

# Aggregate Productivity with Heterogeneous Agents

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## How to Aggregate with Heterogeneous Agents

- ▶ Individuals disagree over how to rank economy-wide allocations.
- ▶ When the economy changes, there are winners and losers.
- ▶ Basic question: how should we define the aggregate value of a change?
- ▶ Joint surplus defined by factor endowments left over if winners were to compensate losers.
- ▶ Every rep. agent welfare result can be translated into a result about aggregate surplus with het. agents — Hulten, misallocation formulas, gains from trade formulas, etc.
- ▶ We characterize aggregate surplus in terms of observables (expenditure shares, price elasticities, no Pareto weights, cardinal utilities, etc.)

## Relation to Prior Work

- ▶ We build on the approach by Allais (1978) and Debreu (1951).
- ▶ Related to traditional cost-benefit, which measures aggregate value of a change by:

**How much money is left over if winners compensate losers?**

- ▶ Usual implementation, sum of comp. variations/real GDP, won't answer question correctly.
- ▶ These can be positive even when winners **cannot** compensate losers.
- ▶ Cost benefit also assumes costless transfers are available, which may not be the case.
- ▶ We provide a GE counterpart to cost benefit.

## Difference to Social Welfare Functions

- ▶ The dominant approach to welfare aggregation is to rely on social welfare functions.
- ▶ Although our definition is not a SWF, it can accommodate SWF-like analysis.
- ▶ If agents' preferences over economy-wide allocations are all the same, then our measure coincides with that.
- ▶ More generally, our measure automatically accounts for equity concerns of agents in the model without requiring that they all agree.
- ▶ No need for analyst to impose externally specified Pareto weights or cardinal utility functions.
- ▶ For most of talk, focus on case with selfish preferences only; altruism at end if time.

# Research Agenda

- ▶ This paper is a general framework paper and part of broader agenda.
- ▶ Our broader research agenda applies this analysis to different settings:
  - ▶ *“Misallocation due to Incomplete Markets”* (in closed & open economies)
  - ▶ *“Welfare Analysis with Discrete Choice in General Equilibrium”*
  - ▶ *“An Approximate General Theory of Second Best”* (optimal policy to max agg prod)
  - ▶ *“Monetary Policy without Redistributive Concerns”* (in HANK environments)

# Agenda

Environment and Definitions

Characterizations

## Economic Environment: Arrow-Debreu with Distortive Wedges

- ▶ Households  $h \in H$  with preferences  $\succeq_h$  over consumption vectors  $\mathbf{c}_h$ .
- ▶ There are  $N$  goods and  $F$  factor endowments.
- ▶ Producer of good  $i$  minimizes costs given general neoclassical production function:

$$y_i = z_i G_i(\{y_{ij}\}, Z\{l_{if}\}).$$

where  $z_i$  is  $i$ 's productivity and  $Z$  is total factor productivity.

- ▶ Distorting implicit or explicit bilateral tax, denoted by  $\mu$ , so  $p_i = \mu_i \times mc_i$ . Revenues rebated.
- ▶ Standard resource constraints for goods and primary factor endowments.
- ▶ Environment is very general and flexible  
(e.g. production networks, incomplete markets, nominal rigidities, international trade, etc.)

## Definition of Aggregate Surplus (given lump-sum transfers)

- ▶ Index techn. and wedges by  $t$ ; let  $t = 0$  be status quo and  $t > 0$  counterfactual of interest.
- ▶ So  $\mathbf{z}(t)$  and  $\boldsymbol{\mu}(t)$  are some counterfactual technologies and distortions.
- ▶ Let  $\mathcal{C}(t, Z)$  be set of allocations supported as equil. given  $(\mathbf{z}(t), \boldsymbol{\mu}(t), Z)$  and some transfers.
- ▶  $A(t)$  is maximum reduction in  $Z$  at  $t$  needed to make everyone indifferent to initial allocation.

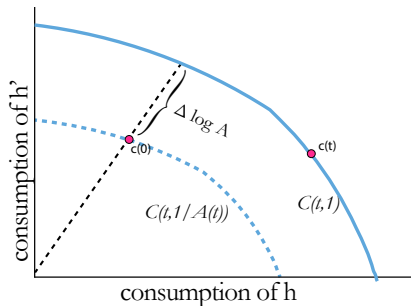
$$A(t) \equiv \max \{ Z \in \mathbb{R} : \text{there is } \mathbf{c} \in \mathcal{C}(t, 1/Z) \text{ and } \mathbf{c}_h \succeq_h \mathbf{c}_h(0) \text{ for every } h \}.$$

- ▶ If  $A(t) > A(0) = 1$ , there exists a potential Pareto improvement.

e.g. if  $A(t) = 1.1$ , it is possible to discard 10% of every factor and keep everyone indifferent.

## Graphical illustration

- ▶ Shock pushes frontier but in equilibrium, one agent is worse off. After transfers, goods left over, so  $A > 1$ .



- ▶ No stance on who gets the left over resources.

## Nesting Other Aggregate Measures

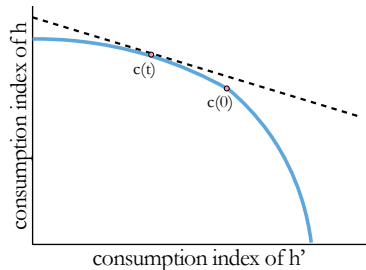
- ▶ Start by considering benchmark where  $A(t)$  coincides with existing popular measures.
- ▶ If everyone has identical, homothetic preferences & faces same prices, then

$$A(t) = \text{consumption equivalent for positive rep agent}(t) = \text{real GDP}(t) = \text{income relative to compensating income}(t).$$

- ▶ But lots of examples violate one of these assumptions (e.g. non-homothetic, incomplete markets, different tastes).
- ▶ Outside of these assumptions, other measures go haywire (violate Pareto criterion)!
- ▶ Rest of the talk focuses on characterizing  $A(t)$  when these conditions do not hold.

## Sum of Compensating Variations with Different Preferences

- Consider transfer from  $h'$  to  $h$  with different preferences.



$$\frac{\text{income at } (t)}{\text{compensating income at } (t)} > 1.$$

- After winners compensate losers, they have money left over (Boadway 1974). But, unfeasible.
- But  $A(t) = 1$ . This economy is Pareto efficient.

## Sum of Compensating Variations with Different Prices

- ▶ Suppose there is unit endowment of consumption good.
- ▶ Agent 1 pays higher markup  $\mu_1$  than agent 2 who pays  $\mu_2$ .
- ▶ A lump-sum transfer from agent 2 to agent 1 raises one's spending share  $\Delta\omega_1$ , and

$$\Delta \log \frac{\text{income at } (t)}{\text{compensating income at } (t)} \approx \bar{\mu} \left[ \frac{\mu_1 - \mu_2}{\mu_1 \mu_2} \right] \Delta\omega_1,$$

- ▶ One unit of 1's consumption is worth  $\mu_1/\mu_2 \geq 1$  units of agent 2's consumption.
- ▶ But  $A(t) = 1$ . This economy is Pareto efficient.
- ▶ Maximizing the sum of compensating variations results in insane policies.

# Agenda

Environment and Definitions

Characterizations

## Compensated Representative Agent

- ▶ In general, aggregate productivity can be computed directly using its definition.
- ▶ But in many cases, easier solve for a dual problem.
- ▶ *Equilibrium with compensated rep agent* is equilibrium with the same technologies & wedges as the original economy, but with a rep agent that has preferences

$$U(\mathbf{c}) = \min \{ \tilde{u}_1(\mathbf{c}_1), \dots, \tilde{u}_H(\mathbf{c}_H) \},$$

where  $\tilde{u}_h(\mathbf{c}_h)$  is  $h$ 's individual consumption-equivalents (invariant to monotone transforms).

- ▶ Agent  $h$ 's consumption-equivalent  $\tilde{u}_h(\mathbf{c}_h)$  solves

$$u_h\left(\frac{\mathbf{c}_h}{\tilde{u}_h}\right) = u_h(\mathbf{c}_h(0))$$

# Computing $A(t)$ via Equilibrium with Representative Agent

## ► Theorem

Given some mild conditions

$$A(t) = U(\mathbf{c}^{comp}(t)) \quad \text{for every } t,$$

at  $t = 0$ , prices and quantities in equil. with comp. rep agent the same as decentralized equil:

$$\mathbf{p}^{comp}(0) = \mathbf{p}(0), \quad \mathbf{c}^{comp}(0) = \mathbf{c}(0).$$

- So we can use representative agent results to study  $A(t)$  in heterogenous agent economies.  
(e.g. Hulten; Baqaee-Farhi; Arkolakis-Costinot-Rodriguez-Clare; Harberger; Hsieh-Klenow, etc.)

# Effect of Productivity Shocks in Competitive Economy

## Hulten's Theorem (1978)

With perfect competition  $\mu = 1$ , in response to  $\Delta \log z_i$ , to a first-order

$$\Delta \log U(\mathbf{c}^{comp}) \approx \frac{sales_i}{GDP} \times \Delta \log z_i \equiv \lambda_i \Delta \log z_i$$

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## Beyond Hulten's Theorem, Baqaee & Farhi (2019)

With perfect competition, in response to  $\Delta \log z_i$ , to a second-order

$$\Delta \log A \approx \lambda_i \Delta \log z_i + \frac{1}{2} \Delta \lambda_i^{comp} \Delta \log z_i.$$

where  $\Delta \lambda_i^{comp}$  is change in sales share with as-if rep agent (paper characterizes  $\Delta \lambda^{comp}$ ).

## Example: Productivity Shocks Across Regions

- ▶ Consider regions  $h$  with C-D preferences over tradeable goods and services.

$$u_h(\mathbf{c}) = c_{hg}^\alpha c_{hs}^{1-\alpha}, \quad \sum_h c_{hg} = z_g, \quad c_{hs} = z_{hs}.$$

- ▶ When productivities change:

$$\Delta \log A = \Delta \log U(\mathbf{c}^{comp}) \approx \alpha \Delta \log z_g + (1 - \alpha) \mathbb{E}_\omega [\Delta \log z_s]$$

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where  $\omega_h$  denotes expenditure share of  $h$ .

- ▶ Shocks to services more costly (or less beneficial) b/c they cannot be shared across regions.
- ▶ Suppose constant factor ownership, then positive rep agent with log utility over agents.  
Consumption-equivalent of this rep agent ignores last term.

## Porting Arkolakis et al. (2012) for Gains from Trade

- ▶ Example application: Losses from autarky with heterogeneous agents.
- ▶ Suppose agent  $h$  spends a share  $s_h$  on domestic goods in status-quo, with elasticity  $\theta_h$ .
- ▶ With one agent, then Arkolakis et al. (2012) show losses from autarky are:

$$\Delta \log A = \frac{\overbrace{\log s_h}^{\text{share of spending on domestic}}}{\underbrace{\theta_h - 1}_{\text{trade elasticity}}}.$$

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- ▶ With heterogeneous agents, we instead get

$$\Delta \log A \approx \underbrace{\mathbb{E}_{\omega} \left[ \frac{\log s_h}{\theta_h - 1} \right]}_{\text{first order}}$$

where  $\omega$  is cross-sectional distribution of spending.

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where  $\omega$  is cross-sectional distribution of spending.

- ▶ Loss greater with heterog. import shares. More exposed agents need more compensation.

## Porting Misallocation Results

- ▶ Use theorem to characterize losses from distortions.

### Harberger's Triangles (Harberger, 1971; Baqaee & Farhi, 2020)

Suppose we eliminate all wedges, to a second-order:

$$\Delta \log A = \Delta \log U(\mathbf{c}^{comp}) \approx - \sum_i \frac{sales_i}{GDP} \times \underbrace{\left( \frac{1}{2} \log \mu_i \Delta \log q_i^{comp} \right)}_{\text{area of deadweight loss triangle}},$$

where  $\Delta q_i^{comp}$  is first-order change in quantity of  $i$  in equilibrium with compensated rep agent.

- ▶ In paper we characterize  $\Delta q_i^{comp}$  in terms of expenditure shares and price elasticities.

## Example: Misallocation with Heterogeneous Agents

- Suppose

$$u_h(\mathbf{c}_h) = \left[ \sum_i c_{hi}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}.$$

- Household  $h$  pays markup  $\mu_{hi}$  on good  $i$ . Goods made from labor endowment.
- If there is a single agent, as in Hsieh-Klenow (2009), then:

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- With heterogeneous agents, misallocation depends on average **conditional** variance:

$$\Delta \log A \approx \frac{\theta}{2} \mathbb{E}_\omega[\text{Var}[\log \mu_h | h]].$$

- No comparison of average markups across individuals — those are purely redistributive.
- Companion paper uses this idea to measure *misallocation due to incomplete markets*.

## Example: Misallocation Due to Incomplete Markets

- ▶ Consider agents with CRRA expected utility

$$u_h(\mathbf{c}_h) = \sum_s \frac{c_h(s)^{1-1/\theta}}{1-1/\theta}$$

- ▶ Financial autarky:  $c_h(s) = a_h + \varepsilon_h(s)$ .
- ▶ Replicate autarky with complete markets and wedges

$$\frac{c_h^0(s)/c_{h'}^0(s)}{c_h^0(s')/c_{h'}^0(s')} = \left[ \frac{\mu_h(s)}{\mu_h(s')} \frac{\mu_{h'}(s')}{\mu_{h'}(s)} \right]^{-\theta}.$$

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- ▶ Gains from completing financial markets, to second order:

$$\Delta \log A \approx \frac{\theta}{2} \mathbb{E}_\omega [\text{Var} [\log \mu_h(s) \mid h]] = \frac{1}{2\theta} \mathbb{E}_\omega [\text{Var} [\log c_h(s) \mid h]],$$

where  $\omega_h = a_h / \sum_{h'} a_{h'}$  is household  $h$ 's expected consumption share.

- ▶ Disregard inequality due to dispersion in persistent component of income.

# Aggregate Productivity with Costly Redistribution

- ▶ Recall that

$A(t)$  is how much we can reduce TFP to make everyone indifferent given lump-sum transfers.

- ▶ But we can handle cases without lump-sum too.

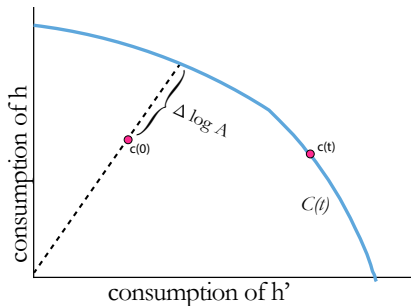
$A^{\text{costly}}(t)$  is how much we can reduce TFP to make everyone indifferent given costly redistribution.

- ▶ Let  $\mathcal{C}^{\text{costly}}(t, Z)$  be set of allocations given  $(\mathbf{z}(t), \boldsymbol{\mu}(t), Z)$ , and some transfers & taxes in  $\mathcal{T}$ .
- ▶  $A^{\text{costly}}(t)$  uses  $\mathcal{C}^{\text{costly}}(t, Z)$  in place of  $\mathcal{C}(t, Z)$ .

# Aggregate Productivity with Costly Redistribution

► Graphical illustration:

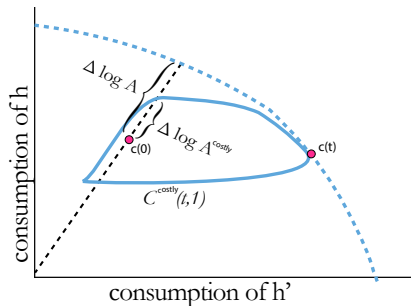
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after transfers, goods left over, so  $\Delta \log A > 0$ .



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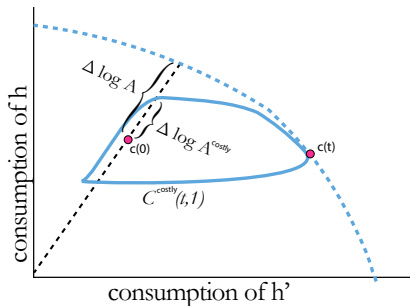
if only distortionary labor tax available, compensating loser causes misallocation.



# Aggregate Productivity with Costly Redistribution

- Graphical illustration:

if only distortionary labor tax available, compensating loser causes misallocation.



- In the paper, we show

$$\Delta \log A^{\text{costly}} \approx \Delta \log A - \text{deadweight loss triangles from redistribution.}$$

## ACR Example Revisited with Distortionary Taxes

- ▶ Consider again the gains from trade relative to autarky with heterogeneous import shares.
- ▶ Now agents have preferences over consumption bundle (of domestic & foreign) and leisure.
- ▶ Substitution elast. between cons-leisure is  $\rho$ , all work same number of hours in status-quo.
- ▶ Suppose compensations must be financed by linear labor income tax.
- ▶ The losses from autarky are:

$$\Delta \log A^{\text{linear tax}} \approx \Delta \log A^{\text{lump-sum}} - \underbrace{\frac{1}{2} \rho \Omega_c (1 - \Omega_c) (d \log \tau^*)^2}_{\text{2nd order losses from distorting taxes}},$$

with two agents, this is  $d \log \tau^* = \frac{\omega_h}{\theta - 1} [\log s_{H'd} - \log s_{hd}] > 0$ .

- ▶ Losses from limited redistrib. growing in  $\rho$  and heterogeneous exposure and falling in  $\theta$ .



## Quantitative Example: Impact of Rise of China on the US

- ▶ Off-the-shelf quant. trade model, (Baqae-Farhi, 2024) — 10 regions & 30 industries, IO.
- ▶ Low-, medium-, high-skill labor & capital in each industry. Each factor-industry is an agent.
- ▶ China grows from 5% of world GDP to 17.5% (2008 to 2023). What is effect on the US?
- ▶ Compensations among US agents from lump-sum transfers or from import tariffs.

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|                                 | National labor market | Industry-level labor market |
|---------------------------------|-----------------------|-----------------------------|
| Available Instruments           | $\Delta \log A$       | $\Delta \log A$             |
| Lump-sum transfers              | 0.010                 | 0.009                       |
| Tariffs with targeted rebates   |                       |                             |
| Tariffs with untargeted rebates |                       |                             |

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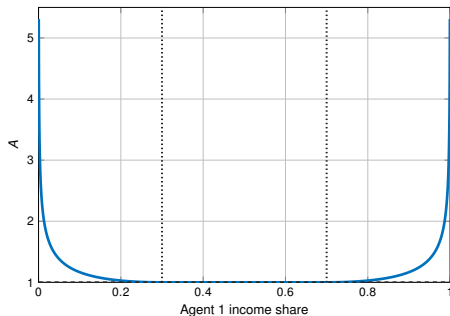
- ▶ Change in aggregate productivity depends on:
  1. How heterogeneous is exposure of workers to the shock (in this case, flexibility of labor market).
  2. How distortionary is redistributive tool.

## Altruism and Misallocation

- ▶ Consider two agents with altruistic preferences:

$$u_i(c_i, c_{-i}) = (1 - \beta) \log c_i + \beta \log c_{-i}.$$

- ▶ Resource constraint:  $c_1 + c_2 = 1$ . Status quo:  $c_1 = z_1$ ,  $c_2 = 1 - z_1$ .
- ▶ Misallocation,  $A > 1$ , only if inequality is large enough relative to altruism:  $z_1 \notin [\beta, 1 - \beta]$ .



- ▶ If  $\beta = 1/2$ , then  $A$  is misallocation according to a utilitarian SWF.

## Summary

- ▶ Household heterogeneity central to much of modern macro.
- ▶ Need aggregate measures that summarize welfare-relevant outcomes & guide policy.
- ▶ Cost-benefit in general equilibrium with policy limits on transfers.
- ▶ Counterfactual question about observables, no Pareto weights or cardinal utility.
- ▶ Characterized in terms of supply and demand curves.

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- ▶ In other papers, we apply this framework to measure the cost of incomplete financial markets within and across borders, to do welfare analysis in spatial models with taste heterogeneity, to study optimal monetary policy in heterogeneous New Keynesian economies.
- ▶ Lots of open questions to explore with this type of measure.

# Appendix Slides

## Formal Economic Environment

- ▶ Households  $h \in \{1, \dots, H\}$  with preferences  $\succeq_h$  over consumption vectors  $\mathbf{c}_h \in \mathbb{R}^N$ :

$$\max u_h(\mathbf{c}_h) \text{ such that } \sum_i p_i c_{hi} \leq \sum_f \omega_{hf} w_f L_f + T_h.$$

- ▶ Producer of good  $i$  minimizes costs given general neoclassical production function:

$$\min \sum_j p_j y_{ij} + \sum_f w_f l_{if}, \text{ such that } y_i = z_i G_i(\{y_{ij}\}, Z\{l_{if}\}).$$

$Z$  is total factor-augmenting productivity.

- ▶ Wedges  $\mu_i$  on  $i$ , explicit (e.g. tax/markup) or implicit (e.g. borrowing constraint).

$$p_i = \mu_i m c_i.$$

## Formal Economic Environment

- ▶ Resource constraint for goods and factors:

$$\sum_j y_{ji} + \sum_h c_{hi} \leq y_i, \quad \text{and} \quad \sum_i l_{if} \leq z_f L_f.$$

- ▶ Aggregate budget balance.

$$\sum_h T_h = \sum_i p_i y_i (1 - \mu_i^{-1}).$$

- ▶ Decentralized equilibrium with wedges:

Utility maximization, cost minimization,  $p_i = \mu_i mc_i$ , budget balance, & resource constraints.

## Formal Definitions of Other Aggregate Measures

- ▶ Chain-weighted real GDP:

$$\log \text{Real GDP}(t) = \int_0^t \sum_i \frac{p_i(s)c_i(s)}{\sum_{i'} p_{i'}(s)c_{i'}(s)} \frac{d \log c_i(s)}{ds} ds.$$

- ▶ Sum of compensating variations (Kaldor-Hicks or cost-benefit analysis)

$$\frac{\text{income at } (t)}{\text{compensating income at } (t)} = \frac{\sum_h e_h(\mathbf{p}(t), u_h(t))}{\sum_h e_h(\mathbf{p}(t), u_h(0))}.$$

- ▶ Consumption-equivalent variation for positive representative agent (if it exists)

$$u^{RA}(\mathbf{c}^{RA}(t)/A^{RA}(t)) = u^{RA}(\mathbf{c}^{RA}(0)).$$