# Aggregate Welfare with Discrete Choice

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# Motivation

- Discrete-choice models capture rich heterogeneity in tastes, skills, choices, and outcomes.
- But they lack a canonical aggregate welfare statistic.
- A common approach to aggregate welfare is to sum willingness-to-pay (Dupuit, Marshall, Harberger).
- Such measures have some desirable properties:
  1. Depend only on revealed preference; invariant to monotone transformations of utility.
  2. Respect Pareto principle: when every household is better off, they rise.
  3. An increase implies that there is some redistribution that can make everyone better off.
- This is common in partial-equilibrium and applied policy analysis, but needs a GE counterpart.

# What We Do

- We develop a measure of this type for general equilibrium models with discrete choice.
- We measure changes in aggregate welfare in TFP-equivalent units (Baqaee-Burstein, 2025):

  **Largest TFP contraction such that everyone can be made at least as well off as before shock.**
- If we can reduce TFP and keep everyone indifferent, then aggregate welfare increased.
- We characterize this notion of economic surplus using only supply & demand curves.
- First, we discuss the problem with the ubiquitous “expected utility” measure.

# Problem with "Expected Utility"

- Choices over locations $r \in \{1, \ldots, R\}$ with associated consumption $c_r$.
- Agent $h$ has ordinal preferences $\succeq_h$ over these choices.
- Utilities $u_h(c_r, r)$ are pinned down only up to monotone transformations.
- Average utility is
$$W(\boldsymbol{c}) = \mathbb{E}[\max_r u_h(c_r, r)].$$
- **Not** vNM expected utility, because it is an average over people not an expectation of a lottery.
- Dynamic discrete choice has same issue.

# Arbitrariness of Average Utility

- In response to a change from $\boldsymbol{c}^0$ to $\boldsymbol{c}$, define individual consumption-equivalent $A_h$ to be

$$\max_r u_h(c_r/A_h, r) = \max_r u_h(c_r^0, r).$$

The number $A_h$ depends on $\succeq_h$ not on $u_h$ representation: $A_h > 1$, iff $\succeq_h$ prefers $\boldsymbol{c}$ to $\boldsymbol{c}^0$.

- Define $A_W$ in the same way using average utility:

$$W(\boldsymbol{c}/A_W) = W(\boldsymbol{c}^0).$$

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## Proposition

*Hold $\{\succeq_h\}$ fixed. Depending on cardinal utility representation, $A_W$ can equal anything in interval:*

$$A_W \in \left(\min_{h\in H} A_h, \max_{h\in H} A_h\right).$$

Social rankings determined by untestable assumptions about mapping ordinal preferences to utils.

# Down to Earth Example

- Example: $\succeq_h$ represented by $u_h(\varepsilon_{hr} c_r)$, where $\varepsilon_{hr}$ is Fréchet with tail parameter $\theta$.

- Increasing functions $\{u_h(\cdot)\}$ have no testable implications.

- Literature assumes $u_h(x) = x$, and gets:

$$A_W = \left[ \sum_r (c_r)^\theta / \sum_r (c_r^0)^\theta \right]^{\frac{1}{\theta}}.$$

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- Observ. equivalent to $u_h(c_r \varepsilon_{hr}) = c_r \frac{\varepsilon_{hr}}{\max_{r'} \varepsilon_{hr'}}$, everyone's highest location-specific taste equals one.
- But now obtain a different function $A_W$, with different policy implications.
- Nothing in data says which is more "reasonable."
- These choices matter for real-world applications.

# Welfare effects of removing the Interstate Highway System

- Replicate Allen-Arkolakis (2014) county-level model with taste heterogeneity.
- They study welfare loss from removing the Interstate Highway System.
- Report consumption-equivalent of average utility, $A_W$, for different cardinalizations.

| | Aggregate Welfare Change |
|---|---|
| $u_h(c_r, r) = c_r \varepsilon_{hr}$ | -1.66 |
| $u_h(c_r, r) = c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$ | -1.42 |
| $u_h(c_r, r) = \exp(c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\})$ | -0.64 |
| $u_h(c_r, r) = -\exp(-c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\})$ | -1.68 |
| $\min_h A_h$ | -11.49 |
| $\max_h A_h$ | 2.81 |

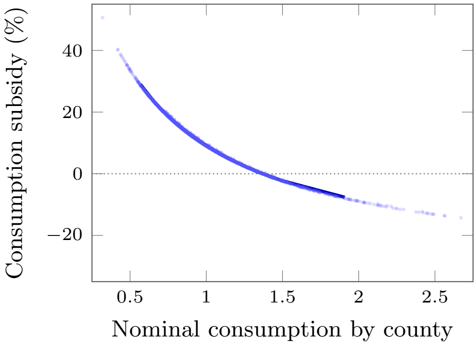
- All specifications are observationally equivalent, but aggregate welfare can rise 2.8% or fall 11%.

# Optimal Place Based Policies

- These choices don't just matter for "positive" effects, they also affect normative policy prescriptions.

- Suppose we give a planner access to place-based subsidies and have it maximize $W(\cdot)$, given IHS.

$$u_h(c_r, r) = c_r \varepsilon_{hr}$$



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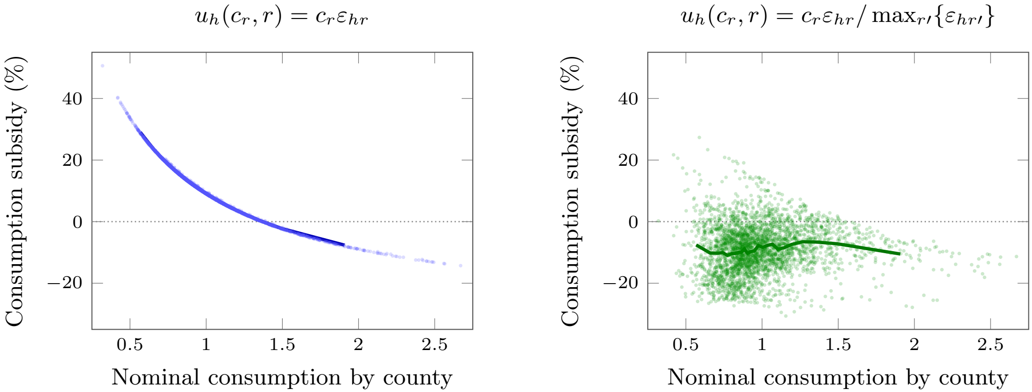


- Massive intervention in markets driven by arbitrary choices, even though equilibrium is Pareto efficient.

# Aggregating Across Households in a Coherent Way

- Average utility depends on arbitrary functional forms, w/ unintuitive redistributive preferences.
- Alternative: use real output as in national accounts — adds dollars without worrying who earned it.
- But real output ignores amenity value: everyone can be better off, and real output can be lower.
- How can we aggregate given these issues?
- Build on cost-benefit analysis — ask what is the total surplus (or deficit) caused by a change.

# Selection of Related Papers

- **Welfare in discrete choice models in partial equilibrium**

  McFadden (1981), Small & Rosen (1981), Anderson, De Palma, & Thisse (1992).

  Dagsvik & Karlstrom (2005), Bhattacharya (2015, 2021), Kim & Vogel (2020).

- **Related measures of aggregate productivity, without discrete choice**

  Allais (1978), Debreu (1951), Luenberger (1996), Baqaee & Burstein (2025).

- **Aggregation using real GDP or average utility**

  Hsieh et al. (2019), Bryan and Morten (2019), Lamadon et al. (2022), Bagga et al. (2025).

  Redding (2016), Caliendo et al. (2019), Fajgelbaum et al. (2019), Dingel & Tintelnot (2020), Allen & Arkolakis (2022), Kleiner & Soltas (2023), Tsivanidis (2026).

- **Decompositions of social welfare functions in discrete choice models**

  Donald, Fukui & Miyauchi (2023), Mongey & Waugh (2025).

# Agenda

Simple Economy

General Setup

Extensions

# Simple Economy: Setup

- Agent $h$ has preferences $\succeq_h$ over location choice $l_h \in \{1, ..., R\}$ and a consumption good $c_h$.
- Budget constraint for $h$, given real wages $\{w_r\}$, TFP shifter $Z$, and transfer $T_h$:

$$c_h = \sum_{r \in R} Z w_r 1[l_h = r] + T_h.$$

- Every region produces good linearly from labor with productivity $z_r$, so $w_r = z_r$.
- Aggregate resource constraint:

$$\sum_h c_h = Z \sum_r z_r L_r \text{ or, equivalently, } \sum_h T_h = 0.$$

  where $L_r = \sum_h 1[l_h = r]$ is the share of households that choose $r$.
- Perfect competition, so equilibrium is Pareto efficient.

# Endowment-Equivalent Aggregate Welfare

- Consider change in technologies from $\boldsymbol{z}^0$ to $\boldsymbol{z}$.
- $A$ is largest contraction in $Z$ to make everyone indifferent to status quo given some transfers.

$$\boxed{A \equiv \max \left\{ Z \in \mathbb{R} : \begin{array}{l} \text{there is an equilibrium } (\boldsymbol{c}, \boldsymbol{l}) \text{ given lump-sum transfers,} \\ \text{and aggregate productivity shifter } 1/Z, \\ \text{such that } (c_h, l_h) \succeq_h (c_h^0, l_h^0) \text{ for every } h \in H \end{array} \right\}.}$$

- If $A > 1$, then there exists a potential Pareto improvement.
- e.g. If $A = 1.01$, it is possible to keep everyone indifferent and discard $1\%$ of every factor,

  No stance on how surplus $10\%$ should be divided.

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  No stance on how surplus 10% should be divided.
- Unlike average utility, it is invariant to monotone transformations of utility.
- Unlike real output, it takes amenity into account and respects the Pareto principle.

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  No stance on how surplus $10\%$ should be divided.

- Unlike average utility, it is invariant to monotone transformations of utility.

- Applied to a single infinitesimal household, $H = \{h\}$, then $A = A_h$.

# First-Order Characterization

- We characterize $A$ nonlinearly. But start with a surprising first-order approximation.

- **Proposition:** The elasticity of aggregate welfare with respect to productivity of $r$ is

$$\boxed{\frac{d\log A}{d\log z_r} = \frac{w_r^0 L_r^0}{\sum_k w_k^0 L_k^0}.}$$

  This holds for any collection of $\succeq_h$.

- Generalizes to perfectly competitive economy with arbitrary technologies & preferences.

- To a first-order, we do not need to solve the model — spending shares suffice.

- Intuition: to a first-order, ignore relocations caused by productivity shocks. Any change in real wage from moving is offset by change in amenity value.

# Relationship between Aggregate Welfare and Real Output

- Real output is the change in total quantity of good:

$$Y = \frac{\sum_h c_h}{\sum_h c_h^0} = \frac{\sum_r z_r L_r}{\sum_r z_r^0 L_r^0}.$$

- To a first-order $\Delta \log A$ coincides with multifactor productivity (e.g. BEA procedures):

$$\underbrace{\Delta \log Y - \sum_r \frac{w_r L_r}{\sum_{r'} w_{r'} L_{r'}} \Delta \log L_r}_{\text{multifactor productivity growth}} \approx \Delta \log A.$$

  Equivalence breaks beyond first-order.

- Changing labor composition implies $\Delta \log Y \neq \Delta \log A$. So, output can fall even if everyone better off.

# Computing Endowment-Equivalent Welfare' Using Compensated Equilibrium

- To go beyond first order, we compute $A$ using a *compensated equilibrium*.
- Shock occurs $\rightarrow$ agents receive compensating transfer $\rightarrow$ How much can we shrink TFP?

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- Note that $T_h(\boldsymbol{w})$ is a partial equilibrium object — easy to solve agent-by-agent.

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- Note that $T_h(\boldsymbol{w})$ is a partial equilibrium object — easy to solve agent-by-agent.
- Equil. wages are $\boldsymbol{z}/A$, so $A$ is pinned down by budget balance:

$$\boxed{\int_h T_h(\boldsymbol{z}/A)dh = 0.}$$

# Solving for Aggregate Welfare using Compensated Supply

- Alternative: $A$ is related to area above compensated labor supply curve (i.e. producer surplus).

- Let $l_h^{\text{comp}}(\boldsymbol{w})$ be compensated individual labor supply choice.

- In words: this is the location $h$ chooses once given the transfer $T_h(\boldsymbol{w})$.

- Define the aggregate compensated labor supply in location $r$ to be:

$$L_r^{comp}(\boldsymbol{w}) = \int_h 1[l_h^{\text{comp}}(\boldsymbol{w}) = r]dh.$$

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- **Proposition:** $A$ solves this equation:

$$\boxed{\int_{\boldsymbol{z}^0}^{\boldsymbol{z}/A} \sum_r L_r^{\text{comp}}(\boldsymbol{x})dx_r = 0.}$$

- $A$ is contraction in real wage such that producer surplus for laborers is zero.

# Computing Compensated Labor Supply

- $L_r^{\text{comp}}(\boldsymbol{w})$ is simple to compute by simulation, but sometimes has closed form solution.

- If $u_h(c,r) = f_h(c + \varepsilon_{hr})$, then compensated and uncompensated labor supply are the same:

$$L_r^{comp}(\boldsymbol{w}) = L_r(\boldsymbol{w}).$$

  Because transfers do not alter choices.

- If $\varepsilon_{hr}$ is type I extreme-value distribution, then

$$L_r(\boldsymbol{w}) = L_r^{\text{comp}}(\boldsymbol{w}) = \frac{e^{\theta w_r}}{\sum_{r'} e^{\theta w_{r'}}}.$$

- We can integrate labor supply analytically, so $A(t)$ solves

$$\log(\sum_r \exp(\theta z_r / A(t))) = \log(\sum_r \exp(\theta z_r^0)).$$

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  Because transfers do not alter choices.
- This is analogue of "quasi-linear" utility in IO.
- *Only* in this case , can use "social surplus function" $U(\boldsymbol{c}) \equiv \mathbb{E}[c_r + \varepsilon_{hr}]$ to compute $A$.

  $$U(\boldsymbol{z}/A) = U(\boldsymbol{z}^0).$$

  Similar spirit to Small & Rosen (1981).

# Approximate Formulas for Aggregate Welfare

- Recall that
$$\int_{\boldsymbol{z}^0}^{\boldsymbol{z}/A} \sum_r L_r^{\text{comp}}(\boldsymbol{x}) dx_r = 0.$$
- Log differentiate once and evaluate at $t = 0$ to get
$$\boxed{d \log A = \sum_r \frac{w_r^0 L_r^0}{\sum_{r'} w_{r'}^0 L_{r'}^0} d \log z_r = \sum_r \lambda_r^0 d \log z_r.}$$

This is the formula we saw before.

# Approximate Formulas for Aggregate Welfare

- Differentiate a second time to get a second order approximation:

$$\boxed{\Delta \log A = \sum_r \lambda_r^0 d \log z_r + \frac{1}{2} \sum_r d\lambda_r^{\text{comp}} d \log z_r.}$$

  If compensated shares rise with productivity, $A$ is convex in $\boldsymbol{z}$.

- In the paper we provide a formula for $d\lambda_r^{\text{comp}}$ in terms of uncompensated supply elasticities (directly estimable, no simulations needed).

# Example: Approximate Formulas for Aggregate Welfare for Frechet

- For Frechet supply system, we have

$$L_r(\boldsymbol{w}) = \frac{w_r^\theta}{\sum_{r'} w_{r'}^\theta},$$

so uncompensated cross elasticities satisfies:

$$\frac{\partial \log L_{r'}}{\partial \log w_r} = \theta \left(1[r' = r] - L_r\right).$$

Using general formula in the paper, assuming $z_r(0)$ is common across $r$, we get

$$\Delta \log A = \mathbb{E}_\lambda [\Delta \log \boldsymbol{z}] + \frac{1}{2}(1+\theta) Var_\lambda (\Delta \log \boldsymbol{z}),$$

the higher $\theta$ is, the more convex is $A$ in productivity shocks.

# Agenda

Simple Economy

General Setup

Extensions

# Generality

- Results from simple model generalize considerably.
- General model is quite flexible, allowing for arbitrary:
  - Many goods, non-traded goods, and home bias,
  - heterogeneity in preferences and skills,
  - different “types” of primary factors (e.g. by education or landlords/labor),
  - input-output linkages,
  - commuting (agents consume & work in different places), as in Monte et al (2018).
- Perfectly competitive benchmark.

# Summary of Generalized Results

- Show that $A(t)$ can be computed using aggregate compensated **supply** and now **demand**.
- Compensated demand maps prices and wages into expenditures in each region.
- To a first order, we still have
$$\Delta \log A \approx \sum_i \frac{sales_i}{GDP} \times \Delta \log z_i.$$
- Higher order terms depend on elasticities of compensated demand and supply.
- Intuition is familiar: more elastic supply and demand means more convexity (convexity amplifies positive shocks and mitigates negative relative to first-order).
- Compensated elasticities can be obtained from uncompensated elasticities.

# Environment – Household Problem

- Choices $r \in \{1, ..., R\}$ index occupation, region, consumption/work location, etc.
- Agents have idiosyncratic skills across occupations, regions.
- Consumption prices vary across locations.
- Budget constraint is

$$\underbrace{\sum_{r \in R} p_r^c c_h 1[l_h = r]}_{\text{spending}} = \underbrace{Z \sum_{r \in R} a_{hr} w_r 1[l_h = r]}_{\text{labor income}} + \underbrace{T_h}_{\text{transfer}},$$

- Household-side summarized by partial equilibrium aggregate labor supply of labor type $r$

$$L_r(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T}) = \int a_{hr} 1[l_h = r] dh,$$

and aggregate demand for consumption good in location $r$

$$E_r(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T}) = w_r L_r + \int T_h 1[l_h = r] dh.$$

# Environment – Firms and Equilibrium

- Producer of $i \in N$ maximizes profits taking prices as given:

$$p_i y_i - \sum_{j \in N} p_j x_{ij} - \sum_{r \in R} w_r L_{ir}, \quad s.t \quad y_i = z_i F_i\left(\{x_{ij}\}_{j \in N}, \{L_{ir}\}_{r \in R}\right).$$

- Prices clear goods market:

$$\sum_r E_r 1[i \text{ is consumption in } r] + \sum_{j \in N} p_i x_{ji} = p_i y_i$$

and wages clear factor market:

$$\sum_{i \in N} L_{ir} = ZL_r.$$

# Solving for Aggregate Welfare using Compensated Equilibrium

- Definition of $A$ is same as in simple setting, compute using compensated equilibrium.
- In this equil., every $h$ receives a $T_h$ to keep them indifferent and TFP is contracted by $A$.
- Expenditure function $T_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c)$ same as before but depends on consumption prices too.

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- Expenditure function $T_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c)$ same as before but depends on consumption prices too.
- Aggregate compensated labor supply function defined as before:

$$L_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c).$$

- Now, we need one more function, aggregate compensated demand:

$$E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c) = w_r L_r^{comp}(\boldsymbol{w}, \boldsymbol{p}^c) + \int_h T_h^{comp}(\boldsymbol{w}, \boldsymbol{p}^c) 1[l_h^{comp} = r] dh.$$

- $L_r^{comp}$ and $E_r^{comp}$ are partial equilibrium objects.

# Solving for Aggregate Welfare using Compensated Equilibrium

- Compensated equilibrium conditions are standard but market clearing uses

$$L_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c) \qquad \text{and} \qquad E_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c),$$

  in place of uncompensated $L_r$ and $E_r$.

- There is one extra unknown, $A$. It is pinned down by total spending equals income:

$$\sum_r E_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c) = \sum_r L_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c) w_r.$$

- Given $E_r^{comp}$ and $L_r^{comp}$, solving for $A$ is straightforward.

# Formulas for Aggregate Welfare in General Setting

- **Proposition:** For any preferences and technologies, $A$ satisfies:

$$\log A = \int_{\boldsymbol{z}^0}^{\boldsymbol{z}} \sum_i \lambda_i^{\text{comp}}(\boldsymbol{x}) d \log x_i,$$

  as area under sales shares in compensated equilibrium, $\lambda_i^{\text{comp}}$.

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- Generalizes simple economy to general technologies and shocks to those technologies.

- Evaluating at $t = 0$ yields generalization of Hulten's Theorem:

$$\boxed{d \log A = \sum_i \lambda_i^0 d \log z_i,}$$

  holds for any perfectly competitive discrete choice economy, no need to solve model.

- Paper provides second-order approximation in terms of supply and demand elasticities.

# Welfare effects of removing the Interstate Highway System

- Replicate Allen-Arkolakis (2014) county-level model with taste heterogeneity.

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| Endowment-equivalent welfare, $A$ | -1.56 |
| *Average-utility welfare* | |
| $u_h(c_r, r) = c_r \varepsilon_{hr}$ | -1.66 |
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- If we give a planner access to place-based subsidies and have it maximize $A$ — leave market allocation alone, it is Pareto efficient.

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# Extensions

- Changes in amenities (e.g. weather in a region, health risk of occupation).

  Index location characteristic by $t$ and apply the same definition of $A(t)$.

- Measure can also be applied to a subset of agents (e.g. by gender, or education).

  When applied to a single agent, collapses to a compensating variation, $A_h$.

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- Measure can be defined with limited redistributive tools (e.g. place-based policies) rather than lump-sum transfers.

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| Endowment-equivalent welfare, place-based compensations | -1.60 |
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| $u_h(c_r, r) = c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$ | -1.42 |
| $u_h(c_r, r) = \exp(c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\})$ | -0.64 |
| $u_h(c_r, r) = -\exp(-c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\})$ | -1.68 |
| $\min_h A_h$ | -11.49 |
| $\max_h A_h$ | 2.81 |

# Extensions

- Changes in amenities (e.g. weather in a region, health risk of occupation).

  Index location characteristic by $t$ and apply the same definition of $A(t)$.

- Measure can also be applied to a subset of agents (e.g. by gender, or education).

  When applied to a single agent, collapses to a compensating variation.

- Measure can be defined with limited redistributive tools (e.g. place-based policies) rather than lump-sum transfers.

- Measure can be applied to distorted equilibria (e.g. with agglomeration externalities).

# Welfare effects of removing the Interstate Highway System

| | No externalities | With externalities |
|---|---|---|
| Endowment-equivalent welfare, $A$ | -1.56 | -1.61 |
| *Average-utility welfare* | | |
| $u_h(c_r, r) = c_r \varepsilon_{hr}$ | -1.66 | -1.68 |
| $u_h(c_r, r) = c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$ | -1.42 | -1.53 |
| $u_h(c_r, r) = \exp(c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\})$ | -0.64 | -1.02 |
| $u_h(c_r, r) = -\exp(-c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\})$ | -1.68 | -1.69 |
| $\min_h A_h$ | -11.49 | -14.00 |
| $\max_h A_h$ | 2.81 | 4.28 |

# Broader Agenda

- Broader agenda studying this measure in different heterogeneous agent settings:
  - *“Aggregate Productivity with Heterog. Agents”* (general framework, assumes cts choice)
  - *“Misallocation due to Incomplete Markets”* (in closed & open economies)
  - *“An Approximate General Theory of Second Best”* (optimal policy to max $A$ with distortions)
  - *“Monetary Policy without Redistributive Concerns”* (in HANK environments)

# Appendix Slides

# Comparison to Sum of Compensating Variations

- Traditional approach in partial equilibrium (Small&Rosen/McFadden) is to do welfare analysis by looking at the sum of compensating variations.
- Sum of CVs ignores the fact that prices are endogenous.
- So, a pure redistribution (movement along Pareto frontier) causes sum of CVs to be positive, even though there is no surplus.
- Intuition: redistributions raise prices for winners and lower them for losers — so it is feasible to compensate losers at ex post prices and have money left-over.
- Positive of sum of CVs is neither necessary nor sufficient for potential Pareto improvement in GE.

# Relation to McFadden's Social Surplus Function

- If indirect utility function is linear in price of good, then "social surplus function" (McFadden 1981) can be used to calculate sum of compensating variations (Small & Rosen 1981).

- We show a counterpart of this result in general equilibrium.

- Define "Social Surplus Function" as $U(\boldsymbol{w}/\boldsymbol{p}^c) = \mathbb{E}[\max_r u_h(w_r/p_r^c, r)] = \mathbb{E}[\max_r g(a_{hr} w_r/p_r^c) + \varepsilon_{hr}]$.

- If $g(c)$ is linear (symmetric second derivatives of $L_r$) and common consumption good, then

$$U(\boldsymbol{w}/(Ap^c)) = U(\boldsymbol{w}^0/p^{c0}),$$

  where real wages are those in decentralized equilibrium with tech. $\boldsymbol{z}$ and TFP $1/A$.

# Approximate Formulas for Aggregate Welfare Statistic, $A$

- To second order, we get

$$\Delta \log A = \sum_r \lambda_r(0) d \log z_r + \frac{1}{2} \sum_r d\lambda_r^{\text{comp}}(0) d \log z_r.$$

- The change in the compensated spending share is

$$d\lambda_r^{\text{comp}} = \lambda_r (d \log(z_r L_r^{\text{comp}}) - \sum_{r'} \lambda_{r'} d \log(z_{r'} L_{r'}^{\text{comp}}))$$

- **Proposition:** Change in compensated supply is

$$dL_r^{\text{comp}} = \sum_{r' \neq r} dL_{r' \to r}^{\text{comp}} - \sum_{r' \neq r} dL_{r \to r'}^{\text{comp}},$$

where $dL_{r' \to r}^{\text{comp}}$ is share of switches from $r'$ to $r$ in compensated equilibrium

$$dL_{r \to r'}^{\text{comp}} = \frac{\partial L_r}{\partial w_{r'}} \left( d(\frac{z_r}{A}) - d(\frac{z_{r'}}{A}) \right) \times 1 \left[ d(\frac{z_r}{A}) \leq d(\frac{z_{r'}}{A}) \right],$$

need uncomp. cross-price aggregate supply elasticities (from regression, no simulation).

# Aggregate Welfare with Place-Based Compensations

- We defined $A$ as factor savings after winners compensate losers using lump-sum transfers.
- Now extend definition to cases where only place-based redistributive policies are available.
- $A^{pb}$ is maximum reduction in $Z$ needed to make everyone at least indifferent to status quo given location-level consumption taxes.
- We characterize it for the simple economy.
- To a first-order approximation, $A^{pb}$ and $A$ agree and obey Hulten's theorem, because distortions from place-based policies are second-order around laissez-faire.

# Aggregate Welfare with Externalities

- Consider simple model with agglomeration externalities.
- Labor productivity in location $r$ is $z_r L_r^\gamma$. Benchmark assumes $\gamma = 0$.
- Equilibrium has wage equal to private marginal product, $w_r = z_r L_r^\gamma$, and transfers $T$.
- Same definition of $A$. Pinned down by budget balance in compensated equilibrium:

$$\int_h T_h^{\text{comp}}(\boldsymbol{w})dh = 0 \qquad \text{where} \qquad w_r = z_r(L_r^{\text{comp}}(\boldsymbol{w}))^\gamma / A.$$

Only difference: wages in compensated equilibrium depend on compensated labor supply.

# Allen–Arkolakis Spatial Model

- Each location $r$ produces a differentiated good using labor with productivity $z_r$; iceberg trade costs $\tau_{rr'}$.
- Consumption good is CES aggregator across origins. Price (excluding amenity value) and expenditure shares:

$$p_r^c = \left[\sum_{r'} \left(\frac{\tau_{rr'} w_{r'}}{z_{r'}}\right)^{1-\sigma}\right]^{1/(1-\sigma)}, \qquad \lambda_{rr'} = \frac{(\tau_{rr'} w_{r'}/z_{r'})^{1-\sigma}}{\sum_{r''}(\tau_{rr''} w_{r''}/z_{r''})^{1-\sigma}}.$$

- Goods-market clearing: $w_{r'} L_{r'} = \sum_r \lambda_{rr'} w_r L_r$.
- Productivity and amenities depend on population: $z_r = \bar{z}_r L_r^{\alpha}$ and $B_r = \bar{B}_r L_r^{\beta}$.
- Fréchet location tastes imply iso-elastic labor supply:

$$L_r = \frac{L_r^{\theta\beta}(\bar{B}_r w_r/p_r^c)^{\theta}}{\sum_{r'} L_{r'}^{\theta\beta}(\bar{B}_{r'} w_{r'}/p_{r'}^c)^{\theta}}.$$

# Calibration and Highway Counterfactual

- 3,109 counties in continental US, Armington elasticity $\sigma = 9$, and labor-supply elasticity $\theta = 3$.
- AA estimate bilateral trade costs using transportation networks & Commodity Flow Survey trade flows.
- Set county productivities and amenities to match 2000 Census wages and population.
- Consider AA baseline specifications: no externalities, $(\alpha, \beta) = (0, 0)$, and $(\alpha, \beta) = (0.1, -0.3)$.
- For each specification, calibrate $\{\bar{z}_r, \bar{B}_r\}$ to match same initial equilibrium.
- Counterfactual: replace baseline bilateral trade costs with no-Interstate-highway-system trade costs.
    - Remove speed advantage of Interstate highways, leaving road, rail, water, and air networks unchanged.