c) = 0.$$

- Hence, given $\overline{T}(\boldsymbol{w}/A, \boldsymbol{p}^c)$, $E_r^{comp}$ and $L_r^{comp}$, solving for $A$ is straightforward.

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# Solving for Aggregate Welfare

- We can get $E_r^{comp}$ and $L_r^{comp}$ either by summing up micro or, as before, from derivatives of $\overline{T}$:

$$\boldsymbol{L}_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c) = -\frac{\partial \overline{T}(\boldsymbol{w}, \boldsymbol{p}^c)}{\partial w_r}, \qquad \boldsymbol{E}_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c) = \frac{\partial \overline{T}(\boldsymbol{w}, \boldsymbol{p}^c)}{\partial \log p_r^c}.$$

- So, aggregate expenditure function $\overline{T}(\boldsymbol{w}, \boldsymbol{p}^c)$ summarizes household side of the model.
- As before, there is a mapping from $\boldsymbol{L}(\boldsymbol{w}/\boldsymbol{p}^c)$ into $\overline{T}(\boldsymbol{w}, \boldsymbol{p}^c)$ .
- Hence uncompensated $\boldsymbol{L}(\boldsymbol{w}/\boldsymbol{p}^c)$ is all you need from household side to solve for aggregate surplus $A$.

# Hulten for Aggregate Welfare in General Setting

- Once again, differentiating $\overline{T}(\boldsymbol{w}/A, \boldsymbol{p}^c) = 0$ at the status quo, using envelope, yields

$$\boxed{d\log A = \sum_i \frac{\text{sales}_i}{GDP} d\log z_i,}$$

  holds for any perfectly competitive discrete choice economy, no need to solve model.

- Integrating this gives the nonlinear version of Hulten's theorem:

$$\log A = \int_{\boldsymbol{z}^0}^{\boldsymbol{z}} \sum_i \lambda_i^{\text{comp}}(\boldsymbol{x}) d\log x_i,$$

  $A$ is the area under compensated sales shares $\lambda_i^{\text{comp}}$.

- Paper provides 2nd-order approximation in terms of uncompensated supply and demand elasticities.

- Now let's go back to the quantitative examples and study them using $A$.

# Welfare effects of removing the Interstate Highway System

- Study welfare loss from removing the Interstate Highway System, raising trade costs between counties.

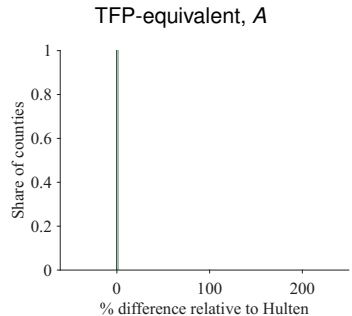
| | Aggregate Welfare Change |
|---|---|
| Endowment-equivalent welfare, $A$ | -1.55 |
| *Average-utility welfare* | |
| $u_h(c_r, r) = B_r c_r \varepsilon_{hr}$ | -1.66 |
| $u_h(c_r, r) = B_r c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$ | -1.42 |
| $u_h(c_r, r) = \exp(B_r c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\})$ | -0.64 |
| minimum possible | -11.49 |
| maximum possible | 2.81 |

- Removing IHS equivalent to a TFP loss of 1.5%, regardless of monotone transformation of utility.

# Effect of County-level Productivity Shocks

- Histogram shows deviations from Hulten's theorem for productivity shocks in different counties $r$:

$$\frac{\partial \log A_W / \partial \log TFP_r - sales_r / GDP}{sales_r / GDP} \times 100$$



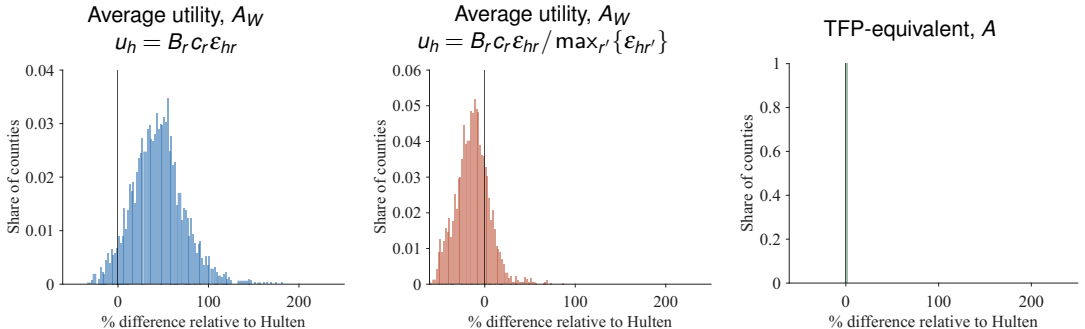

Log-productivity increases of 0.01, one county at a time.

- As expected $A$ obeys Hulten's theorem, regardless of monotone transformation of utility.

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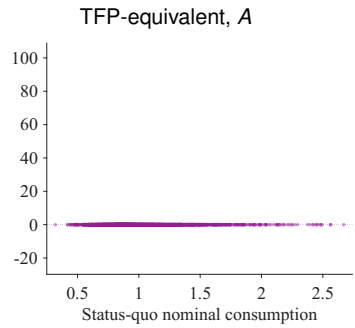



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# Optimal place-based policies

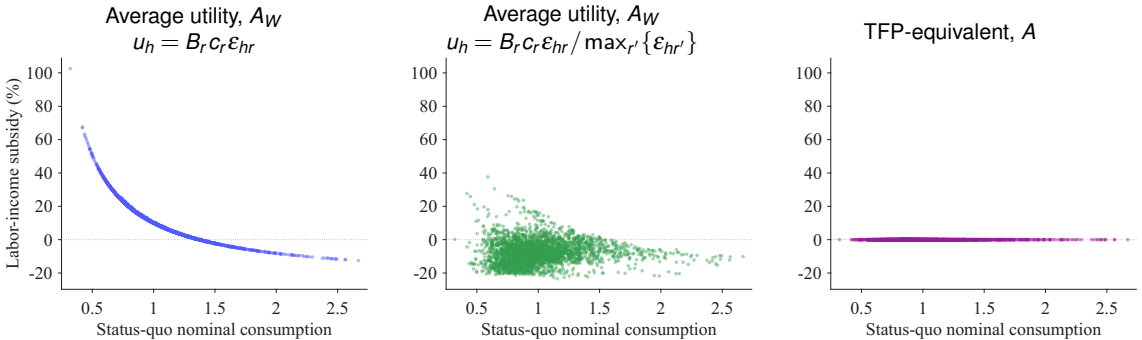
- Suppose we give a policymaker access to place-based subsidies and have it maximize $A_W$ or $A$.




- Maximizing $A$ leaves the competitive equilibrium unchanged because it is Pareto efficient.

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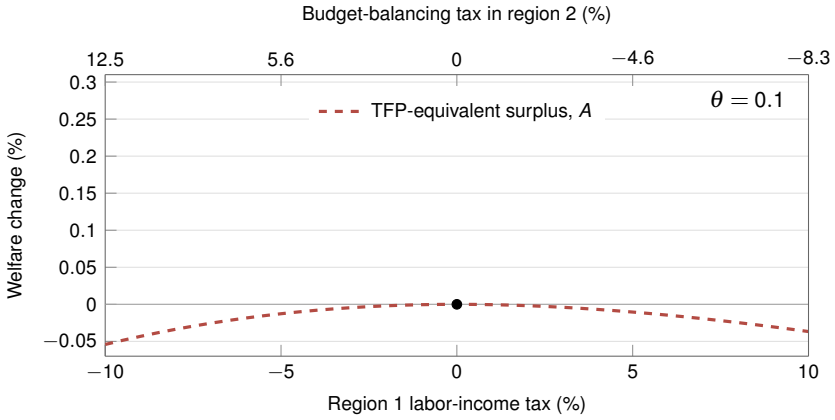
Extensions

# Comparison with Cost-Benefit Analysis

- In partial equilibrium, aggregate surplus is often measured by cost-benefit analysis.
- Typically, by summing up compensating variations (Small & Rosen, 1981; McFadden, 1981).
- In general equilibrium, this is problematic because relative prices are endogenous.
- Issue: relative prices do not tell you about the economy's ability to transform one good into another.
- A winner that can compensate at fixed prices & income may not be able to do it in general equilibrium.

# Maximizing Sum of Compensating Variations

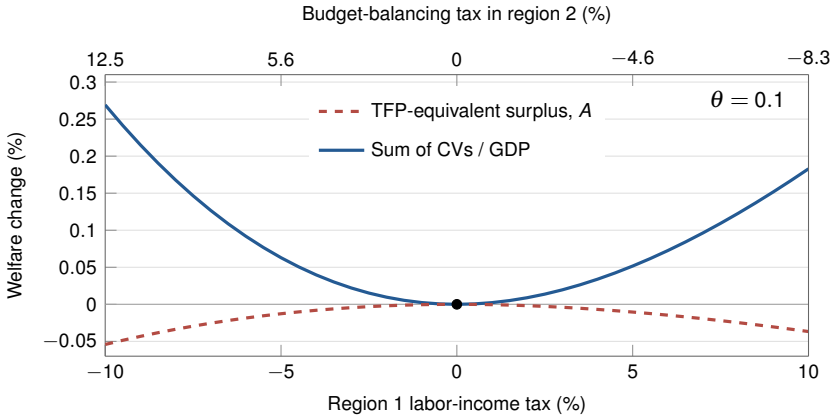
- Two locations with home bias, efficient status quo. Vary budget-balanced location-specific labor taxes.




- Taxes distort location choices, so $A$ falls in either direction from zero taxes.

# Maximizing Sum of Compensating Variations

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- Taxes distort location choices, so $A$ falls in either direction from zero taxes.
- But sum of CVs rises: at new prices, winners can compensate losers and have money left over.

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- Measure can be applied to distorted equilibria (e.g. with agglomeration externalities).

# Example with Externalities

- Suppose that there is a Marshallian externality as in AA (2014):

$$z_r = \tilde{z}_r L_r^{\alpha}.$$

- In this case, the competitive equilibrium is inefficient.
- Consider place-based labor subsidies $\{\tau_r\}$.
- The social marginal revenue product of labor is:

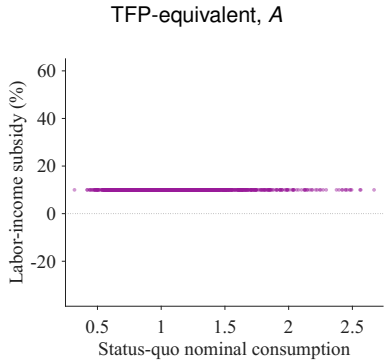
$$MRPL_r = (1+\alpha) p_r z_r.$$

- The wage, inclusive of the subsidy, is

$$w_r = (1+\tau_r) p_r z_r.$$

- Consider $\{\tau_r\}$ that maximize average utility $A_W$, or TFP-equivalent $A$.

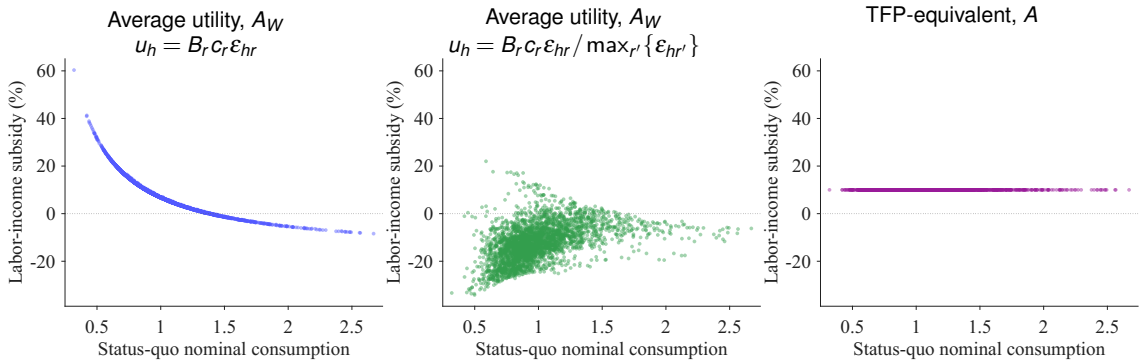
# Optimal place-based policies




- Maximizing $A$: Pigouvian subsidies that correct for externality ($\tau_r = \alpha$) so that wage = MRPL.

  Maximized value of $A$ is deadweight loss from misallocation in terms of wasted factor endowments.

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- Maximizing $A$: Pigouvian subsidies that correct for externality ($\tau_r = \alpha$) so that wage = MRPL.

  Maximized value of $A$ is deadweight loss from misallocation in terms of wasted factor endowments.

- Place-based policies maximizing $A_W$ are contaminated by arbitrary cardinal choices.

# Extensions

- Changes in amenities (e.g. weather in a region, health risk of occupation).
- Measure can also be applied to a subset of agents (e.g. by gender, or education).
- Measure can be defined with limited redistributive tools (e.g. place-based policies) rather than lump-sum transfers. Hulten still holds to a first-order.
- Measure can be applied to distorted equilibria (e.g. with agglomeration externalities).
- Measure and results can be applied to dynamic discrete choice model. Preferences over discrete state contingent plans. Complete markets: get dynamic Hulten. Incomplete markets: add wedges.

# Appendix Slides

# Deriving $\overline{T}(\boldsymbol{w})$ from $\boldsymbol{L}(\boldsymbol{w})$

- Define the following bounds on the compensating transfer:

$$T_{min}(\boldsymbol{w}) \equiv \min_r\{w_r^0 - w_r\}, \qquad T_{max}(\boldsymbol{w}) \equiv \max_r\{w_r^0 - w_r\}.$$

  We know that $T_{min}(\boldsymbol{w}) \leq T_h(\boldsymbol{w}) \leq T_{max}(\boldsymbol{w})$ for every $h$.

- From the tail-sum formula for expectations,

$$\overline{T}(\boldsymbol{w}) = T_{min}(\boldsymbol{w}) + \int_{T_{min}(\boldsymbol{w})}^{T_{max}(\boldsymbol{w})} \Pr(T_h(\boldsymbol{w}) > x)\, dx.$$

- So it remains to recover $\Pr(T_h(\boldsymbol{w}) > x)$.

- For each $x$, let $\widetilde{w}_r = \max\{w_r^0, w_r + x\}$. Then

$$\Pr(T_h(\boldsymbol{w}) > x) = \sum_{r:\, w_r^0 > w_r + x} L_r(\widetilde{\boldsymbol{w}}).$$

  last step uses insight from Bhattacharya (2018).

  Share requiring more than $x$ equals share that still prefer status quo after $x$ added to all counterfactual wages.

# Recovering $\Pr(T_h(\boldsymbol{w}) > x)$

- By the definition of the compensating transfer,

$$T_h(\boldsymbol{w}) > x \quad \Longleftrightarrow \quad \max_r u_h(w_r + x, r) < \max_r u_h(w_r^0, r).$$

- Given $\widetilde{w}_r = \max\{w_r + x, w_r^0\}$ and since $u_h(c, r)$ is strictly increasing in $c$,

$$\max_r u_h(\widetilde{w}_r, r) = \max\left\{\max_r u_h(w_r + x, r), \max_r u_h(w_r^0, r)\right\}.$$

- So, ignoring measure-zero ties,

$$T_h(\boldsymbol{w}) > x \quad \Longleftrightarrow \quad \arg\max_s u_h(\widetilde{w}_s, s) \in \{r : w_r^0 > w_r + x\}.$$

- Integrating across households gives

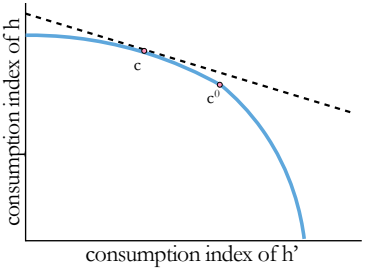
$$\Pr(T_h(\boldsymbol{w}) > x) = \sum_{r:\, w_r^0 > w_r + x} L_r(\widetilde{\boldsymbol{w}}).$$

# Comparison to Sum of Compensating Variations

- Traditional approach in partial equilibrium (Small&Rosen/McFadden) is to do welfare analysis by looking at the sum of compensating variations.
- Sum of CVs is $-\overline{T}(\boldsymbol{w}, \boldsymbol{p}^c)$ evaluated at new equilibrium prices and wages.
- This is money left after winners compensate losers, holding equilibrium prices and wages constant.
- So computing sum of CVs requires same information as computing aggregate surplus $A$.
- But, sum of CVs ignores the fact that prices can change as agents are compensated.
- So, pure redistribution along Pareto frontier can cause sum of CVs $> 0$ even though no surplus.
- Intuition: redistributions raise prices for winners and lower them for losers — so it is feasible to compensate losers at ex post prices and have money left-over.
- Policy that maximizes sum of CVs can be insane because of this.

# Sum of Compensating Variations with Endogenous Prices

- Illustrate in a two agent economy with two consumption goods.
- Consider transfer from $h'$ to $h$.




- After winners compensate losers, they have money left over (Boadway 1974). But, unfeasible.
- Problem becomes more severe in distorted economy.

Back

# Relation to McFadden's Social Surplus Function

- If indirect utility function is linear in price of good, then "social surplus function" (McFadden 1981) can be used to calculate sum of compensating variations (Small & Rosen 1981).
- We show a counterpart of this result in general equilibrium.
- Define "Social Surplus Function" as $U(\boldsymbol{w}/\boldsymbol{p}^c) = \mathbb{E}[\max_r g(a_{hr} w_r / p_r^c) + \varepsilon_{hr}]$.
- If $g(c)$ is linear (symmetric cross derivatives of $\boldsymbol{L}$) and common consumption good, then

$$U(\boldsymbol{w}/(Ap^c)) = U(\boldsymbol{w}^0/p^{c0}),$$

where real wages are those in decentralized equilibrium with technology $\boldsymbol{z}$ and TFP $1/A$.

# Example of 2nd order Approximation

- Consider popular isoelastic supply system:

$$L_r(\boldsymbol{w}) = \frac{w_r^\theta}{\sum_{r'} w_{r'}^\theta},$$

so uncompensated cross elasticities are:

$$\frac{\partial \log L_{r'}}{\partial \log w_r} = -\theta L_r.$$

Using general formula in the paper, assuming $z_r(0)$ is common across $r$, we get

$$\Delta \log A = \mathbb{E}_\lambda[\Delta \log \boldsymbol{z}] + \frac{1}{2}(1+\theta)\, Var_\lambda(\Delta \log \boldsymbol{z}),$$

the higher $\theta$ is, the more convex is $A$ in productivity shocks.

# Approximate Formulas for Aggregate Welfare Statistic, $A$

- To second order, we get

$$\Delta \log A = \sum_r \lambda_r(0) d\log z_r + \frac{1}{2}\sum_r d\lambda_r^{\text{comp}}(0) d\log z_r.$$

- The change in the compensated spending share is

$$d\lambda_r^{\text{comp}} = \lambda_r(d\log(z_r L_r^{\text{comp}}) - \sum_{r'} \lambda_{r'} d\log(z_{r'} L_{r'}^{\text{comp}}))$$

- **Proposition:** Change in compensated supply is

$$dL_r^{\text{comp}} = \sum_{r' \neq r} dL_{r' \to r}^{\text{comp}} - \sum_{r' \neq r} dL_{r \to r'}^{\text{comp}},$$

where $dL_{r' \to r}^{\text{comp}}$ is share of switches from $r'$ to $r$ in compensated equilibrium

$$dL_{r \to r'}^{\text{comp}} = \frac{\partial L_r}{\partial w_{r'}} \left( d(\frac{z_r}{A}) - d(\frac{z_{r'}}{A}) \right) \times 1 \left[ d(\frac{z_r}{A}) \leq d(\frac{z_{r'}}{A}) \right],$$

need uncomp. cross-price aggregate supply elasticities (from regression, no simulation).

# Aggregate Welfare with Place-Based Compensations

- We defined $A$ as factor savings after winners compensate losers using lump-sum transfers.
- Now extend definition to cases where only place-based redistributive policies are available.
- $A^{pb}$ is maximum reduction in $Z$ needed to make everyone at least indifferent to status quo given location-level consumption taxes.
- We characterize it for the simple economy.
- To a first-order approximation, $A^{pb}$ and $A$ agree and obey Hulten's theorem, because distortions from place-based policies are second-order around laissez-faire.

# Welfare effects of removing the Interstate Highway System

| | Aggregate Welfare Change |
|---|---|
| Endowment-equivalent welfare, lump-sum transfers | -1.55 |
| Endowment-equivalent welfare, place-based compensations | -1.60 |

# Allen–Arkolakis Spatial Model

- Each location $r$ produces a differentiated good using labor with productivity $z_r$; iceberg trade costs $\tau_{rr'}$.
- Consumption good is CES aggregator across origins. Price (excluding amenity value) and expenditure shares:

$$p_r^c = \left[ \sum_{r'} \left( \frac{\tau_{rr'} w_{r'}}{z_{r'}} \right)^{1-\sigma} \right]^{1/(1-\sigma)}, \qquad \lambda_{rr'} = \frac{(\tau_{rr'} w_{r'} / z_{r'})^{1-\sigma}}{\sum_{r''} (\tau_{rr''} w_{r''} / z_{r''})^{1-\sigma}}.$$

- Goods-market clearing: $w_{r'} L_{r'} = \sum_r \lambda_{rr'} w_r L_r$.
- Productivity depends on population, $z_r = \bar{z}_r L_r^{\alpha}$; amenities $B_r$ are fixed.
- Fréchet location tastes imply isoelastic labor supply:

$$L_r = \frac{(B_r w_r / p_r^c)^{\theta}}{\sum_{r'} (B_{r'} w_{r'} / p_{r'}^c)^{\theta}}.$$

# Calibration and Highway Counterfactual

- 3,109 counties in continental US, Armington elasticity $\sigma = 9$, and labor-supply elasticity $\theta = 3$.
- AA estimate bilateral trade costs using transportation networks & Commodity Flow Survey trade flows.
- Set county productivities and amenities to match initial population and expenditure shares by county.
- Consider no externalities, $\alpha = 0$, and agglomeration externalities, $\alpha = 0.1$.
- For each specification, calibrate $\{\bar{z}_r, B_r\}$ to match the same initial equilibrium.
- Counterfactual: replace baseline bilateral trade costs with no-Interstate-highway-system trade costs.
  - Remove speed advantage of Interstate highways, leaving road, rail, water, and air networks unchanged.

# Arbitrariness of Average Utility

- In response to a change from $\boldsymbol{c}^0$ to $\boldsymbol{c}$, define individual Lucas consumption-equivalent $A_h$ to be

$$\max_r u_h(c_r/A_h, r) = \max_r u_h(c_r^0, r).$$

  The number $A_h$ depends on $\succeq_h$ not on $u_h$ representation: $A_h > 1$ iff $\succeq_h$ prefers $\boldsymbol{c}$ to $\boldsymbol{c}^0$.

- Define utilitarian welfare $A_W$ as the consumption-equivalent for average utility:

$$W(c_1/A_W, \ldots, c_R/A_W) = W(c_1^0, \ldots, c_R^0).$$

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## Proposition

*Hold $\{\succeq_h\}$ fixed. Depending on cardinal utility representation, $A_W$ can equal anything in interval:*

$$A_W \in \left( \min_{h \in H} A_h, \max_{h \in H} A_h \right).$$

Social rankings determined by untestable assumptions about mapping ordinal preferences to utils.

# Proof Sketch: Arbitrariness of Average Utility

- Fix any $b \in (\min_h A_h, \max_h A_h)$, and define

$$d_h(b) \equiv \max_r u_h(c_r/b, r) - \max_r u_h(c_r^0, r).$$

  By definition, $A_W$ solves

$$\int_h d_h(A_W)\, dh = 0.$$

- By the definition of $A_h$,

$$d_h(b) \gtrless 0 \quad \Longleftrightarrow \quad A_h \gtrless b.$$

  Thus, $A_W$ cannot lie outside $[\min_h A_h, \max_h A_h]$.

- To attain $b$, multiply each household's utility by a positive weight $\omega_h$, with $\omega_h = \omega_+$ if $A_h > b$ and $\omega_h = \omega_-$ if $A_h < b$, and choose

$$\frac{\omega_+}{\omega_-} = \frac{-\int_{A_h<b} d_h(b)\, dh}{\int_{A_h>b} d_h(b)\, dh} > 0.$$

- Under the rescaled utilities, $\int_h \omega_h d_h(b)\, dh = 0$. Thus, $b$ solves the defining equation and $A_W = b$.

# Average Utility in Dynamic Models

- Averaging utilities across households in dynamic model has same issues as in static model.
- If all households have same preferences over the allocations being compared, averaging utilities is harmless: consumption equivalent for average utility is the same as one constructed agent-by-agent.
- Suppose that, conditional on entering market $r$ at $t$ before new tastes realized, all households have the same preferences over the future. Then, the consumption equivalent $A_r$ solves

$$\begin{aligned} & u_h\left(\bar{c}_{t-1}(s^{t-1}), \bar{l}_{t-1}(s^{t-1}), A_r c_t, r, A_r c_{t+1}(s^{t+1}), l_{t+1}(s^{t+1}), \ldots\right) \\ & \quad = u_h\left(\bar{c}_{t-1}(s^{t-1}), \bar{l}_{t-1}(s^{t-1}), c'_t, r, c'_{t+1}(s^{t+1}), l'_{t+1}(s^{t+1}), \ldots\right). \end{aligned}$$

  The same $A_r$ applies to all households, regardless of their past consumption and location.

  See paper for how this assumption holds in Caliendo, Dvorkin & Parro (2019).

- This preference agreement doesn't typically extend across starting markets or to comparisons that change the market initially entered. Our framework accommodates these heterogeneous comparisons.