Aggregate Welfare with Discrete Choice

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Abstract

Discrete-choice general-equilibrium models are widely used to study how people sort across places, sectors, and jobs, but they lack a canonical aggregate welfare statistic. This paper proposes a TFP-equivalent welfare measure: the largest uniform reduction in factor productivity such that everyone can be at least as well off as in the status quo. This measures the resource surplus left after winners compensate losers, allowing prices, wages, and choices to adjust in general equilibrium. We characterize this measure using compensated supply and demand functions. A main result is a version of Hulten's theorem for discrete-choice economies: under perfect competition, the first-order welfare effect of a productivity shock to producer i is given by its sales relative to GDP, regardless of the distribution of preferences and technologies. Beyond first order, we provide approximations in terms of observable income and expenditure shares and uncompensated supply and demand elasticities. We compare this measure with common alternatives: Real output ignores amenity value; average utility depends on arbitrary cardinalizations of individual utility; and the sum of compensating variations can rise after pure redistributions. Our measure avoids these problems while preserving the logic of cost-benefit analysis in general equilibrium. In an application to the U.S. Interstate Highway System, we show that average-utility estimates vary widely across observationally equivalent representations of the same preferences, while our welfare measure is invariant to these arbitrary cardinalizations.

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1 Introduction

General-equilibrium models in which agents make discrete decisions—where to live, which sector to work in, or what job to take—are now standard in trade, spatial, labor, and macroeconomics. These models are popular because they capture rich heterogeneity in preferences, skills, and individual outcomes. Yet they make it surprisingly hard to answer a basic question: what is the aggregate welfare value of a change in the economy when agents are affected differently by that change? A meaningful answer is important for summarizing the overall welfare consequences of economic change and for guiding policy.

In economics, a common approach to aggregate welfare is to add up money-metric measures of individual welfare — sums of willingness-to-pay in the tradition of Harberger (1971). In their ideal form, measures constructed this way have the following attractive properties. First, they depend only on revealed preference information, and are invariant to arbitrary cardinalizations of utility. Second, they respect Pareto improvements: when every household is better off, they rise. Third, an increase in these types of measures means that there is a redistribution possible such that every household can be made better off.

In this paper, we develop a measure of this type for discrete choice economies. It is defined as the fraction of endowments left over after winners from a change compensate the losers. Our contribution is to characterize this general-equilibrium notion of economic surplus in terms of supply and demand curves.

We compare our measure to other common aggregate measures in discrete-choice settings: real output/consumption, average utility, and the sum of compensating variations. As we describe below, not only do these measures fail to satisfy at least one of the three properties above, they all have serious flaws.

i. **Chain-weighted real output**: This is how real output and consumption are measured in the national accounts. However, as is well-known, these measures can be an inappropriate gauge for welfare because they ignore amenity value (Roback, 1982). For example, if an agent moves from a high- to a low-wage location/job, then real output and consumption fall, even though that agent may be better off once we take into account her location preferences. Indeed, it is simple to construct examples where a shock makes every agent better off, but causes real output to decline (e.g. if productivity rises in a low-wage but high-amenity location, then real output can fall because workers move there).[1]

[1] This measure is nevertheless popular in the literature, partly because it is consistent with objects in

ii. **Average utility.** This measure is the cross-sectional average of individual utilities. A fundamental issue with this measure is that it is not invariant to monotone transformations of individual utility functions. Replacing a utility function with a monotone transformation that generates exactly the same individual choice behavior can nonetheless alter the social welfare ranking of allocations. For instance, in the spatial literature, a standard utility specification is $c\varepsilon_{hr}$, where c is consumption and ε_{hr} is h's idiosyncratic taste for choosing r.[2] This specification is observationally equivalent to $c\varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$, which normalizes each agent's highest location-specific taste to one. Although both specifications imply the same individual rankings, and hence choices, they imply different rankings of social allocations and different optimal policies. The issue is that there is no canonical basis for choosing between these normalizations. Normalizing tastes so that every agent assigns a taste of one to their favorite location is just as plausible as allowing some agents to have higher tastes in their favorite location than others. The data do not distinguish between these cardinalizations, and the resulting aggregate welfare measure hence depends on untestable and nonintuitive assumptions about the interpersonal scale of utility.[3]

iii. **Sum of compensating variations**: This measure is also called Kaldor-Hicks or cost-benefit efficiency, and has been analyzed by Small and Rosen (1981) for settings with discrete choice. It measures the amount of money left over after winners compensate the losers holding fixed post-shock equilibrium prices and wages.[4,5] While this

national income accounting. Some recent examples that report real output include Hsieh et al. (2019), Lamadon et al. (2022), and Bagga et al. (2025) in models with occupation choice, and Bryan and Morten (2019) and Heise and Porzio (2022) in spatial migration models.

[2]The use of average utility to measure aggregate welfare is widespread in the spatial literature. Examples include, among many others, Redding (2016), Caliendo et al. (2019), Fajgelbaum et al. (2019), Dingel and Tintelnot (2020), Allen and Arkolakis (2022), Monte et al. (2023), and Tsivanidis (2026). Average utility is also widely used outside spatial models; see, for example, Kleiner and Soltas (2023) in a model of occupational choice.

[3]See Section 6.2 for a discussion and worked-out example. Although this measure is sometimes called "expected utility," average utility is not a von Neumann–Morgenstern expected utility function. Expected utility functions represent ordinal preference relations over lotteries; they are not expectations of utility values across households with different preferences.

[4]The pioneers in the discrete choice literature, like McFadden (1981) and Anderson et al. (1992), emphasize that consumer welfare in discrete choice models should be measured using compensating or equivalent variations of the individual agents, as in standard consumer theory. Dagsvik and Karlström (2005) show how to calculate the distribution of compensating variation across agents using compensated choice probabilities. Bhattacharya (2015) and Bhattacharya (2021) study non-parametric identification of the distribution of compensating and equivalent variation.

[5]While this approach is common in the industrial organization literature, it is less common in general equilibrium spatial and occupational choice models. One exception is Kim and Vogel (2020), who study a model where workers choose among a discrete number of sectors including non-employment. They show that the elasticity of the sum of compensating variations with respect to wages is given by the income share

measure does not have the issues of the previous two, it can nevertheless be unreliable in general equilibrium. In particular, it may not be technologically feasible for winners to compensate the losers even though the sum of compensating variations is positive. The problem is that the fixed-price compensation exercise may describe a set of transfers that cannot be implemented once the transfers themselves change equilibrium prices, wages, and choices. We show, for example, that in response to pure transfers that move the economy along the Pareto-efficient frontier, the sum of compensating variations can become positive.

Our approach follows Harberger (1971) and Small and Rosen (1981) in using willingness-to-pay as the basis for welfare measurement, but not at fixed prices. Instead, following Baqaee and Burstein (2025) and inspired by Debreu (1951), we measure the increase in aggregate welfare by the fraction of primary factors that can be saved while keeping every household at least indifferent, allowing equilibrium choices and prices to adjust. The analyses in Debreu (1951) and Baqaee and Burstein (2025) do not carry over to discrete-choice economies because individual choice sets are nonconvex due to discreteness. For example, this means that the second welfare theorem may not hold in these economies.

Our definition restores the three cost-benefit properties discussed above, accounting for general equilibrium forces. First, because the statistic is defined by households' indifference conditions rather than by utility levels, it is invariant to monotone transformations of individual utility (unlike average utility). And because the answer is expressed in TFP-equivalent units, it has a simple interpretation: it is the amount of aggregate productive capacity that can be given up after compensating losers. Second, because it takes amenity into account, it respects the Pareto principle (unlike real output). Finally, by construction, it is positive only if a potential Pareto improvement is possible. In this sense, it preserves the distribution-neutral economic surplus logic of cost-benefit analysis — adding costs and benefits without taking a stance on whose gains count more — while avoiding the inconsistency of the fixed-price sum of compensating variations.

A payoff of this approach is that this measure is tractable and amenable to revealed preference reasoning. For example, in perfectly competitive economies, the first-order effect of a productivity shock to producer i is equal to producer i's sales divided by total income. Thus, despite the heterogeneity of preferences and the non-convexity created by discrete choices, aggregate welfare obeys a Hulten theorem just as in continuous-choice economies. To a first order, one does not need to solve for how households switch locations or occupations in response to the shock. Intuitively, when a household switches

of each sector.

choices, the pecuniary gain from switching is exactly offset at the margin by the amenity loss required to make the household willing to switch.

Our baseline model abstracts from distortions and externalities (see, e.g., Fajgelbaum and Gaubert, 2020). Focusing on a perfectly competitive benchmark helps clarify how our approach differs from standard practice in the literature. We sketch how to extend our approach beyond perfectly competitive economies in Appendix G, allowing for externalities and wedges. Similarly, when calculating aggregate welfare, we focus on the benchmark case where compensations can be implemented through individual-specific lump-sum transfers, abstracting from limitations on transfers arising, for example, from private information as discussed in Schulz et al. (2023). Appendix H discusses how to adjust our measure when individual-specific lump-sum transfers are not available and compensations must instead be contingent on location choices (i.e. place-based policies).

The structure of the paper is as follows. We set up the preferences and technologies and define perfectly competitive equilibrium in Section 2. The economy is static, and agents choose consumption and the location and/or industry in which they live and work. Agents can differ flexibly in their skills and tastes over locations and occupations. Production uses different types of primary factors and intermediate inputs, and accommodates input-output networks and costly trade.

In Section 3, we define our aggregate welfare statistic and show that it can be characterized using compensated supply and demand functions. Compensated supply functions map relative wages and prices to labor supply across locations or occupations, holding each agent on a fixed indifference curve. Compensated demand maps wages and prices to total spending on different consumption goods, holding each agent on a fixed indifference curve. We show how to compute these compensated supply and demand functions using simulation methods.

In Section 4, we provide some analytical (as opposed to simulation-based) characterizations of our measure. First, we show that our measure of TFP-equivalent aggregate welfare, to a first-order approximation, obeys Hulten's theorem in perfectly competitive economies. In particular, given sales shares, one does not need to know anything about the underlying preferences or technologies. This is in contrast to average utility and real output, both of which require solving a general equilibrium model, even to a first order.

In this section, we also discuss a special case where compensated and uncompensated supply functions coincide because compensating transfers do not alter discrete choices. This allows us to nonlinearly solve for aggregate welfare using the uncompensated supply system. Under these assumptions, aggregate welfare can also be computed using the "Social Surplus Function" of McFadden (1981).

In Section 5, we investigate nonlinearities in cases where compensations do affect discrete choices. We provide a second-order approximation of changes in aggregate welfare. We show that the first-order approximation understates positive shocks and overstates negative shocks if compensated sales shares rise for producers with positive shocks. The reverse is true if compensated sales shares fall in response to positive shocks. We express the elasticities of compensated sales shares in terms of observables in the initial equilibrium: income and expenditure shares, and price elasticities of uncompensated (market-level) supply and demand systems. Given these statistics, the distribution of tastes and the functional forms of utility functions do not matter.

In Section 6, we compare our approach to the three popular alternative measures of aggregate efficiency/welfare discussed above.

i. **Chain-weighted real output**: Real output differs from our measure of TFP-equivalent aggregate welfare even to a first-order (despite the fact that the equilibrium is Pareto-efficient). Whereas our measure differs from real output, it does coincide, to a first-order approximation, with measures of multi-factor productivity growth (or the Solow residual) computed using a quality-adjusted labor input. That is, our analysis provides a theoretical justification for those statistics in efficient economies with heterogeneous tastes and discrete choice.[6]

ii. **Average utility**: This utilitarian approach does not coincide with our measure, even to a first-order. Using a model calibrated to regional U.S. data, we show how different arbitrary but equally valid monotone transformations of individual utility functions have very different implications for policy.

iii. **Sum of compensating variations**: Our measure does not generally coincide with the sum of compensating variations (Kaldor-Hicks/cost-benefit efficiency) because our measure accounts for the fact that compensating transfers can alter equilibrium prices and wages. We provide an example where lump-sum transfers strictly raise the sum of compensating variations, but a potential Pareto improvement is impossible. Intuitively, using post-transfer prices and wages, the winners can compensate losers and still come out ahead because prices fall for goods consumed principally by the losers.

In Section 7, we provide a real-world illustration using a well-known example. We revisit the aggregate welfare cost of removing the Interstate Highway System (IHS) in the United States studied by Allen and Arkolakis (2014). We use the interpretation of their

[6] See, e.g., chapter 3 of the OECD's manual on measuring productivity for quality-adjusted labor input.

model that allows for taste heterogeneity by location and show that the welfare cost of removing the IHS implied by average utility depends on untestable assumptions about how ordinal preferences are mapped to cardinal utilities. Hence, the magnitude of welfare losses under an average utility metric can differ substantially from the cost implied by our measure.

Our paper complements the analysis of social welfare functions in spatial models by Donald et al. (2023). They decompose changes in a class of social welfare functions in response to changes in the equilibrium allocation. The most important difference between our papers is that we ask a different question. Instead of decomposing changes in the value of a social welfare function, we define and characterize a general equilibrium counterpart to cost-benefit analysis. Our analyses do not coincide even to a first order since, for example, our measure obeys Hulten's theorem whereas the class of social welfare functions they study does not.[7]

This difference is important because, as with cost-benefit analysis, our measure does not take a stance on optimal redistribution. In contrast, social welfare functions encode distributional preferences (see the discussion by Fajgelbaum and Gaubert, 2025). Therefore, policymakers that maximize a social welfare function may intervene in Pareto-efficient economies in pursuit of redistributive objectives. Which interventions are desirable (for example, whether workers should be induced to move from high-productivity to low-productivity locations) then depend on arbitrary assumptions about the monotone transformation used to represent individual utility (see Example 7 in Section 6).[8] In contrast, our measure is always maximized if the status quo is Pareto-efficient. Moreover, if a policy yields a higher value of TFP-equivalent aggregate welfare, then it must be that this policy can make every household weakly better off than in the status quo given appropriate transfers.

[7] More concretely, Donald et al. (2023) consider a model with a set of types $\theta \in \{\theta_1, \ldots, \theta_S\}$. Within each type, agent h has idiosyncratic tastes ϵ^{θ}_{hr} for location r. The social surplus function for type θ is: $U^{\theta}(\boldsymbol{c}^{\theta}) = \mathbb{E}\left[\max_r g(c^{\theta}_r) + \epsilon^{\theta}_{hr}\right]$, where c^{θ}_r is consumption and g is an increasing function. They study the class of social welfare functions that can be written as: $W = \mathcal{W}\left[U^{\theta_1}(\boldsymbol{c}^{\theta_1}), \ldots, U^{\theta_S}(\boldsymbol{c}^{\theta_S})\right)$. That is, whereas $\mathcal{W}$ is a flexible function, U^{θ} embeds a specific cardinal assumption about the representation of individual utility functions (within θ).

[8] Mongey and Waugh (2024) provide an alternative interpretation where average utility is interpreted as the expected utility of individuals who are ex-ante identical (before they know their tastes) and lack access to insurance markets for their idiosyncratic tastes. However, if such an ex-ante state does not exist, then the results of this type of analysis rest on an untestable cardinalizing assumption. That is, if households cannot make choices in the ex-ante stage that reveal their risk preferences about how they rank one set of taste parameters against another, then choice data identifies utility up to a monotone transformation so that the ex-ante expected utility function cannot be recovered from such data. See also the discussion of this issue by Davis and Gregory (2021).

2 Environment and Equilibrium

In this section we define the environment and equilibrium. We consider perfectly competitive economies without externalities (e.g. positive or negative spillovers).

Commodity space. There is a set C of consumption bundles, a set N of intermediate goods, and a set R of primary factors. Commodities are indexed by a comprehensive set of characteristics that define their uniqueness including region (e.g. haircuts in LA and New York can be treated as different commodities).[9]

Primary factors can be arbitrarily finely differentiated within R. For example, R can distinguish factors by region, occupation, skill-level, and type (e.g. high-skilled accountants in LA and commercial real estate in New York are different factors). Agents differ in their ability to provide these factor services. For example, some agents, called low-skilled workers, can only supply low-skilled labor, or some agents, called landowners in region x, can only supply land to firms in that region. Each agent h makes a discrete choice $l_h \in R$ that we refer to as the *location* of h, with the understanding that it may index location in space as well as occupation, industry, and so on.[10]

For simplicity, the set of consumption bundles C has the same size as the set of primary factors R, with a one-to-one correspondence between consumption bundles and locations. Thus, for each $r \in R$, there is a primary factor ("labor in location r") and an associated consumption bundle $c(r) \in C$. By letting consumption bundles differ by location, we allow for the possibility that some consumption goods are nontradeable (e.g. barbers in Los Angeles consume a different bundle of goods from waiters in New York).

Households' problem. There is a set H of agents. Each agent $h \in H$ chooses a location $l_h \in R$ (e.g. being a barber in Los Angeles or a waiter in New York, etc.) and consumes a scalar quantity of a homothetic bundle c_h (composed of goods and services) in that location.

Agent h has preferences $\succeq_h$ over the location choice l_h and the quantity of the consumption bundle consumed. These preferences are represented by a well-behaved utility function $u_h(c_h, l_h)$. Almost all the results in the paper hold without putting any additional functional form requirements on $u_h(c_h, l_h)$. However, for concreteness, we suppose pref-

[9] Similarly, if there are trade costs, then traded goods are indexed by both origin and destination. For example, oranges produced in California and consumed in New York are different from oranges produced in California and consumed in Illinois.

[10] This setup is general enough to capture settings that feature commuting, as in Monte et al. (2018), where agents consume and work in different locations. In this case, we let r index pairs of locations instead.

erences are *ordinally additively separable* between location choices and consumption.[11] This means that the utility function can be written as

$$u_h(c_h, l_h) = f_h(g(c_h) + \epsilon_{hl_h}),$$

where g is a strictly increasing function, ϵ_{hl_h} is a taste parameter for household h's preferences for location l_h, and f_h is any strictly increasing (potentially household-specific) function. Since utility is only pinned down up to monotone increasing transformations, the specific function f_h has no testable implications and can vary at the level of individual agents in arbitrary ways so long as it is increasing.

However, the shape of f_h does affect average utility, and including it allows us to contrast our approach to the utilitarian approach in Section 6.2. In contrast to f_h, the shape of the function g does have testable implications and its shape can be recovered from choice data.[12,13]

Households' behavior. Each agent maximizes utility choosing consumption c_h and location l_h. Recall that the choice of location encompasses both spatial location and choice of occupation. The household's budget constraint is that consumption expenditures are equal to income plus transfers:

$$\sum_{r \in R} p_r^c c_h \mathbf{1}[l_h = r] = \sum_{r \in R} a_{hr} Z w_r \mathbf{1}[l_h = r] + T_h,$$

[11] To be explicit, only Propositions 3, 5 and 13 require this ordinally additive specification of preferences. The rest of the results can be proven without relying on this assumption.

[12] The shape of $g(c_h)$ has testable implications because it controls the way income and substitution effects interact with each other. In particular, $g(\cdot)$ affects how households switch choices in response to lump-sum transfers. For example, if $g(\cdot)$ is an affine function, then households do not switch their choices if we add a constant amount to their consumption in every location. By contrast, if $g(\cdot)$ is log, then households do not switch their choices if we multiply their consumption by a constant in every location. These statements are true regardless of the distribution of tastes. See Appendix B for more information on how the function $g(\cdot)$ can be identified from behavior (see also Allen and Rehbeck, 2019).

[13] Since location choices are discrete, agents' preferences are nonconvex. This means that theorems that require convexity, like the second welfare theorem, cannot be used. The nonconvexity arises because the convex combination of two location choices is undefined. To convexify preferences, we could consider lotteries over location choices (e.g. with probability π_r consumption c_r and location r is realized). If we augment the model with lotteries, and impose the von-Neumann & Morgenstern assumptions, then the function $f_h(\cdot)$ is identifiable because it disciplines h's degree of risk-aversion. However, if such lotteries are not observed, then f_h cannot be elicited from observed choices. Furthermore, if we augment the model with lotteries so that f_h is pinned down by choices, then h's utility for lottery $\boldsymbol{\pi}$ is $u_h(\boldsymbol{\pi}) = v_h(\mathbb{E}_\pi[f_h(g(c_h) + \epsilon_{hr})|h])$ where $v_h(\cdot)$ is any strictly increasing function. That is, even in models with lotteries, utilitarian social welfare functions still depend on cardinal properties of the utility function and are not invariant to monotone transformations that do not alter individual choices even though $f_h(\cdot)$ is pinned down.

where p_r^c denotes the price of the consumption bundle in location r, $a_{hr}Z$ is agent h's efficiency units of factor type r, w_r is the wage per efficiency unit of factor r, and T_h is a lump-sum transfer to agent h. We anticipate that firms earn zero profits in equilibrium. The scalar Z is an aggregate factor-augmenting productivity shifter that uniformly scales the efficiency of factors in every location for all agents.[14] We use Z to define the TFP-equivalent variation later.

Each agent h chooses location $r \in R$ to maximize $g((Za_{hr}w_r + T_h)/p_r^c) + \epsilon_{hr}$. Denote the resulting *location choice* by $l_h(Z\boldsymbol{w}, \boldsymbol{p}^c, T_h)$, where $\boldsymbol{w} = (w_r)_{r\in R}$ and $\boldsymbol{p}^c = (p_r^c)_{r\in R}$ are the vectors of wages and consumption prices. We refer to the quantity of factor type r supplied by h as *labor* supplied by h, with the understanding that this is just a shorthand and factors could in principle include non-labor inputs like land. Given this shorthand, efficiency units of labor supplied in location r (not including the aggregate shifter Z) are

$$L_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T}) = \int a_{hr}\mathbf{1}[l_h(Z\boldsymbol{w}, \boldsymbol{p}^c, T_h) = r]dh,$$

where $\boldsymbol{T} = (T_h)_{h\in H}$ is the vector of transfers. We call the function $L_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})$ the aggregate *labor supply function*. Efficiency units of labor supplied in location r, and available to firms, are $ZL_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})$. Let $\boldsymbol{\chi}$ denote the vector of final consumption spending shares in each location:

$$\chi_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T}) = \frac{Zw_rL_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T}) + \int T_h\mathbf{1}[l_h(Z\boldsymbol{w}, \boldsymbol{p}^c, T_h) = r]dh}{\sum_{r'} Zw_{r'}L_{r'}(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})}.$$

We call the function $\boldsymbol{\chi}(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})$ the *final demand* function. Note that these functions are partial equilibrium objects that can be evaluated for any vectors of wages, prices, and lump-sum transfers.

Producers' problem. The set of commodities is the union of consumption goods, C, and intermediate goods, N, denoted by $C + N$. Each commodity i is produced by a perfectly competitive representative firm that maximizes its profit:

$$p_i y_i - \sum_{j\in N} p_j x_{ij} - \sum_{r\in R} w_r L_{ir},$$

subject to a constant-returns technology

$$y_i = z_i F_i\left(\{x_{ij}\}_{j\in N}, \{L_{ir}\}_{r\in R}\right), \tag{1}$$

[14] Equivalently, we can model Z as a shifter that scales value added productivity of all producers.

where z_i is a Hicks-neutral productivity shifter for producer i, F_i is a constant returns production technology, x_{ij} is producer i's use of intermediate good j, and L_{ir} is its use of efficiency units of primary factor r. The price of the consumption bundle in location $r \in R$ is $p_r^c = p_{c(r)}$.

Resource Constraints. The resource constraint requires that consumption and intermediate input usage be equal to production. Recall that commodities are split into pure consumption goods ($i \in C$) and pure intermediates ($i \in N$). Hence, if i is a consumption bundle, then i is not used as an intermediate $x_{ji} = 0$. On the other hand, if $i \in N$, then i is not directly consumed by any household. The resource constraints are therefore:

$$\int c_h \mathbf{1}[l_h = r] dh = y_{c(r)} \text{ if } r \in R \qquad \text{and} \qquad \sum_{j \in C+N} x_{ji} = y_i \text{ if } i \in N. \tag{2}$$

The resource constraint for factor $r \in R$ requires that total factor demand from producers is equal to total factor supply from households:

$$\sum_{i \in C+N} L_{ir} = Z L_r. \tag{3}$$

Finally, lump-sum transfers add up to zero:

$$\int T_h dh = 0. \tag{4}$$

Definition 1 (Equilibrium with Discrete Choice)**.** An equilibrium is a collection of consumptions, c_h, location choices, l_h, outputs, y_i, input choices, $\{x_{ij}, L_{ir}\}$, prices, p_i, and wages, w_r, such that each agent chooses consumption and location to maximize utility subject to their budget constraint, producers maximize profits subject to technology taking prices as given, transfers add up to zero, and all resource constraints are satisfied.

We focus on decentralized equilibria where transfers are equal to zero: $\boldsymbol{T} = 0$.

Solving for Equilibrium. To solve for the equilibrium, we introduce some useful accounting variables. Let $\boldsymbol{\lambda}$ be the vector of Domar (1961) weights, whose ith component is commodity i's sales or factor i's income divided by total income:

$$\lambda_i = \frac{p_i y_i}{\sum_r Z w_r L_r} \mathbf{1}[i \in C + N] + \frac{Z w_i L_i}{\sum_r Z w_r L_r} \mathbf{1}[i \in R].$$

Let Ω be the $|C+N+R| \times |C+N+R|$ input-output matrix. The first $|C|$ rows and columns are the consumption bundles, the next $|N|$ are the intermediates, and the final $|R|$ are the primary factors. For $i \in C+N$, the ijth element of Ω is the expenditure share of i on inputs from j relative to i's total sales:

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}\mathbf{1}[j \in N] + \frac{w_j L_{ij}}{p_i y_i}\mathbf{1}[j \in R].$$

The rows corresponding to primary factors are zero. Using this notation, we can now describe the equilibrium conditions. Perfect competition and cost minimization by producer $i \in C+N$ imply that p_i is equal to its marginal cost:

$$p_i = z_i^{-1}\mathrm{mc}_i(\boldsymbol{p}, \boldsymbol{w}), \tag{5}$$

where mc_i is producer i's marginal cost function when $z_i = 1$ and depends on input prices and factor wages.

Market clearing for $i \in C+N+R$ requires that commodity i's sales, or factor i's income, equal total spending on i by households and producers:

$$\lambda_i = \sum_{r \in R} \chi_r \mathbf{1}[i = c(r)] + \sum_{j \in C+N} \lambda_j \Omega_{ji}, \tag{6}$$

where $c(r)$ is the consumption bundle associated with location r, and χ_r is the share of final consumption spending in location r. The factor market clearing condition states that spending on factor r by firms is equal to earnings by agents that supply that factor:

$$\lambda_r = \frac{Z w_r L_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})}{\sum_{r'} Z w_{r'} L_{r'}(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})}. \tag{7}$$

Finally, when lump-sum transfers are zero, $\boldsymbol{T} = 0$, the share of spending in each location is equal to the share of income earned in that location, $\chi_r = \lambda_r$. Using this fact, and given the aggregate supply function $L_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})$, one can solve for equilibrium prices, wages, and quantities using (5), (6), and (7).

The example below provides a concrete illustration of the structure of the model using Cobb-Douglas and logit functional forms.

Example 1 (Cobb-Douglas with Logit). Suppose location r produces a single intermediate good linearly using only factor r, with productivity z_r. Its price is therefore w_r/z_r. The consumption bundle is the same in every location and is a Cobb-Douglas aggregator with constant expenditure share parameters $\{\Omega_{cr}\}_{r\in R}$ that sum to one. Thus, $p^c_r = p^c$ for every $r \in R$, where

$$p^c = \prod_{r\in R} \left(\frac{w_r}{z_r}\right)^{\Omega_{cr}}. \tag{8}$$

Suppose there is a unit mass of agents with homogeneous skills, $a_{hr} = 1$ for every $h \in H$ and $r \in R$, and preferences take the form $u_h(c_h, l_h) = f_h(c_h + \epsilon_{hl_h})$, where f_h is any strictly increasing function and the ϵ_{hr} are independently drawn from type-I extreme-value distributions. Because consumption enters linearly inside f_h and its price is common across locations, lump-sum transfers do not affect location choices. Following McFadden (1973), the labor supply function is

$$L_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T}) = \frac{\exp(\theta Z w_r / p^c + B_r)}{\sum_{r'} \exp(\theta Z w_{r'} / p^c + B_{r'})}, \tag{9}$$

where θ and $\{B_r\}$ are parameters of the distribution of ϵ_{hr}.

Because the good produced in location r uses only factor r and all households consume the same Cobb-Douglas bundle, (6) implies that factor r's income share is $\lambda_r = \Omega_{cr}$. Combining this condition with labor supply yields

$$\frac{w_r \exp(\theta Z w_r / p^c + B_r)}{\sum_{r'} w_{r'} \exp(\theta Z w_{r'} / p^c + B_{r'})} = \Omega_{cr}, \tag{10}$$

Equations (8) and (10) jointly pin down wages and the consumption price index in the decentralized equilibrium, up to the choice of a numeraire. Given prices and wages, quantities are then straightforward to calculate. Example 8 in the appendix provides a more involved example with intermediate-input linkages.

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3 Defining and Characterizing Aggregate Welfare

In this section, we define our measure of aggregate welfare and then characterize it. For convenience, we parameterize technologies by a scalar t and write them as $\boldsymbol{z}(t)$. We assume that $t = 0$ corresponds to an initial equilibrium, which we call the *status quo*. We calibrate the status quo to the allocation observed in the data. Counterfactual technologies of interest are indexed by $t > 0$. As t increases, productivity parameters change. We normalize Z to be constant and equal to one as a function of t.[15]

Our treatment is fairly abstract and general, so readers looking for a concrete example are encouraged to focus on the running example introduced in Example 2. Throughout the rest of the paper, we always return to this same example to illustrate the general results.

3.1 Definition of TFP-Equivalent Aggregate Welfare

We define the set of feasible allocations to be

$$\mathcal{C}(t, Z) \equiv \{\{c_h, l_h\}_{h\in H} \text{ supported via competitive equilibrium given } \boldsymbol{z}(t), Z, \text{and some transfers}\}.$$

Feasible here means allocations that can be implemented as competitive equilibrium allocations given some transfers (not just technologically feasible).

Inspired by Debreu (1951), we measure changes in aggregate welfare using equivalent changes in factor-augmenting productivity Z.

Definition 2 (TFP-Equivalent Aggregate Welfare)**.** Aggregate welfare in TFP-equivalent terms at t is

$$A(t) = \max\left\{Z^{-1} : \{c_h, l_h\}_{h\in H} \in \mathcal{C}(t, Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h\right\}, \quad (11)$$

where $c_h(0)$ and $l_h(0)$ are the consumption and location of h in the status quo.

In words, to measure aggregate welfare, we consider a hypothetical rescaling of factor productivity by $1/A(t)$. We define $A(t)$ as the largest contraction such that every agent can be made at least as well off as in the status quo given some transfers.[16] Intuitively, if $A(t) = 1.01$, then this means that we can shrink the productivity of factors by roughly 1% (more precisely, $1 - 1.01^{-1}$) and still keep every household indifferent to the initial equilibrium. Equivalently, because Z uniformly scales the production possibility set, every

[15] To model changes in aggregate factor-augmenting productivity, we can introduce intermediaries that sell primary factor services to other producers and instead scale their productivities.

[16] This is not exactly the definition in Debreu (1951) because we restrict the set of feasible allocations to be the ones that can be reached using individual-specific lump-sum transfers as opposed to any technologically feasible allocation. This distinction matters because, in this economy, the second welfare theorem need not hold. In Appendix H, we discuss how to further restrict the set of feasible allocations to be the ones that can be supported using place-based, rather than individual-specific, transfers. We show that aggregate welfare using place-based compensations is weakly less than aggregate welfare using individual-specific lump-sum transfers, since compensations using place-based policy are distortionary, but these distortions are second-order around laissez-faire.

household can be kept indifferent to the status quo with roughly 1% less of every good — that is, there is a 1% TFP-equivalent surplus.

Throughout, we assume that at the solution to (11), every agent can be made exactly indifferent to the status quo. This rules out pathological cases where agents are completely disconnected in goods and factor markets.[17]

Before proceeding with the analysis, we collect some remarks on how this measure can be extended.

Remark (Changes in amenities). Since t only indexes productivity parameters $\boldsymbol{z}$, our analysis focuses on the effect of productivity shocks only. However, the same definition of aggregate welfare and many of the results we present can be extended to cover changes in other parameters as well. For example, we can allow for changes in location characteristics that affect amenity values (e.g. changes in the weather in a region or in the health risk associated with particular occupations). In that case, we index the value of the relevant characteristic by t and apply Equation (11) without any other modification. If the amenity value of choosing location r can be written as $u_h(B_r c_r, r)$, where B_r is a location-specific quality shifter, then an increase in B_r has the same effect on aggregate welfare $A(t)$ as the same proportional increase in the productivity of the consumption good in that location, $z_{c(r)}$.

Remark (Welfare for a subset of agents). Definition (11) can also be applied to measure collective welfare for a subset of agents, such as agents of a given gender, race, or age. In that case, we contract the endowments of agents in that subset subject to indifference conditions and balanced-budget transfers within the subset. If the subset consists of a single infinitesimal agent h, then the TFP-equivalent welfare collapses to that agent's compensating variation,[18] denoted by $A_h(t)$, and defined by

$$\max_r u_h\left(\frac{a_{hr} w_r(t)}{A_h(t) p_r^c(t)}, r\right) = u_h^0. \tag{12}$$

The value of $A_h(t)$ depends only on household h's ordinal preferences $\succeq_h$ and is invariant to strictly increasing transformations of utility.

Remark (Imperfect competition and externalities). Although in the body of the paper we

[17] For example, all households are immobile across locations and only consume goods produced by their own location (i.e. autarky in both goods and factor markets). In such a case, making households exactly indifferent to the status quo can become impossible because transfers between locations are not possible. In these cases the measure is still well-defined, but some of our theorems do not go through.

[18] Technically, this is a compensating variation in ratios—the proportional reduction in agent h's productivity—rather than in differences. See Appendix I for a comparison of compensating variations in ratios and in differences.

restrict attention to perfectly competitive economies, in Appendix G we show how to calculate aggregate welfare in the presence of distortions and externalities. This extension can be used to study the effects of policies (modelled as implicit or explicit tax-like wedges) or to quantify misallocation.

3.2 Exact Characterization of $A(t)$

Solving for $A(t)$ by brute-force is challenging. To make progress, we break the problem into pieces and characterize $A(t)$ using the aggregate *compensated* supply and demand functions.[19] As in classical consumer theory, the compensated supply and demand functions map wages and prices into labor supplied and spending in each location, holding each agent on a given indifference curve using lump-sum transfers.

3.2.1 Expenditure Function and Compensated Choices

Begin by defining the expenditure function of each consumer.

Definition 3 (Expenditure function)**.** Define $e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ to be the (net) expenditure function for agent h given wages $\boldsymbol{w}$, consumption prices $\boldsymbol{p}^c$, and status quo utility $u_h^0 = u_h(c_h(0), l_h(0))$:

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = \min\{T_h : u_h(c_h, l_h) \geq u_h^0, \text{ and } \sum_r p_r^c c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

We call the location choice $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ associated with this optimization problem *the compensated choice* of agent h.

In words, $e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ is the minimum lump-sum transfer agent h requires to be made at least indifferent to some reference utility level u_h^0, given prices and wages. The location choice that the agent makes, given that transfer, is what we call the compensated choice. This definition parallels the classical definition of expenditure functions and compensated demand in consumer theory.[20]

To characterize the compensated choice of each agent, we first define the consumption-equivalent variation (Lucas, 1987).

[19] In Appendix C, we consider a special case in which agents are homogeneous in both preferences and skill levels. In this case, lump-sum transfers are not needed to evaluate the compensated supply and demand functions, since all agents are affected symmetrically by shocks. When $g(c) = \log(c)$, $A(t)$ equals the proportional change in real consumption per capita in any location occupied in both equilibria.

[20] See also Small and Rosen (1981) for a related analysis. Whereas they consider discrete consumption choices, given fixed income, here the level of income depends also on the discrete choice (via the wage).

Definition 4 (Consumption-equivalent variation)**.** For each $l_h \in R$, define $\bar{c}_{hl_h}$ as the solution to

$$u_h(\bar{c}_{hl_h}, l_h) = u_h^0,$$

where u_h^0 is utility of h in the status quo. In words, $\bar{c}_{hl_h}$ is the consumption agent h must have in location l_h to be indifferent to the status quo.

Given the consumption-equivalent variation, the following proposition shows how to compute the compensated choices and the expenditure function agent-by-agent.

Proposition 1 (Compensated Choices and Expenditure Function)**.** *The compensated choice* $l_h^{comp}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ *satisfies*

$$l_h^{comp}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) \in \arg\max_{l \in R} \left[\sum_r (a_{hr} w_r - p_r^c \bar{c}_{hr}) \mathbf{1} \left[l = r \right] \right].$$

The expenditure function, or transfer needed to ensure indifference to u_h^0*, is*

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = p^c_{l_h^{comp}} \bar{c}_{hl_h^{comp}} - a_{hl_h^{comp}} w_{l_h^{comp}}.$$

In words, $(a_{hr} w_r - p_r^c \bar{c}_{hr})$ is the surplus, in dollars, agent h receives from being sent to location r. The compensated choice maximizes this surplus, and the expenditure function is equal to the negative of the surplus. Proposition 1 gives a straightforward way to calculate the compensated choice and the compensating transfer for each agent given any vector of consumption prices $\boldsymbol{p}^c$ and wages $\boldsymbol{w}$.

We aggregate agent-level location choices and expenditure functions, given by Proposition 1, to get location-level variables. The compensated labor supply function, the efficiency of units of labor in location r (not including the aggregate shifter Z) given compensating transfers, is:

$$L_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0) = \int a_{hr} \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = r] dh.$$

By duality, compensated and uncompensated labor supply are connected by the identity

$$L_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0) = L_r(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{e}(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0)).$$

Similarly, the compensated spending by households that choose location r is the sum of their labor income and net transfer payments:

$$E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0) = w_r L_r^{\text{comp}} + \int e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = r] dh,$$

where $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ and $e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ are given by Proposition 1. We can also denote the compensated share of total spending by households in location r to be

$$\chi_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0) = \frac{E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0)}{\sum_{r'} E_{r'}^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0)}.$$

Proposition 1 provides a way to evaluate the compensated labor supply $\boldsymbol{L}^{\text{comp}}$ and expenditure $\boldsymbol{E}^{\text{comp}}$ functions given any arbitrary vector of wages, prices, and utility levels.

3.2.2 Calculating $A(t)$ using Compensated Choices and Spending

The next result uses the compensated supply and demand functions to solve for $A(t)$.

Theorem 1 (Aggregate Welfare using Compensated Equilibrium)**.** *TFP-equivalent aggregate welfare satisfies*

$$\sum_r E_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0) = \sum_r (w_r/A(t)) L_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0), \tag{13}$$

The vector of commodity prices $\boldsymbol{p}$ satisfies the marginal-cost conditions (5)*, given wages $\boldsymbol{w}$ and productivities $\boldsymbol{z}(t)$, and consumption prices satisfy $p_r^c = p_{c(r)}$. The vector of wages $\boldsymbol{w}$ satisfies the market-clearing conditions* (6) *and* (7)*, given compensated labor supply $\boldsymbol{L}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0)$ and spending shares $\boldsymbol{\chi}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0)$. The wages, prices, labor supplies, and quantities that satisfy these conditions are not the same as the ones in the decentralized equilibrium. We call them variables in the* compensated *equilibrium instead.*

In the compensated equilibrium, every agent h receives a transfer of $e_h(\boldsymbol{w}/A(t), \boldsymbol{p}^c, u_h^0)$ to keep them indifferent, and aggregate factor productivity is set to $1/A(t)$. The value of $A(t)$ is pinned down, according to Equation (13), by the condition that total spending equals income in the compensated equilibrium or, equivalently, that lump-sum transfers add up to zero.

Solving for $A(t)$ using Theorem 1 is relatively straightforward. To see the logic, consider the following iterative procedure. First, conjecture a vector of wages in the compensated equilibrium, $\boldsymbol{w}$, and aggregate TFP shifter $1/A$. (For example, $A = 1$ and $\boldsymbol{w}$ equal to its status quo values.) Use the wages and the productivities, $\boldsymbol{z}(t)$, to solve for prices by setting prices equal to marginal costs, as in (5). Then draw a population of agents with different ϵ's and apply Proposition 1 to get compensated supplies $\boldsymbol{L}^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, \boldsymbol{u}^0)$ and spending $\boldsymbol{E}^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, \boldsymbol{u}^0)$ using conjectured prices, wages, and aggregate TFP shifter. Given $\boldsymbol{L}^{\text{comp}}$ and $\boldsymbol{\chi}^{\text{comp}} \equiv \boldsymbol{E}^{\text{comp}} / \sum_r E_r^{\text{comp}}$, use equations (6) and (7) to update wages, and given $\boldsymbol{L}^{\text{comp}}$ and $\boldsymbol{E}^{\text{comp}}$ use equation (13) to update the TFP shifter A. In a compensated equilibrium, these new wages and TFP shifter coincide with the initial conjectures.

We illustrate Theorem 1 using a simple example.

Example 2 (One-Good Economy). Suppose there is a single consumption good produced linearly from labor. The production technology in location r is

$$y_r = Zz_r(t) \int a_{hr} \mathbf{1}[l_h = r] dh = Zz_r(t) L_r,$$

where z_r is the productivity and L_r is the supply of labor in location r. Consumption goods produced in each location are perfectly substitutable and freely traded. The aggregate resource constraint is therefore total consumption equals total production:

$$\int c_h dh = Z \sum_r z_r(t) L_r.$$

The feasible set of equilibrium allocations is given by

$$\mathcal{C}(t, Z) = \Big\{ \{c_h, l_h\}_{h \in H} : \int T_h \, dh = 0, \quad c_h = Za_{hl_h} z_{l_h}(t) + T_h, \\ l_h \in \arg\max_{r \in R} u_h \left(Za_{hr} z_r(t) + T_h, r \right) \Big\}.$$

The first condition is that transfers must be budget-balanced, the second are households' budget constraints, and the last states that households choose their locations to maximize utility. Applying Theorem 1 is simple in this economy since the real wage (per efficiency unit) in each location in the compensated equilibrium is equal to exogenous parameters: $z_r(t)$. Hence, (13) can be rearranged as

$$A(t) = \frac{\sum_r \int z_r(t) a_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh}, \tag{14}$$

which is the ratio of production (given the technologies at $\boldsymbol{z}(t)$ but compensated locations) and the amount of consumption necessary to keep everyone indifferent at those locations. By Proposition 1, the compensated choices satisfy:

$$l_h^{\text{comp}} \in \arg\max_{r \in R} \left\{ z_r(t) a_{hr} / A(t) - \bar{c}_{hr} \right\}.$$

This is a simple numerical problem. For example, we first solve for l_h^{comp} agent-by-agent given a conjectured value of $A(t)$, then use (14) to update the value of $A(t)$.

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4 Analytical Results for Aggregate Welfare

In this section, we provide a more analytical characterization of aggregate welfare using compensated supply and demand curves (e.g. their elasticities and integrals). Section 4.1 provides a differential exact-hat algebra approach to computing $A(t)$. This has the virtue of being computationally simpler, since it does not require solving a system of nonlinear equations, and it also provides us with some analytical results, including a version of Hulten's theorem. Section 4.2 focuses on a special case where the uncompensated supply system coincides with the compensated one, simplifying the analysis.

Throughout, for any variable x, we use $x(t)$ and $x^{\text{comp}}(t)$ to refer to its value in the decentralized equilibrium (where there are no transfers and aggregate factor productivity $Z = 1$) and the compensated equilibrium (where there are compensating transfers and $Z = 1/A(t)$). When it is clear from context, we suppress the comp superscript to simplify the notation.

4.1 Sufficient Statistics for Aggregate Welfare

We begin with the following result showing that aggregate welfare is exactly the chained sum of microeconomic productivity shocks, weighted by Domar weights in the compensated equilibrium.

Theorem 2 (Welfare as Area Under Compensated Sales Shares)**.** *The change in TFP-equivalent aggregate welfare satisfies*

$$\log A(t) = \int_0^t \sum_i \lambda_i^{comp}(s) \frac{d \log z_i}{ds} ds, \tag{15}$$

where $\lambda_i^{comp}(s)$ are i's sales divided by GDP in the compensated equilibrium with productivities

$\boldsymbol{z}(s)$. *Equivalently, for any* $t \geq 0$,

$$\frac{d \log A(t)}{dt} = \sum_i \lambda_i^{comp}(t) \frac{d \log z_i}{dt}. \tag{16}$$

with boundary condition $\log A(0) = 0$.

As shown by Baqaee and Burstein (2025), the exact same formula also holds in economies with continuous choice. In this sense, there is nothing conceptually different about measuring aggregate welfare when choices are discrete.

Theorem 2 characterizes $A(t)$ as the solution to a differential equation. Solving $A(t)$ nonlinearly requires knowledge of compensated sales shares, $\lambda_i^{\text{comp}}(s)$, for every $s \in [0, t]$. We characterize $\lambda_i^{\text{comp}}(s)$ below, but first we make an important observation. In the status quo, $t = 0$, compensated Domar weights coincide with decentralized Domar weights $\lambda_i^{\text{comp}}(0) = \lambda_i(0)$.[21] This allows us to establish the following useful corollary of (16).

Corollary 1 (First-Order Approximation)**.** *To a first-order approximation, at* $t = 0$*, the change in TFP-equivalent aggregate welfare satisfies*

$$\Delta \log A \approx \sum_i \lambda_i(0) \Delta \log z_i,$$

where $\lambda_i(0)$ *is the sales of* i *divided by GDP in the decentralized equilibrium at the status quo.*

Corollary 1 is a generalization of Hulten's theorem to economies with discrete choice. Using Theorem 2 beyond first-order approximations requires knowledge of sales shares, $\lambda_i^{\text{comp}}(t)$, in the compensated equilibrium away from status quo $t > 0$. This is easy to do, using standard methods, if we know the compensated location choices, $\boldsymbol{L}^{\text{comp}}$ and compensated final demand $\boldsymbol{\chi}^{\text{comp}}$.

Given these, computing $\lambda^{\text{comp}}(s)$ follows from the results in Baqaee and Farhi (2019). For example, assume that all production and consumption functions are CES, with the ith producer in the input-output matrix Ω corresponding to a CES aggregator with elasticity of substitution θ_i. This structure is flexible enough to capture any nested-CES production and consumption functions through relabeling.[22] Then the following proposition determines sales shares in both the compensated and uncompensated equilibrium.

[21] This is because the decentralized equilibrium is Pareto efficient (the first welfare theorem holds), and hence, $A(0) = 1$ and the decentralized equilibrium supports the compensated allocation with no transfers needed.

[22] One can extend to non-nested CES along similar lines to Baqaee and Farhi (2019).

Proposition 2 (Prices and Expenditures in Equilibrium)**.** *In the decentralized equilibrium, given labor supplies $\boldsymbol{L}(t)$ and final demand $\boldsymbol{\chi}(t)$, the following differential equations determine the rest of the equilibrium variables:*

$$d\log\Omega_{ij} = (1-\theta_i)\left[d\log p_j \mathbf{1}[j\in N] + d\log w_j \mathbf{1}[j\in R] - \sum_{k\in N}\Omega_{ik} d\log p_k - \sum_{r\in R}\Omega_{ir} d\log w_r\right], \tag{17}$$

$$d\log p_i = -d\log z_i + \sum_{j\in N}\Omega_{ij} d\log p_j + \sum_{r\in R}\Omega_{ir} d\log w_r, \tag{18}$$

$$d\log\lambda_r = d\log w_r + d\log L_r - \sum_{r'}\lambda_{r'}\left[d\log w_{r'} + d\log L_{r'}\right], \quad (r\in R) \tag{19}$$

$$d\log\lambda_i = \lambda_i^{-1}\left(\sum_{r\in R} d\chi_r \mathbf{1}[i=c(r)] + \sum_{j\in C+N}\left[d\lambda_j\Omega_{ji} + \lambda_j d\Omega_{ji}\right]\right), \tag{20}$$

where all the variables are indexed by t. If we swap $\boldsymbol{L}$ and $\boldsymbol{\chi}$ for $\boldsymbol{L}^{comp}$ and $\boldsymbol{\chi}^{comp}$, we get the evolution of variables in the compensated equilibrium instead. The boundary conditions are the same for both the compensated and decentralized equilibria — spending shares are equal to their values in status quo, and the initial level of prices and wages is irrelevant.

Proposition 2 can be used to calculate either the decentralized equilibrium, given $d\log\chi_r$ and $d\log L_r$, or the compensated equilibrium, given $d\log\chi_r^{\mathrm{comp}}$ and $d\log L_r^{\mathrm{comp}}$. The changes $d\log\chi_r^{\mathrm{comp}}$ and $d\log L_r^{\mathrm{comp}}$ can be computed numerically using Proposition 1. This approach is useful both computationally and analytically. Computationally, it obviates the need to solve a system of nonlinear equations to compute $A(t)$ (i.e. as in Theorem 1). Instead, it requires repeatedly solving a system of linear equations and updating variables (Appendix F provides additional information). Analytically, it allows us to prove some useful results.

Before using Proposition 2, we briefly describe the intuition for equations (17)-(20). The expression in (17) is log-linearized input demand by i for input j, which depends on the elasticity of substitution, θ_i, and on how the price of j changes relative to the marginal cost of i. The expression in (18) is Shephard's lemma, relating the marginal cost of i to the expenditure-share weighted change in input prices and the productivity shock to i. Equation (19) log-linearizes the definition of λ_r, and (20) gives the log-linearized market-clearing conditions (6).

A simple corollary of Theorem 2 and Proposition 2 is the following exact characterization of $A(t)$.

Corollary 2 (Cobb-Douglas Economies with Common Consumption)**.** *Suppose that all production functions are Cobb-Douglas and that the consumption bundle is a common Cobb-Douglas aggregator in every location, so that* $\theta_i = 1$ *for every* $i \in C + N$*. Then, for every* t*,*

$$\log A(t) = \sum_i \lambda_i(0)[\log z_i(t) - \log z_i(0)],$$

regardless of the shape of the $g(c)$ *function and distribution of taste and productivity shifters.*

Under these assumptions, the right-hand sides of (17) and (20) are zero.[23] Thus, changes in location choices and spending shares do not affect the sales shares of goods and factors with productivity shifters, which are constant over t. The sales shares of the consumption aggregators may vary with χ_r, but do not enter Theorem 2. Since compensated variables are equal to decentralized variables in the status quo, $\lambda_i^{\text{comp}}(0) = \lambda_i(0)$, and the integral in Theorem 2 can be solved easily.

4.2 Equivalence of Compensated and Uncompensated Supply

Theorem 2 shows that computing $A(t)$ nonlinearly requires knowledge of compensated Domar weights $\boldsymbol{\lambda}^{\text{comp}}$. In this section, we consider a tractable special case where compensated Domar weights are simple to solve for, because the compensated and uncompensated supply systems coincide.

Assumption 1 (Common Consumption Good and Affine g)**.** The consumption aggregator in every location is the same as every other location, so that the consumption price p_r^c is the same in every location r, denoted by p^c. Preferences can be represented by $u_h(c_h, l) = f_h(c_h + \epsilon_{hl})$, where f_h is an arbitrary increasing function (i.e. $g(c)$ is affine).

Under this assumption, lump-sum transfers do not change agents' location choices. To see this, note that if preferences are $f_h(c_h + \epsilon_{hl})$ then a constant increase in consumption in every location does not alter household h's location choice. Moreover, if the price of consumption is the same in every location, then a lump-sum transfer gives the same amount of consumption in every location. That is, under Assumption 1, agent h chooses the location with the highest $(Za_{hr}w_r + T_h)/p^c + \epsilon_{hr}$ — and this choice is independent of the value of T_h. This implies the following.

[23] When $\theta_i = 1$ for every i, (17) implies $d\Omega = 0$. Because the common consumption good has the same input shares in every location and $\sum_r d\chi_r = 0$, it follows from (20) that $d\lambda_i = 0$ for every $i \in N + R$.

Proposition 3 (Characterization Using Differential Equations)**.** *If Assumption 1 holds, then the compensated supply function coincides with the uncompensated supply function:*

$$L^{comp}(\boldsymbol{w}/A, p^c, \boldsymbol{u}^0) = L(\boldsymbol{w}/A, p^c) = L(\boldsymbol{w}/(Ap^c), 1)$$

Therefore, along the compensated-equilibrium path,

$$dL_r^{comp} = \sum_{r'} \frac{\partial L_r}{\partial \left[w_{r'}/(Ap^c) \right]} d(\frac{w_{r'}}{Ap^c}), \tag{21}$$

where prices and wages are evaluated in the compensated equilibrium at t. Moreover, because there is a common consumption good, equation (20) *does not depend on compensated demand χ^{comp}. Hence, equations* (16) *to* (21)*, together with the status quo values of the endogenous variables at $t = 0$, fully pin down $A(t)$.*

Under Assumption 1, the compensated and uncompensated supply functions coincide. Furthermore, the distribution of spending χ^{comp} is irrelevant for determining compensated sales shares λ^{comp} because the input shares of the common consumption aggregator are the same in every location r. In particular, the distribution of spending χ^{comp} only shows up in (20) and has no effect (since $\Omega_{c(r)i} = \Omega_{ci}$ for every $r \in R$). Therefore, Proposition 3 allows us to compute $A(t)$ as the solution to a system of ordinary differential equations.

The following example solves these differential equations in the one-good economy and relates $A(t)$ to the area under compensated supply curves.

Example 3 (One-Good Economy under Assumption 1). Consider Example 2 again: there is one common and freely-traded consumption good made linearly from labor in each location. In this economy, since Domar weights always sum to one, $\sum_r \lambda_r \equiv 1$, we can rewrite (16) as

$$\sum_r \lambda_r^{\text{comp}}(t)\, d\log\left(z_r(t)/A(t)\right)/dt = 0.$$

By definition of λ_r^{comp}, and since $w_r(t) = z_r(t)$, this is

$$\sum_r L_r^{\text{comp}}(t) \frac{d\left[z_r(t)/A(t)\right]}{dt} = 0.$$

Since this equation holds for every $t > 0$, we can integrate both sides to get

$$\int_0^t \sum_r L_r^{\text{comp}}(s) \frac{d\left[z_r(s)/A(s)\right]}{ds} ds = 0. \tag{22}$$

In words, (22) shows that $A(t)$ is the reduction in TFP such that the area under the compensated labor supply curve is zero. Suppose that Assumption 1 holds, so that we can use the uncompensated supply function in place of the compensated supply function, and hence

$$\int_0^t \sum_r L_r\left(\frac{\boldsymbol{z}(s)}{A(s)}, 1\right) \frac{d\left[z_r(s)/A(s)\right]}{ds} ds = 0.$$

If we know the functional form for the uncompensated supply function, then the equation above is a single nonlinear equation that pins down $A(t)$. For example, suppose that workers have homogeneous skills ($a_{hr} = 1$ for every $h \in H$ and $r \in R$), and that the uncompensated supply system is logit, as in Example 1. In this case, the uncompensated supply is

$$L_r\left(\frac{\boldsymbol{z}(s)}{A(s)}, 1\right) = \frac{\exp(\theta \frac{z_r(s)}{A(s)} + B_r)}{\sum_{r'} \exp(\theta \frac{z_{r'}(s)}{A(s)} + B_{r'})}.$$

With this functional form, we can evaluate the integral in closed-form as:

$$\log \sum_r \exp\left(\theta \frac{z_r(t)}{A(t)} + B_r\right) = \log \sum_r \exp\left(\theta z_r(0) + B_r\right). \tag{23}$$

This nonlinear equation pins down $A(t)$.

The general equilibrium structure in this example is simple since real wages must equal labor productivity in each location (i.e. labor demand is infinitely elastic at a fixed real wage). Example 9 in the appendix provides a slightly richer multi-industry example where relative wages are endogenous.

Relation to social surplus function. In Appendix E we show that under Assumption 1, $A(t)$ can also be computed using the utilitarian social welfare function $U\left(\boldsymbol{w}/\boldsymbol{p}^c\right) = \mathbb{E}[\max_{l_h} g(a_{hl_h} w_{l_h}/p^c_{l_h}) + \epsilon_{h,l_h}]$ where $a_{hr} w_r / p^c_r$ is real consumption of household h when choosing location r. The function U is called the "*Social Surplus Function*" in the discrete choice literature (see McFadden, 1981). The reason is that, by the Williams-Daly-Zachary theorem, the social surplus function is the integral of uncompensated labor supply. When

Assumption 1 holds, compensated and uncompensated labor supply coincide, so the social surplus function is also the integral of compensated supply. Hence, by a similar logic to (22), $U(\boldsymbol{w}/\boldsymbol{p}^c)$ can be used to determine $A(t)$. However, if Assumption 1 does not hold, then the social surplus function can no longer be used to compute $A(t)$ because compensated and uncompensated labor supply are no longer the same.

5 Second-Order Approximation of Aggregate Welfare

In this section, we investigate nonlinearities in cases where transfers do affect location choices (i.e. Assumption 1 does not hold). We do this by relating derivatives of compensated labor supply and demand to derivatives of their uncompensated counterparts. This allows us to characterize changes in aggregate welfare to a second-order approximation in terms of uncompensated elasticities and expenditure shares. Throughout this section, we explicitly use the fact that preferences are ordinally additively separable. We also assume that idiosyncratic tastes $\boldsymbol{\epsilon}$ have an absolutely continuous density $f(\boldsymbol{\epsilon})$.

The previous section shows that to a first-order approximation, $A(t)$ is a Domar-weighted sum of microeconomic productivity shocks. Differentiating (16) once more and applying a second-order Taylor expansion at $t = 0$ yields the following approximation for $A(t)$.

Corollary 3 (Second-Order Approximation)**.** *To a second-order approximation, the change in TFP-equivalent aggregate welfare satisfies*

$$\Delta \log A \approx \sum_i \lambda_i(0) \Delta \log z_i + \frac{1}{2} \sum_i \Delta \lambda_i^{comp}(0) \Delta \log z_i,$$

where $\Delta\lambda_i^{comp}$ is the (first-order) change in the compensated sales of i divided by GDP at the status quo.

The intuition is very similar to the formulas derived by Baqaee and Farhi (2019) for an economy without discrete choice. If sales shares in the compensated equilibrium rise in response to positive productivity shocks, then aggregate welfare rises more quickly than its first-order approximation would suggest. If sales shares fall, then the reverse happens.

Calculating $\Delta\lambda_i^{\text{comp}}(0)$ in Corollary 3 is a straightforward application of Proposition 2 if we know the derivatives of compensated labor supply $dL^{\text{comp}}(0)$ and demand $d\chi^{\text{comp}}(0)$. When Assumption 1 holds, this is simple to do using Proposition 3, since dL^{comp} can be computed using the uncompensated supply system and $d\chi^{\text{comp}}$ is irrelevant. In this section, we focus on characterizing dL^{comp} and $d\chi^{\text{comp}}$ at the status quo, without imposing Assumption 1, and we compare these to their decentralized uncompensated counterparts.

To simplify the analysis, we focus on the case with only preference heterogeneity and abstract from skill-heterogeneity. This implies that agents make different choices only due to differences in tastes.[24]

Assumption 2 (Homogeneous Idiosyncratic Skills)**.** Agents have homogeneous idiosyncratic skill, which we normalize to one: for every agent $h \in H$ and location $r \in R$, we have $a_{hr} = 1$.

We now proceed to characterizing $d\chi^{\text{comp}}$ (in Proposition 4) and dL^{comp} (in Proposition 5) at the status quo. Given these derivatives, we can compute $d\lambda^{\text{comp}}$ using Proposition 2, and hence, we can approximate $\log A(t)$ to a second-order using Corollary 3.

Proposition 4 (First-Order Change in Spending Shares)**.** *For any variable that varies by location, x_r, define $\mathbb{E}_\chi[x] \equiv \sum_r \chi_r(0) x_r$. In the decentralized equilibrium, at $t = 0$, the spending share in location r satisfies*

$$d\log \chi_r = d\log w_r - \mathbb{E}_\chi \left[d \log w\right] + d\log L_r - \mathbb{E}_\chi \left[d\log L\right],$$

where the changes in wages $\boldsymbol{w}$ and locations $\boldsymbol{L}$ are in the decentralized equilibrium. In the compensated equilibrium, at $t = 0$, we instead have

$$d\log \chi_r^{comp} = d\log p_r^c - \mathbb{E}_\chi \left[d\log p^c\right] + d\log L_r^{comp} - \mathbb{E}_\chi \left[d\log L^{comp}\right],$$

where the changes in prices $\boldsymbol{p}^c$ and locations $\boldsymbol{L}^{comp}$ are in the compensated equilibrium.

The intuition is the following. In the decentralized equilibrium, the share of spending by each location must move in line with the share of income earned by households in that location. This is the product of wages w_r and population shares L_r. A location's share of spending rises if wages in that location increase quickly or if households flock to that location. On the other hand, in the compensated equilibrium, the share of spending in each location depends on the rate at which the cost of consumption in location r has increased and the population shares in the compensated equilibrium. If consumption prices rise quickly in a location, or households flock to a location, in the compensated equilibrium, then that location's share of total spending rises.

[24] Assumption 2 can be slightly relaxed to allow for the possibility that some agents h have skill $a_{hr} = 0$ for some locations r. This is because such an extension is isomorphic to giving those agents ϵ_{hr} values that are so low that those agents would never choose r in equilibrium. Since we do not impose strong assumptions on the distribution of ϵ's, we model such scenarios by picking an appropriate distribution of ϵ.

The next proposition provides a general characterization of dL^{comp} in terms of the derivatives of the uncompensated labor supply function at the status quo $\partial L_r / \partial (w_{r'}/p^c_{r'})$. This is important because the derivatives of the uncompensated supply system are estimable using exogenous variation in the decentralized equilibrium.[25]

Proposition 5 (First-Order Changes in Location Choices)**.** *The change in the share of households in location r can be decomposed into inflows minus outflows:*

$$dL_r = \sum_{r' \neq r} dL_{r' \to r} - \sum_{r' \neq r} dL_{r \to r'}, \tag{24}$$

where $dL_{r' \to r}$ is the share of households switching from r' to r. To a first order, the flows satisfy:

$$dL_{r \to r'} = \left[\frac{\partial L_{r'}}{\partial (w_r / p^c_r)} d(\frac{w_r}{p^c_r}) - \frac{\partial L_r}{\partial (w_{r'} / p^c_{r'})} d(\frac{w_{r'}}{p^c_{r'}}) \right] \mathbf{1} \left[\frac{\partial L_r}{\partial (w_{r'} / p^c_{r'})} d(\frac{w_{r'}}{p^c_{r'}}) \leq \frac{\partial L_{r'}}{\partial (w_r / p^c_r)} d(\frac{w_r}{p^c_r}) \right], \tag{25}$$

where wages and prices are those of the decentralized equilibrium.

The compensated supply function instead satisfies:

$$dL^{comp}_r = \sum_{r' \neq r} dL^{comp}_{r' \to r} - \sum_{r' \neq r} dL^{comp}_{r \to r'},$$

where $dL^{comp}_{r' \to r}$ is the share of households switching from r' to r in the compensated equilibrium. At $t = 0$, these flows satisfy:

$$dL^{comp}_{r \to r'} = \frac{\partial L_r}{\partial \left[w_{r'} / p^c_{r'} \right]} \frac{1}{p^c_{r'}} \left(p^c_r d(\frac{w_r}{A p^c_r}) - p^c_{r'} d(\frac{w_{r'}}{A p^c_{r'}}) \right) \times \mathbf{1} \left[p^c_r d(\frac{w_r}{A p^c_r}) \leq p^c_{r'} d(\frac{w_{r'}}{A p^c_{r'}}) \right], \tag{26}$$

where wages and prices are those of the compensated equilibrium and, at $t = 0$, $d \log A$ is given by Corollary 1.

In the decentralized equilibrium, Equation (25) expresses changes in location choices as pairwise movements across indifference boundaries. Changes in real wages determine the direction in which the boundary between r and r' moves, while the cross-derivatives of labor supply between r and r' determine the mass of households that crosses it. The overall change in the share of households choosing r, given by (24), is the sum of inflows into r minus outflows from r.

[25] Formally, in the decentralized equilibrium, where transfers are zero, the supply function $L_r(\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})$ is a function only of the vector of real wages per efficiency unit in each location w_r / p^c_r. Hence, $L_r(\boldsymbol{w}, \boldsymbol{p}^c, \mathbf{0}) = L_r(\boldsymbol{w}/\boldsymbol{p}^c, 1, \mathbf{0})$. This allows us to define $\partial L_r / \partial (w_{r'}/p^c_{r'})$, which can be estimated using the observed changes in market-level labor supply in response to exogenous variation in real wages.

Equation (26) is the counterpart to (25) in the compensated equilibrium. Household-specific transfers keep every household indifferent to the status quo, while aggregate factor productivity is adjusted so that these transfers satisfy budget balance. For a household initially indifferent between r and r', the term $p_r^c d\left(\frac{w_r}{Ap_r^c}\right)$ measures the first-order reduction in the transfer required for indifference if the household chooses r. The household chooses the location for which this reduction is larger. The size of the resulting flow is determined by the cross-derivative of labor supply between the two locations. Crucially, the partial derivatives in (26) are those of the uncompensated supply system evaluated at the status quo in the decentralized economy.

We illustrate Corollary 3 and Proposition 5 with a simple example.

Example 4 (Second-Order Approximation of Aggregate Welfare with Fréchet tastes). Consider again the one-good economy described in Example 2. Applying Corollary 3 to the one-good economy we get that, to a second-order,

$$\Delta \log A \approx \mathbb{E}_\lambda[\Delta \log z] + \frac{1}{2}\left[Var_\lambda\left(\Delta \log z\right) + Cov_\lambda(\Delta \log L^{\text{comp}}, \Delta \log z)\right], \tag{27}$$

where the variance and covariance operators use λ as the probability weights and we use the fact that $d\lambda_r^{\text{comp}} = \lambda_r(d\log(z_r L_r^{\text{comp}}) - \mathbb{E}_\lambda[d\log(zL^{\text{comp}})])$. In words, the nonlinear change in aggregate welfare depends on the variance of the shocks and the covariance of compensated migrations with the shocks. The variance term reflects the fact that, even if households are not mobile, an increase in productivity in one location raises that location's sales share, which magnifies the value of positive productivity growth in that location relative to the loglinear approximation. The covariance reflects the fact that if, in the compensated equilibrium, households move to locations experiencing more rapid productivity growth, then this further boosts the compensated sales shares of producers in that location, and raises aggregate welfare.

To solve out for $\Delta \log \boldsymbol{L}^{\text{comp}}$, suppose that $u_h(c_h, r) = f_h(\log(c_h) + \epsilon_{hr})$ and that the ϵ_{hr} are i.i.d. type-I extreme-value random variables with inverse scale parameter θ (so that $\exp(\epsilon_{hr})$ are Fréchet distributed with shape parameter θ). The share of households that choose location r in the decentralized equilibrium takes the familiar isoelastic form

$$L_r(\boldsymbol{z}) = \frac{(B_r z_r)^\theta}{\sum_{r'}(B_{r'} z_{r'})^\theta}, \tag{28}$$

where the parameters $\{B_r\}$ control the desirability of living in each location. The uncompensated cross-elasticities of labor supply are

$$\frac{\partial \log L_{r'}}{\partial \log w_r} = \theta \left(\mathbf{1}[r' = r] - L_r \right),$$

where we use the fact that $w_r = z_r$ in the one-good economy. For simplicity, suppose that initial productivities, $z_r(0)$, are the same in every location. Then applying Proposition 5 yields

$$\Delta \log L_r^{\text{comp}} = \theta \left(\Delta \log z_r - \mathbb{E}_\lambda \left[\Delta \log z \right] \right).$$

In this case, changes in population shares in the compensated equilibrium, $d \log L^{\text{comp}}$, are the same as the ones in the decentralized equilibrium, $d \log L$, up to a first-order approximation. Substituting this into the approximation for $\Delta \log A$ in (27) yields:

$$\Delta \log A \approx \mathbb{E}_\lambda[\Delta \log z] + \frac{1}{2}(1+\theta) Var_\lambda \left(\Delta \log z \right).$$

Hence, the higher is θ, the more convex is aggregate welfare.[26]

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6 Comparison to Other Aggregate Measures

In this section, we compare our measure of aggregate welfare with three alternative aggregate measures that are popular in the literature. In Section 6.1, we relate our measure to real output (GDP) and multi-factor productivity growth as measured in the national accounts. In Section 6.2, we contrast our measure with the popular utilitarian or average utility measure. Finally, in Section 6.3, we compare our results to the sum of compensating variations (i.e. Kaldor-Hicks efficiency) previously analyzed by Small and Rosen (1981) in a discrete-choice context. Throughout the section, we quantify the differences between the different measures using the one-good economy calibrated to U.S. data.

[26] If instead the share function were logit (i.e. $g(c_h)$ is linear and ϵ_{hr} are type I extreme value with parameter θ^{logit}), then $\Delta \log A \approx \mathbb{E}_\lambda[\Delta \log z] + \frac{1}{2}(1+\theta^{\text{logit}} z(0)) Var_\lambda \left(\Delta \log z \right)$, where we abuse notation and treat $z(0)$ as a scalar because productivity in every location is the same in the status quo. Calibrating the derivative of L_r with respect to real wages in r' to be the same as in the Fréchet model implies $\theta^{\text{logit}} = \theta / z(0)$. Hence, the implications for $\Delta \log A$ in the two models are the same up to a second-order approximation as long as we match the same cross-derivatives of supply.

6.1 Real Output

We contrast changes in aggregate welfare in TFP equivalent terms, defined via $A(t)$, with changes in a chain-weighted (Divisia) index of real output, as measured in the national accounts.

Definition 5 (Real Output)**.** The change in real output is the share-weighted sum of changes in final consumption quantities:

$$\log Y(t) = \int_0^t \sum_{r \in R} \frac{p_r^c(s) C_r(s)}{\sum_{r' \in R} p_{r'}^c(s) C_{r'}(s)} \frac{d \log C_r}{ds} ds,$$

where $C_r(t) = \int c_h(t) \mathbf{1}[l_h(t) = r] dh$ is total consumption in location r at t.

Using Hulten (1978), we can write real output as a Domar-weighted sum of technology and labor supply changes:

$$\log Y(t) = \int_0^t \left[\sum_i \lambda_i(s) \frac{d \log z_i}{ds} + \sum_{r \in R} \lambda_r(s) \frac{d \log L_r}{ds} \right] ds.$$

Differentiating this with respect to t and evaluating at $t = 0$ yields the first order approximation:

$$\Delta \log Y \approx \sum_i \lambda_i(0) \Delta \log z_i + \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r.$$

Real output responds positively to productivity growth, and — holding productivities fixed — benefits from relocations of labor towards higher-wage locations. These labor relocations, ΔL_r, are endogenous and require solving the general equilibrium model.

Unlike $A(t)$, real output does not respect the Pareto principle. To see this, consider a positive productivity shock to a low-wage location. In this case, output can fall if the shock induces large flows out of high-wage locations into the still-low-wage location.[27] Intuitively, because agents care about amenities, they may move to a lower-wage region and this reduces production even though every agent is made (weakly) better off.[28]

[27] In the one-good economy with isoelastic labor supply, as in Example 4, the elasticity of real output with respect to a productivity shock is

$$\frac{\Delta \log Y}{\Delta \log z_r} \approx \frac{w_r(0) L_r(0)}{\sum_k w_k(0) L_k(0)} + \theta \left[\frac{w_r(0)}{\sum_k w_k(0) L_k(0)} - 1 \right] L_r(0).$$

This elasticity can be negative if θ is high and w_r relatively low.

[28] There is a strong analogy to economies with labor-leisure choice. In such economies, real output can fall even as welfare rises because of the endogenous response of leisure. This happens because GDP does not include the value of leisure. In contrast, because of the envelope theorem, in an economy with continuous labor-leisure choice, $\Delta \log A$ still obeys Hulten's theorem (as does $\Delta \log A^{MFP}$).

In contrast, aggregate welfare, $A(t)$, does not respond to relocations (see Corollary 1). If households choose to move from high- to low-wage locations, this has no first-order effect on aggregate welfare. The change in wages experienced by movers is exactly offset by the change in the amenity value of the move. Hence, whereas real output changes due to the changes in location choices, aggregate welfare does not. We summarize this in the proposition below.

Proposition 6 (First-Order Difference between Real Output and Welfare). *To a first-order approximation, the difference between real output and aggregate welfare in terms of TFP equivalents is given by the change in income caused by relocation:*

$$\Delta \log Y - \Delta \log A \approx \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r.$$

Hence, real output and aggregate welfare can differ, even in sign, to a first-order approximation. The following example quantifies these differences.

Example 5 (Quantitative Difference Between Output and Aggregate Welfare). We compute the quantitative difference between the response of aggregate welfare and real output to productivity shocks using a calibrated one-good economy with isoelastic labor supply, as in Equation (28), with elasticity parameter $\theta = 1.5$. We consider an economy with 588 locations calibrated to US commuting zones.[29] We choose the parameters $\{z_r\}$ and $\{B_r\}$ to match both the observed GDP share and labor-force share of each commuting zone in 2024.

Table 1 displays the change in aggregate welfare and output in response to a 1% rise in productivity in selected commuting zones.[30] The gap between output and aggregate welfare can be substantial. In response to a productivity shock in a high-productivity location such as New York, the increase in output is almost twice as large as the increase

[29]We define locations using the USDA Economic Research Service commuting-zone classification based on county boundaries (U.S. Department of Agriculture, Economic Research Service, 2026). The ERS crosswalk assigns each county or county-equivalent to exactly one commuting zone; commuting zones may contain multiple counties. After excluding Puerto Rico, this leaves 588 commuting zones in our sample. County GDP is from BEA's GDP by County series (U.S. Bureau of Economic Analysis, 2026), and county labor force is from the BLS Local Area Unemployment Statistics program (U.S. Bureau of Labor Statistics, 2026). For each commuting zone r, we sum GDP and labor force across all counties assigned to that commuting zone.

[30]The change in aggregate welfare is very well approximated by the shocked region's sales share, also displayed in the table, which shows that the first-order Hulten approximation in Corollary 1 is very accurate for a 1% shock. Table 5 in Appendix H reports the corresponding elasticities for a 10% shock, where nonlinearities become more visible.

Table 1: Aggregate effects of a 1 percent productivity increase in selected locations

Location of shock	$\Delta \log A / \Delta \log z_r$	Sales share λ_r	$\Delta \log Y / \Delta \log z_r$
San Francisco–Oakland, CA	0.031	0.031	0.050
New York, NY	0.057	0.057	0.091
Seattle–Tacoma, WA	0.024	0.024	0.034
Brownsville, TX	0.001	0.001	-0.0002
McAllen, TX	0.002	0.002	0.000
Daytona Beach, FL	0.001	0.001	0.000

Notes: Entries are elasticities with respect to the regional productivity increase. Sales share is λ_r in the initial equilibrium. Results use $\theta = 1.5$ and 400,000 simulated households.

in aggregate welfare, consistent with Proposition 6. In response to a shock in a low-productivity location such as Brownsville, TX, output falls even though every agent is weakly better off in the new equilibrium.

Multi-factor productivity growth is defined as real output growth minus the growth in quality-adjusted labor:

$$\Delta \log A^{MFP} \approx \Delta \log Y - \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r,$$

Hence, Proposition 6 implies that $\Delta \log A^{MFP} \approx \Delta \log A$ to a first-order.[31] The intuition here is that if we separate changes in output due to technology shocks from changes in output due to relocation, then multi-factor productivity, as measured by a national income accountant, coincides to a first-order with our measure of aggregate welfare.

We note an important and tractable special case. Suppose there is no amenity value associated with different locations, and that there is a single consumption good. Agents can have idiosyncratic skills by location, so that a_{hr} can vary as a function of both h and r. In this case, real output, $Y(t)$, and our measure of aggregate welfare, $A(t)$, coincide. These assumptions hold in many models of occupational choice, but are less likely to hold in spatial models

Proposition 7 (Coincidence of $Y(t)$ and $A(t)$)**.** *Suppose that ϵ_{hr} does not vary by h and r (i.e. $\epsilon_{hr} = \epsilon$), and that there is a common consumption good. Then*

$$A(t) = A^{MFP}(t) = Y(t).$$

[31]The nonlinear definition of $A^{MFP}(t)$ is $\log A^{MFP}(t) = \log Y(t) - \int_0^t \sum_r \lambda_r(s)(d \log L_r/ds)ds$.

This follows from the fact that under these assumptions, $\boldsymbol{L} = \boldsymbol{L}^{\text{comp}}$, and changes in location spending shares, $\boldsymbol{\chi}^{\text{comp}}$ have no effect on relative prices since all households spend income on the same consumption good (e.g. $d\chi$ drops out of equation (20)). Hence, $\lambda_i^{\text{comp}}(s) = \lambda_i(s)$. This implies that $A(t) = A^{MFP}(t)$. Furthermore, since every household chooses location to maximize nominal income, marginal relocations do not contribute to the change in maximized nominal income (by the envelope theorem), which implies that $d\log Y = d\log A^{MFP}$. Since this holds for any $s > 0$, integration gives $Y(t) = A^{MFP}(t)$.

6.2 Average Utility/Utilitarian Social Welfare

In economies where households have heterogeneous preferences, by far the most common way to evaluate aggregate welfare is a utilitarian social welfare function (sometimes called "ex-ante expected utility").

Definition 6. Given real wages across locations, define the utilitarian social welfare function as

$$W(\boldsymbol{w}/\boldsymbol{p}^c) = \mathbb{E}\left[\max_{l_h \in R} u_h\left(a_{hl_h} w_{l_h} / p^c_{l_h}, l_h\right)\right], \tag{29}$$

where $\mathbb{E}$ denotes the population-weighted cross-sectional average and $a_{hl_h} w_{l_h} / p^c_{l_h}$ is household h's consumption if location l_h is chosen. Although (29) is sometimes called "expected utility," the expectation is across households, not over lotteries faced by any household, so the label is misleading.[32]

The consumption-equivalent variation for the utilitarian social welfare function W is a scalar $A^W(t)$ that solves:

$$W(\boldsymbol{w}(t)/(\boldsymbol{p}^c(t)A^W(t))) = W(\boldsymbol{w}(0)/\boldsymbol{p}^c(0)). \tag{30}$$

Unlike the TFP-equivalent aggregate welfare measure $A(t)$, the utilitarian welfare measure $A^W(t)$ is not invariant to increasing transformations of individual utility functions. In particular, its value is not pinned down by any observables. To see this, recall that $u_h(c_h, l) = f_h(g(c_h) + \epsilon_{hl})$, where f_h is any arbitrary strictly increasing function. The function f_h has no testable implications for households' choices: any choice of f_h represents the same underlying preference relation $\succeq_h$. Nevertheless, changing f_h has important implications for $A^W(t)$ as the following proposition demonstrates.

[32] A von Neumann–Morgenstern expected utility function represents an ordinal preference relation over lotteries of allocations; it is not an average of utility levels across households with different preferences. See Mas-Colell et al. (1995, Chapter 6) and Footnote 13. Here, each household's tastes are fixed, and its preferences are defined over consumption-location bundles rather than lotteries over its own tastes.

Proposition 8 (Arbitrariness of average utility). *Hold fixed households' ordinal preferences $\{\succeq_h\}$ and hence their individual compensating variations $A_h(t)$ defined in* (12). *Then $A^W(t)$ can be made equal to any value in the interval*

$$\left(\min_{h\in H} A_h(t),\, \max_{h\in H} A_h(t)\right),$$

depending on the cardinal utility representations $\{f_h\}$, but cannot lie outside this interval.

This proposition shows that $A^W(t)$ can take any value inside the range spanned by individual compensating variations, depending on the choice of $\{f_h\}$.[33] Thus, if the shock creates both winners and losers, then whether average utility rises, falls, or stays the same depends on untestable assumptions about how ordinal preferences are mapped into cardinal utilities.

One might imagine that if we restrict ourselves to a priori "reasonable" transformations that are symmetric across households, then $A^W(t)$ may be pinned down, but this is not true. The following example uses the popular isoelastic location choice specification to show that alternative seemingly reasonable and symmetric choices of f_h cause $A^W(t)$ to move in unpredictable ways. Of course, all choices of f_h are observationally equivalent models and consistent with the same underlying preference relations.

Example 6 (Arbitrariness of average utility under Fréchet tastes). Consider an economy like in Example 4 with a single good and homogeneous skills. Agent h has utility function

$$u_h(c_r, r) = f_h\big(\log(c_r) + \epsilon_{hr}\big),$$

where $\varepsilon_{hr} = \exp(\epsilon_{hr})$ is distributed according to a Fréchet distribution and f_h is an arbitrarily chosen increasing function. Typically, the literature assumes that $f_h(x) = \exp(x)$ so that

$$u_h(c_r, r) = c_r \varepsilon_{hr}.$$

Under this popular cardinalization, $A^W(t)$ has a familiar closed-form solution:

$$A^W(t) = \left[\frac{\sum_r (B_r z_r(t))^\theta}{\sum_r (B_r z_r(0))^\theta}\right]^{1/\theta}. \tag{31}$$

[33] Under mild conditions—satisfied, for example, in the standard case with Fréchet tastes—the lower and upper bounds in Proposition 8 equal the smallest and largest proportional changes in regional real wages between the equilibria at 0 and t.

This is a widely used measure of aggregate welfare in spatial models.[34,35]

Now consider a different cardinalization of utility:

$$u_h(c_r, r) = c_r \frac{\varepsilon_{hr}}{\max_{r'} \varepsilon_{hr'}}.$$

This has no implication for choices, but normalizes each agent's vector of location-specific tastes so that its highest value is one and all other values are below one.[36] That is, each agent assigns a taste of unity to their favorite location and a value below one to every other location. One can argue that this is a more "reasonable" cardinalization of utility because it does not allow for the possibility that some agents have high tastes in every location while others have low tastes in every location. Under this cardinalization, however, $A^W(t)$ does not obey (31).

To see this difference, we use the same calibrated economy as in Example 5. Table 2 quantifies the difference between A^W and A for different cardinalizations f_h. We report A^W under four observationally equivalent choices of the cardinal utility representations $\{f_h\}$. The first is the commonly used $u_h(c_r, r) = c_r \varepsilon_{hr}$, labelled as A_1^W. The second rescales utility by the maximum taste parameter $u_h(c_r, r) = c_r \varepsilon_{hr} / (\max_r \varepsilon_{hr})$, and is labelled A_2^W. The third and fourth use $u_h(c_r, r) = (c_r \varepsilon_{hr} / (\max_r \varepsilon_{hr}))^2$ and $u_h(c_r, r) = \sqrt{(c_r \varepsilon_{hr} / (\max_r \varepsilon_{hr}))}$, and are labelled A_3^W and A_4^W respectively. All these specifications are observationally equivalent but imply very different welfare gains according to the average utility metric. More generally, Proposition 8 shows that the arbitrary choice of cardinalization can make $A^W(t)$ take any value between the smallest and largest individual compensating variations.

The following example considers optimal place-based policies chosen by a policymaker who maximizes utilitarian welfare $W(\boldsymbol{c})$. We show that the policymaker intervenes in a Pareto-efficient equilibrium and chooses Pareto-inefficient policies. Furthermore, the direction and magnitude of these policies depend on the chosen cardinalization of individual utilities.

[34]An odd feature of this cardinalization is that average utility cannot be calculated when the mobility elasticity θ is less than or equal to one. In this case, $W(\boldsymbol{c})$ diverges because the Fréchet taste parameters ε_{hr} are sufficiently fat-tailed that their expectation is infinite. In contrast, our measure of welfare is well-defined for any value of θ.

[35]If one restricts the set of admissible monotone transformations, then the common Fréchet specification has an invariance property. However, these restrictions are arbitrary from the standpoint of observed choices; see Appendix D.

[36]Formally, this cardinalization sets $f_h(x) = \exp(x) / (\max_{r'} \varepsilon_{hr'})$.

Table 2: Welfare effects of a 1 percent productivity increase in selected commuting zones

Location of shock	$\Delta \log A$	$\Delta \log A_1^W$	$\Delta \log A_2^W$	$\Delta \log A_3^W$	$\Delta \log A_4^W$
San Francisco–Oakland, CA	0.031	0.018	0.026	0.039	0.022
New York, NY	0.057	0.034	0.049	0.071	0.041
Seattle–Tacoma, WA	0.024	0.017	0.022	0.028	0.019
Brownsville, TX	0.001	0.001	0.001	0.000	0.001
McAllen, TX	0.002	0.004	0.003	0.002	0.003
Daytona Beach, FL	0.001	0.002	0.001	0.001	0.002

Notes: Entries are elasticities with respect to the regional productivity increase. The columns $\Delta \log A_i^W$ report consumption-equivalent utilitarian welfare under observationally equivalent cardinalizations of utility. A_1^W uses $u_h(c_r, r) = c_r \varepsilon_{hr}$. The remaining specifications normalize tastes by $\max_{r'}\{\varepsilon_{hr'}\}$. A_2^W uses $u_h(c_r, r) = c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$, while A_3^W and A_4^W square and take the square root of this expression, respectively. All four specifications imply identical household choices and equilibrium observables. Results use $\theta = 1.5$ and 400,000 simulated households.

Example 7 (Optimal policy for average utility). Consider again the one-good economy in Example 6. Suppose that the policymaker chooses place-based consumption levels to maximize a utilitarian social welfare function

$$\max_{\boldsymbol{c}} \qquad \mathbb{E}[u_h(c_{l_h}, l_h)]$$

subject to feasibility,

$$\sum_r L_r c_r = \sum_r L_r z_r,$$

and household location choice satisfies,

$$l_h \in \arg\max_r [\varepsilon_{hr} c_r].$$

These location-choice conditions imply implementability constraints of the form

$$L_r = \frac{c_r^\theta}{\sum_{r'} c_{r'}^\theta},$$

where for exposition simplicity we assume that $B_r = 1$ in every location. Under the common assumption that $u_h(c_r, r) = c_r \varepsilon_{hr}$, the optimal c_r is a convex combination of productivity in region r and average consumption:

$$c_r = \frac{\theta}{\theta+1} z_r + \frac{1}{\theta+1} \left(\sum_k L_k c_k \right). \tag{32}$$

See the appendix for the derivation. This policy redistributes consumption from high-productivity to low-productivity regions.

However, other observationally equivalent choices of the cardinal utility representations $\{f_h\}$, all of which generate the same mapping from wages to labor allocations, imply different optimal place-based policies. Using the calibrated economy from Examples 5 and 6, Table 3 displays the labor allocation induced by the optimal place-based policy under the four observationally equivalent utility representations described in Example 6. The optimal allocations differ substantially across these cardinalizations, even though all four are consistent with the same individual choice behavior.

Table 3: Labor-supply allocation implied by optimal place-based policies

Commuting zone	L_r^{initial}	$L_r^{W,1}$	$L_r^{W,2}$	$L_r^{W,3}$	$L_r^{W,4}$
San Francisco–Oakland, CA	0.018	0.013	0.016	0.024	0.014
New York, NY	0.034	0.025	0.031	0.047	0.028
Seattle–Tacoma, WA	0.017	0.014	0.016	0.022	0.014
Brownsville, TX	0.001	0.002	0.001	0.001	0.002
McAllen, TX	0.004	0.005	0.004	0.003	0.005
Daytona Beach, FL	0.002	0.003	0.002	0.001	0.002

Notes: The first column reports the initial labor share. The remaining columns report the labor shares that maximize the corresponding utilitarian social welfare functions defined in the notes to Table 2.

Moreover, whereas the decentralized equilibrium in the first column is Pareto efficient, the "optimal" allocations chosen by the policymaker are Pareto inefficient. Equivalently, starting from those optimal allocations, $A(t) > 1$: there exists another feasible allocation, implemented with lump-sum transfers, that makes every household strictly better off. Thus, the utilitarian planner sacrifices aggregate efficiency to pursue redistributive goals that depend on an arbitrary cardinalization of utility.

6.3 Sum of Compensating Variations / Kaldor-Hicks Efficiency

In partial equilibrium contexts, where there is an outside good, sometimes called "money", a common measure of aggregate welfare is the sum of compensating variations (Small and Rosen, 1981). This measure, also known as the Kaldor-Hicks measure of efficiency, can be defined, using our notation, as:

$$S(t) = -\int e_h(\boldsymbol{w}(t), \boldsymbol{p}^c(t), u_h^0) dh,$$

where e_h is the net expenditure function — the transfer h needs to attain u_h^0, given wages and consumption prices. Note that if h prefers $(\boldsymbol{w}(t), \boldsymbol{p}^c(t))$ to status quo prices and wages, then e_h is a negative number. So, the scalar $S(t)$ measures the amount of money left, in terms of the numeraire, after winners exactly compensate the losers, holding prices and wages constant at t.[37]

We discuss this measure in more detail in Appendix I. This measure is very similar to $A(t)$ as long as prices and wages are exogenously given (e.g. as in the one-good economy example). However, $S(t)$ has undesirable properties when wages and prices are endogenous. In particular, $S(t)$ can be strictly positive in response to pure redistributions that move the economy along the Pareto frontier. Intuitively, this is because redistributions cause prices to fall for goods consumed primarily by losers relative to winners. Hence, at post-shock prices, the winners can compensate the losers and have money left over, which implies that $S(t) > 0$. See Appendix I for a worked-out example.[38]

7 Application: Removing the Interstate Highway System

In this section, we illustrate the difference between TFP-equivalent aggregate welfare, A, and the various measures of aggregate welfare using a real-world application. We revisit a counterfactual studied in Allen and Arkolakis (2014): the aggregate welfare effect of removing the Interstate Highway System in the United States when households have heterogeneous tastes.[39]

[37] An unimportant difference between $S(t)$ and $A(t)$ is that $S(t)$ is measured in dollars, whereas $A(t)$ is unit-free, being defined as a ratio. In Appendix I we use an alternative, ratio-based version of Kaldor-Hicks efficiency — defined as total income at t relative to total income required to compensate households at those prices — and show that the conclusions of this section are unchanged.

[38] This phenomenon also occurs in general equilibrium models with continuous choice, where it is known as the Boadway (1974) paradox. See Baqaee and Burstein (2025) for more information in a continuous choice setting.

[39] The baseline model in Allen and Arkolakis (2014) features perfectly elastic location choice and homogeneous preferences. However, they show that their equilibrium conditions can also be interpreted in a way

Sketch of model. Each location, r, produces a differentiated good using labor with productivity z_r. Goods from r' sold to r are subject to iceberg cost $\tau_{rr'}$. Consumers in each location have CES preferences over goods from different origins with elasticity of substitution σ. The price of the consumption bundle in location r, not including amenity value, and the expenditure share in location r on goods produced in region r' are

$$p_r^c = \left[\sum_{r'} \left(\frac{\tau_{rr'} w_{r'}}{z_{r'}}\right)^{1-\sigma}\right]^{1/(1-\sigma)}, \qquad \lambda_{rr'} = \frac{(\tau_{rr'} w_{r'}/z_{r'})^{1-\sigma}}{\sum_{r''} (\tau_{rr''} w_{r''}/z_{r''})^{1-\sigma}}.$$

Goods-market clearing requires

$$w_{r'} L_{r'} = \sum_r \lambda_{rr'} w_r L_r.$$

Productivity and amenities depend on regional population:

$$z_r = \bar{z}_r L_r^{\alpha}, \qquad B_r = \bar{B}_r L_r^{\beta},$$

where α governs productivity spillovers and β governs amenity spillovers; $\beta < 0$ captures congestion.[40]

We introduce heterogeneous tastes using the Fréchet preferences in Example 4 and Example 6, where θ denotes the shape parameter. Labor supply in the decentralized equilibrium, normalizing aggregate population to one, satisfies

$$L_r = \frac{L_r^{\theta\beta} (\bar{B}_r w_r / p_r^c)^{\theta}}{\sum_{r'} L_{r'}^{\theta\beta} (\bar{B}_{r'} w_{r'} / p_{r'}^c)^{\theta}}. \tag{33}$$

Calibration. We follow the calibration strategy in Allen and Arkolakis (2014). The model has 3,109 counties in the continental United States ($R = 3,109$), and the Armington elasticity is $\sigma = 9$. We consider two specifications of the externality parameters: $\alpha = \beta = 0$ and the Allen–Arkolakis values, $\alpha = 0.1$ and $\beta = -0.3$. Relative to Allen and Arkolakis (2014), our model allows the labor-supply elasticity to be finite; for illustration, we set

that accommodates heterogeneous tastes, where the parameter controlling congestion externalities now captures heterogeneity in tastes. We use the latter interpretation of their model in this section, but explicitly incorporate taste heterogeneity so that we have separate parameters controlling congestion and dispersion in tastes. Although the labor supply equations are similar with taste heterogeneity and congestion, the welfare implications are different. We discuss the special case where households have homogeneous preferences in Appendix C.

[40]When either α or β is nonzero, then the model features externalities and is technically not a special case of the perfectly competitive benchmark model in Section 2. However, in Appendix G we show how to extend the analysis to models with distortions and externalities.

$\theta = 3$. For each specification, we calibrate $\bar{z}_r$ and $\bar{B}_r$ so that the model exactly reproduces the same initial equilibrium.

Quantitative exercise. We repeat the counterfactual exercise in Allen and Arkolakis (2014). We replace the baseline bilateral trade-cost matrix with a no-Interstate-highway-system version of this matrix. This matrix is constructed by removing the speed advantage associated with interstate highways while leaving the remaining road, rail, water, and air networks unchanged.

Table 4: Aggregate welfare effects of removing the Interstate Highway System

	No externalities $\alpha = \beta = 0$	With externalities $\alpha = 0.1,\ \beta = -0.3$
Panel A.		
TFP-equivalent welfare, A	-1.563	-1.612
Panel B.		
Consumption-equivalent for average utility, A^W		
For the normalized specifications, let $c_r \equiv w_r / p_r^c$ and define $x_{hr} \equiv c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$.		
$u_h(c_r, r) = c_r \varepsilon_{hr}$	-1.659	-1.679
$u_h(c_r, r) = x_{hr}$	-1.424	-1.527
$u_h(c_r, r) = x_{hr}^{1/2}$	-1.544	-1.605
$u_h(c_r, r) = x_{hr}^2$	-1.150	-1.350
$u_h(c_r, r) = \exp(x_{hr})$	-0.638	-1.022
$u_h(c_r, r) = -\exp(-x_{hr})$	-1.676	-1.690
Minimum consumption-equivalent change	-11.490	-14.002
Maximum consumption-equivalent change	2.814	4.280

Notes: All entries are 100 times log changes between the baseline and the no-highway economy. All calculations use $\theta = 3$ and $\sigma = 9$.

Results. The results are shown in Table 4. Panel A reports the change in TFP-equivalent aggregate welfare, $A(t)$, which falls by around 1.6% both with and without externalities.[41] Panel B reports the consumption equivalent for average utility, A^W, under alternative cardinal representations of the same ordinal preferences. This panel shows that $A^W(t)$ can vary substantially across different cardinalizations, all of which are consistent with identical household choice behavior and observables. The first row, which has a welfare loss of around 1.7%, uses the common cardinalization $u_h(c_r, r) = c_r \varepsilon_{hr}$. The second

[41] If we instead set $\theta = \infty$, then we recover the Allen-Arkolakis results. In this case, aggregate welfare declines by 1.1% without externalities and 1.6% with externalities.

row normalizes each household's taste vector by $\max_{r'}\{\varepsilon_{hr'}\}$, so that each household's taste for their most-preferred location is one. This yields a welfare loss between 1.4% and 1.5%. The fact that $A(t)$ lies between the values implied by these two cardinalizations is a numerical accident rather than a theoretical result. The remaining rows consider different strictly increasing transformations. The resulting aggregate welfare number moves around in erratic ways depending on the arbitrary transformation used. The panel also reports the maximum and minimum attainable values for $A^W(t)$ established by Proposition 8, showing that the bounds are quite wide, going from very negative (-11% and -14%) to positive (2.8% and 4.3%). Within these bounds, the data provide no discipline on which value should be preferred.

Although not reported in the table, we also compute other measures for comparison. For example, real output falls by 1.2% or 1.3% depending on the externality parameters. The loss is smaller than $A(t)$ because removing the Interstate Highway System induces agents to move toward higher-real-wage, lower-amenity locations. For completeness, we also compute TFP-equivalent aggregate welfare when compensation is restricted to place-based instruments, denoted by $A^{pb}(t)$ and defined in Appendix H. In this application, $A^{pb}(t)$ and $A(t)$ are very similar. The appendix shows that the two measures are first-order equivalent in the absence of externalities.

8 Conclusion

We generalize the cost-benefit approach of Harberger (1971) and Small and Rosen (1981) to measure aggregate welfare in general equilibrium environments with discrete choice. Our measure converts shocks into a welfare-equivalent change in total factor productivity.

We show that, to a first-order approximation, aggregate welfare in discrete-choice economies obeys a Hulten-type Domar-weighted formula, and we provide second-order corrections expressed in terms of uncompensated supply and demand elasticities. We contrast our welfare measure with chain-weighted real output, average utility, and the sum of compensating variations (Kaldor-Hicks efficiency), and show that these popular alternatives have serious flaws. Real output can fall when every household is better off. Rankings based on average utility depend on arbitrary cardinalizations of individual utilities and can recommend Pareto-inefficient place-based policies that move the economy away from efficiency in unintuitive ways. Kaldor-Hicks efficiency can rise in response to pure redistributions that only move the economy along the Pareto frontier, attributing "efficiency gains" to changes that are purely distributional. In contrast, our measure only rises when Pareto improvements are possible, is invariant to monotone transformations of utility, and stays constant in response to pure redistributions.

Our benchmark environment is perfectly competitive, Pareto-efficient, and features lump-sum transfers that can be used for compensation. The appendices generalize this basic approach to settings with distortions, externalities, and more restricted redistributive tools. A natural next step is to use our measure of aggregate welfare A as the objective function in optimal policy problems in these settings.

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Appendix

Appendix A Proofs of propositions and theorems in the body of the paper

Proof of Proposition 1. The expenditure function is defined to be

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = \min_{T_h, c_h, l_h} \{T_h : u_h(c_h, l_h) \geq u_h^0, \text{ and } \sum_r p_r^c c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

Since utility is increasing in consumption, the utility constraint binds at the optimum. Hence, conditional on choosing l_h, consumption must be $\bar{c}_{hl_h}$, where

$$u_h(\bar{c}_{hl_h}, l_h) = u_h^0.$$

Substituting this into the definition gives

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = \min_{T_h, l_h}\{T_h : \sum_r p_r^c \bar{c}_{hr}\mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

The budget constraint also binds at an optimum (otherwise the transfer could be reduced), so substituting out for T_h yields

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = -\max_{l_h}\{\sum_r a_{hr} w_r \mathbf{1}[l_h = r] - \sum_r p_r^c \bar{c}_{hr}\mathbf{1}[l_h = r]\}.$$

The maximizer of this problem (or a set if there are multiple solutions) is the compensated choice $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$, which satisfies the first expression in Proposition 1. Evaluating the transfer at that choice gives the second expression in the proposition.

□

Proof of Theorem 1. Recall that $\mathcal{C}(t, Z)$ is the set of allocations supported by a competitive equilibrium given $\boldsymbol{z}(t)$, Z, and some transfers. TFP-equivalent welfare solves

$$A(t) = \max\left\{Z^{-1} : \{c_h, l_h\}_{h\in H} \in \mathcal{C}(t, Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h\right\}.$$

Let $A = A(t)$ and $Z^* = 1/A$. Let $\{c_h^*, l_h^*\}_{h\in H}$ be an allocation attaining this maximum. By the definition of $\mathcal{C}(t, Z^*)$, there is a budget-balanced transfer vector $\boldsymbol{T}^*$ that supports this allocation. Given the maintained assumption that every household can be made exactly indifferent at the solution, $u_h(c_h^*, l_h^*) = u_h^0$. Therefore,

$$c_h^* = u_h^{-1}\left(u_h^0; l_h^*\right) = \bar{c}_{hl_h^*},$$

where the inverse is with respect to consumption, holding location fixed.[42] Let $\boldsymbol{w}$ and $\boldsymbol{p}^c$ denote wages and consumption prices in this equilibrium. The household budget constraint becomes

$$p_{l_h^*}^c \bar{c}_{hl_h^*} = \frac{a_{hl_h^*} w_{l_h^*}}{A} + T_h^*.$$

Hence,

$$T_h^* = p_{l_h^*}^c \bar{c}_{hl_h^*} - \frac{a_{hl_h^*} w_{l_h^*}}{A}.$$

Because the location chosen at the optimum minimizes the required transfer, Proposi-

[42]If we impose ordinal additive separability, which is not required for this proof, then $g(c_h^*) + \epsilon_{hl_h^*} = f_h^{-1}(u_h^0)$, so $c_h^* = g^{-1}\left(f_h^{-1}(u_h^0) - \epsilon_{hl_h^*}\right) \equiv \bar{c}_{hl_h^*}$.

tion 1 implies

$$l_h^* = l_h^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, u_h^0), \qquad T_h^* = e_h(\boldsymbol{w}/A, \boldsymbol{p}^c, u_h^0).$$

Summing the household budget constraints over households choosing location r gives

$$\begin{aligned}\int_h p_r^c \bar{c}_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh &= \frac{w_r}{A} L_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, \boldsymbol{u}^0) \\ &\quad + \int_h e_h(\boldsymbol{w}/A, \boldsymbol{p}^c, u_h^0) \mathbf{1}[l_h^{\text{comp}} = r] dh \\ &= E_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, \boldsymbol{u}^0).\end{aligned}$$

Since transfers are budget-balanced,

$$0 = \int_h T_h^* dh = \sum_r E_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, \boldsymbol{u}^0) - \sum_r \frac{w_r}{A} L_r^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, \boldsymbol{u}^0).$$

This proves that the solution to $A(t)$ satisfies (13). The remaining conditions in Theorem 1 follow because $\boldsymbol{w}$ and $\boldsymbol{p}$ are equilibrium wages and prices at the optimizing allocation.

Conversely, suppose that A, wages, prices, and quantities satisfy the compensated-equilibrium conditions in Theorem 1. Give each household a transfer

$$T_h = e_h(\boldsymbol{w}/A, \boldsymbol{p}^c, u_h^0), \qquad l_h = l_h^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}^c, u_h^0), \qquad c_h = \bar{c}_{hl_h}.$$

By Proposition 1, households optimize and are indifferent to the status quo. Equation (13) implies that transfers are budget-balanced, while the remaining compensated-equilibrium conditions ensure that firms optimize and markets clear. Hence, $\{c_h, l_h\}_{h \in H} \in \mathcal{C}(t, 1/A)$, so A is feasible in the definition of $A(t)$. Thus, the feasible values of A are precisely those supported by the compensated-equilibrium conditions, and the largest such value equals $A(t)$.

□

Proof of Theorem 2. Throughout the proof, wages, prices, quantities, and expenditure shares refer to the compensated equilibrium, so we suppress the superscript comp. By Proposition 1, household h's expenditure function can be written as

$$e_h(\boldsymbol{w}/A, \boldsymbol{p}^c, u_h^0) = \min_{r \in R} \left\{ p_r^c \bar{c}_{hr} - a_{hr} w_r / A \right\}.$$

Except on the measure-zero set of households indifferent between locations, the envelope theorem therefore gives

$$\frac{\partial e_h}{\partial (w_r/A)} = -a_{hr}\mathbf{1}[l_h^{\text{comp}} = r], \qquad \frac{\partial e_h}{\partial p_r^c} = \bar{c}_{hr}\mathbf{1}[l_h^{\text{comp}} = r].$$

These expressions depend only on the expenditure necessary to attain u_h^0 and therefore do not require a particular cardinalization of utility.

Budget balance in the compensated equilibrium requires

$$\int_h e_h(\boldsymbol{w}/A, \boldsymbol{p}^c, u_h^0)\, dh = 0. \tag{34}$$

Totally differentiating (34) and applying the two envelope conditions yields

$$\begin{aligned}
0 &= -\sum_{r\in R} L_r^{\text{comp}} d(w_r/A) + \sum_{r\in R}\left(\int_h \bar{c}_{hr}\mathbf{1}[l_h^{\text{comp}} = r]dh\right) dp_r^c \\
&= -\sum_{r\in R} L_r^{\text{comp}} d(w_r/A) + \sum_{r\in R}\frac{E_r^{\text{comp}}}{p_r^c} dp_r^c,
\end{aligned}$$

where the second equality uses $E_r^{\text{comp}} = p_r^c \int_h \bar{c}_{hr}\mathbf{1}[l_h^{\text{comp}} = r]dh$. Hence,

$$\sum_{r\in R}(w_r/A)L_r^{\text{comp}} d\log(w_r/A) = \sum_{r\in R} E_r^{\text{comp}} d\log p_r^c. \tag{35}$$

Because aggregate transfers sum to zero,

$$\sum_{r\in R} E_r^{\text{comp}} = \sum_{r\in R}(w_r/A)L_r^{\text{comp}}.$$

Dividing (35) by this common value, using $d\log(w_r/A) = d\log w_r - d\log A$, and recalling the definitions of λ_r and χ_r, gives

$$d\log A = \sum_{r\in R}\lambda_r d\log w_r - \sum_{r\in R}\chi_r d\log p_r^c. \tag{36}$$

It remains to connect wages and consumption prices to productivity. For every produced commodity $i \in C+N$, Shephard's lemma and perfect competition imply

$$d\log p_i = -d\log z_i + \sum_{j\in N}\Omega_{ij} d\log p_j + \sum_{r\in R}\Omega_{ir} d\log w_r.$$

Multiplying by λ_i and summing over $i \in C + N$ gives

$$\sum_{i\in C+N} \lambda_i d\log p_i = -\sum_{i\in C+N} \lambda_i d\log z_i + \sum_{j\in N}\left(\sum_{i\in C+N}\lambda_i\Omega_{ij}\right) d\log p_j + \sum_{r\in R}\left(\sum_{i\in C+N}\lambda_i\Omega_{ir}\right) d\log w_r.$$

Market clearing implies that the two expressions in parentheses are λ_j and λ_r, respectively. Moreover, $\lambda_{c(r)} = \chi_r$ and $p_{c(r)} = p_r^c$. Separating the consumption bundles from the intermediate goods on the left-hand side and cancelling the intermediate-price terms therefore gives

$$\sum_{r\in R} \chi_r d\log p_r^c = -\sum_i \lambda_i d\log z_i + \sum_{r\in R} \lambda_r d\log w_r.$$

Substitution into (36) yields

$$d\log A = \sum_{i\in C+N} \lambda_i d\log z_i.$$

Restoring the compensated-equilibrium superscript and integrating along the path from 0 to t gives

$$\log A(t) = \int_0^t \sum_{i\in C+N} \lambda_i^{\text{comp}}(s) \frac{d\log z_i(s)}{ds}\, ds,$$

as required. □

Proof of Proposition 2. Equation (17) follows by log-differentiating the CES input-demand equations. Equation (18) follows from Shephard's lemma, constant returns to scale, and the condition that price equals marginal cost. Equation (19) follows by log-differentiating the definition of λ_r for $r \in R$, and (20) follows by differentiating the goods-market-clearing condition (6). These cost-minimization, pricing, and market-clearing conditions hold in both the decentralized and compensated equilibria. □

Proof of Proposition 3. The first part follows from:

$$
\begin{aligned}
L_r^{\text{comp}}(\boldsymbol{w}/A, p^c, \boldsymbol{u}^0) &= \int a_{hr}\mathbf{1}\left[r \in \arg\max_i \left\{a_{hi}\frac{w_i}{Ap^c} + \frac{e_h(\boldsymbol{w}/A, p^c, u_h^0)}{p^c} + \epsilon_{hi}\right\}\right] dh \\
&= \int a_{hr}\mathbf{1}\left[r \in \arg\max_i \left\{a_{hi}\frac{w_i}{Ap^c} + \epsilon_{hi}\right\}\right] dh \\
&= L_r(\boldsymbol{w}/A, p^c) = L_r(\boldsymbol{w}/(Ap^c), 1),
\end{aligned}
$$

where the compensating transfer drops out of the location ranking because it provides the same amount of consumption in every location. Equation (21) is the total differential of this expression along the compensated-equilibrium path. Together with equations (16) to (20), it pins down the compensated equilibrium and $A(t)$, except potentially for the terms involving $\boldsymbol{\chi}^{\text{comp}}$ in (20). Since the consumption good is common across locations, define its common input share by $\Omega_{ci} \equiv \Omega_{c(r)i}$ for every r. The terms in (20) involving compensated spending shares satisfy

$$
\sum_r d\chi_r^{\text{comp}}\Omega_{c(r)i} + \sum_r \chi_r^{\text{comp}} d\Omega_{c(r)i} = \Omega_{ci}\sum_r d\chi_r^{\text{comp}} + d\Omega_{ci}\sum_r \chi_r^{\text{comp}} = d\Omega_{ci},
$$

because $\sum_r d\chi_r^{\text{comp}} = 0$ and $\sum_r \chi_r^{\text{comp}} = 1$. Thus, compensated spending shares drop out of (20), completing the result. □

Proof of Proposition 4. In the decentralized equilibrium, transfers are zero, so expenditure in location r is

$$
E_r = w_r L_r.
$$

Log-differentiating $\chi_r = E_r / \sum_{r'} E_{r'}$ at $t = 0$ therefore gives

$$
d\log\chi_r = d\log w_r + d\log L_r - \mathbb{E}_\chi\left[d\log w + d\log L\right].
$$

Now consider the compensated equilibrium. Let $B_r^{\text{comp}}(t)$ denote the set of households whose compensated choice at t is r. Expenditure in location r is

$$
E_r^{\text{comp}}(t) = \int_{B_r^{\text{comp}}(t)} p_r^c(t)\bar{c}_r(\epsilon) f(\epsilon) d\epsilon.
$$

By definition, $\bar{c}_r(\epsilon)$ is the consumption required in location r to make the household indifferent to the status quo, so it does not vary with t. At the status quo, $p_r^c\bar{c}_r(\epsilon) = w_r$ for all households in $B_r^{\text{comp}}(0)$ and on its boundary. Assumption 2 implies that

$$L_r^{\text{comp}}(t) = \int_{B_r^{\text{comp}}(t)} f(\epsilon) d\epsilon.$$

Under the regularity conditions ensuring differentiability of the choice shares, applying Leibniz's rule to the moving choice set therefore gives

$$dE_r^{\text{comp}} = w_r L_r \, d\log p_r^c + w_r \, dL_r^{\text{comp}},$$

where $d\log p_r^c$ is the change in consumption prices in the compensated equilibrium. The first term is the change in expenditure by households initially choosing r, while the second is the contribution from households entering or leaving r. Using

$$E_r^{\text{comp}}(0) = w_r L_r \qquad \text{and} \qquad L_r^{\text{comp}}(0) = L_r,$$

it follows that

$$d\log E_r^{\text{comp}} = d\log p_r^c + d\log L_r^{\text{comp}}.$$

Finally, log-differentiating

$$\chi_r^{\text{comp}} = \frac{E_r^{\text{comp}}}{\sum_{r'} E_{r'}^{\text{comp}}}$$

gives

$$d\log \chi_r^{\text{comp}} = d\log p_r^c + d\log L_r^{\text{comp}} - \sum_{r'} \chi_{r'}^{\text{comp}}(0) \left[d\log p_{r'}^c + d\log L_{r'}^{\text{comp}} \right].$$

Since the compensated and decentralized equilibria coincide at the status quo, $\chi^{\text{comp}}(0) = \chi(0)$. Hence,

$$d\log \chi_r^{\text{comp}} = d\log p_r^c + d\log L_r^{\text{comp}} - \mathbb{E}_\chi \left[d\log p^c + d\log L^{\text{comp}} \right],$$

as required. □

Proof of Proposition 5. We first derive the expression for pairwise flows in the uncompensated equilibrium and relate them to derivatives of the uncompensated supply system. We then derive the compensated flows and express them in terms of those derivatives.

Uncompensated Equilibrium. Fix two distinct locations i and j. Under the regularity conditions ensuring differentiability of the choice shares, households close to more than one pairwise choice boundary do not affect the first-order calculation. We can therefore study the flow across the i-j boundary in isolation. Suppressing the household index, consider the portion of the status-quo choice boundary where households are indifferent between i and j and strictly prefer these two locations to every other location:

$$g\left(\frac{w_i}{p_i^c}\right) + \epsilon_i = g\left(\frac{w_j}{p_j^c}\right) + \epsilon_j > g\left(\frac{w_k}{p_k^c}\right) + \epsilon_k, \qquad k \notin \{i, j\}.$$

The cutoff for $\epsilon_j - \epsilon_i$ is

$$g\left(\frac{w_i}{p_i^c}\right) - g\left(\frac{w_j}{p_j^c}\right).$$

Equality corresponds to indifference between i and j. Above the cutoff, j is strictly preferred to i; below the cutoff, i is strictly preferred to j. Let κ_{ij} denote the density, measured with respect to $\epsilon_j - \epsilon_i$, of households on this portion of the boundary.

Let d denote the right derivative at the status quo, $t = 0$, and undated variables are evaluated at that point. The change in the cutoff is

$$g'\left(\frac{w_i}{p_i^c}\right) d\left(\frac{w_i}{p_i^c}\right) - g'\left(\frac{w_j}{p_j^c}\right) d\left(\frac{w_j}{p_j^c}\right).$$

If this expression is negative, the cutoff falls, and the mass moving from i to j equals κ_{ij} times the fall in the cutoff. Hence,

$$\begin{aligned} dL_{i\to j} = \kappa_{ij} &\left[g'\left(\frac{w_j}{p_j^c}\right) d\left(\frac{w_j}{p_j^c}\right) - g'\left(\frac{w_i}{p_i^c}\right) d\left(\frac{w_i}{p_i^c}\right)\right] \\ &\times \mathbf{1}\left[g'\left(\frac{w_i}{p_i^c}\right) d\left(\frac{w_i}{p_i^c}\right) \leq g'\left(\frac{w_j}{p_j^c}\right) d\left(\frac{w_j}{p_j^c}\right)\right]. \end{aligned} \tag{37}$$

We now express κ_{ij} in terms of cross-derivatives of the uncompensated supply system. All such derivatives are evaluated at the decentralized status quo. If w_j/p_j^c increases while all other real wages remain fixed, the cutoff falls and households leave i for j. Therefore,

$$\frac{\partial L_i}{\partial\left(w_j/p_j^c\right)} = -\kappa_{ij} g'\left(\frac{w_j}{p_j^c}\right).$$

Similarly, if w_i/p_i^c increases while all other real wages remain fixed, households leave j

for i, so

$$\frac{\partial L_j}{\partial\left(w_i/p_i^c\right)}=-\kappa_{ij}g'\left(\frac{w_i}{p_i^c}\right).$$

Differentiability makes these equations valid for changes of either sign. Substituting them into (37) gives

$$\begin{aligned}dL_{i\to j}=&\left[\frac{\partial L_j}{\partial\left(w_i/p_i^c\right)}d\left(\frac{w_i}{p_i^c}\right)-\frac{\partial L_i}{\partial\left(w_j/p_j^c\right)}d\left(\frac{w_j}{p_j^c}\right)\right]\\&\times\mathbf{1}\left[\frac{\partial L_i}{\partial\left(w_j/p_j^c\right)}d\left(\frac{w_j}{p_j^c}\right)\leq\frac{\partial L_j}{\partial\left(w_i/p_i^c\right)}d\left(\frac{w_i}{p_i^c}\right)\right],\end{aligned}$$

which is equation (25). Summing pairwise inflows and subtracting pairwise outflows gives

$$dL_i=\sum_{j\neq i}dL_{j\to i}-\sum_{j\neq i}dL_{i\to j}.$$

Compensated Equilibrium. Now let the changes in wages, prices, and A be those in the compensated equilibrium. Consider households initially choosing i and close to the same status-quo i-j boundary. Let $T_i(t)$ denote the transfer required to keep such a household indifferent to the status quo if it continues to choose i. Under Assumption 2, all households initially choosing i have the same status-quo consumption. Indifference requires

$$\frac{w_i(t)/A(t)+T_i(t)}{p_i^c(t)}=\frac{w_i(0)}{p_i^c(0)}.$$

Since $A(0)=1$ and $T_i(0)=0$, differentiating at $t=0$ gives

$$0=d\left(\frac{w_i}{Ap_i^c}\right)+\frac{dT_i}{p_i^c},$$

and hence

$$dT_i=-p_i^c d\left(\frac{w_i}{Ap_i^c}\right).$$

Under this same transfer, consumption in j changes by

$$d\left(\frac{w_j/A+T_i}{p_j^c}\right)=d\left(\frac{w_j}{Ap_j^c}\right)+\frac{dT_i}{p_j^c}$$
$$=\frac{1}{p_j^c}\left[p_j^c d\left(\frac{w_j}{Ap_j^c}\right)-p_i^c d\left(\frac{w_i}{Ap_i^c}\right)\right]. \tag{38}$$

To determine which households switch, compare $T_i(t)$ with the transfer required for indifference in j. Under the transfer $T_i(t)$, a household weakly prefers choosing j to its status-quo bundle if and only if

$$g\left(\frac{w_j(t)/A(t)+T_i(t)}{p_j^c(t)}\right)+\epsilon_j\geq g\left(\frac{w_i(0)}{p_i^c(0)}\right)+\epsilon_i.$$

Equivalently,

$$\epsilon_j-\epsilon_i\geq g\left(\frac{w_i(0)}{p_i^c(0)}\right)-g\left(\frac{w_j(t)/A(t)+T_i(t)}{p_j^c(t)}\right).$$

Thus, the expression on the right is the threshold for $\epsilon_j-\epsilon_i$ above which the transfer required in j is no greater than the transfer required in i. By Proposition 1, j is then the compensated location choice. For households on this portion of the boundary, the other locations remain strictly dominated for sufficiently small changes. At $t=0$, $A(0)=1$ and $T_i(0)=0$, so the threshold becomes

$$g\left(\frac{w_i(0)}{p_i^c(0)}\right)-g\left(\frac{w_j(0)}{p_j^c(0)}\right).$$

This is the status-quo cutoff from the uncompensated part of the proof. The density of households at this threshold is therefore κ_{ij}. Using (38), the change in the threshold $\epsilon_j-\epsilon_i$ is

$$-g'\left(\frac{w_j}{p_j^c}\right)\frac{1}{p_j^c}\left[p_j^c d\left(\frac{w_j}{Ap_j^c}\right)-p_i^c d\left(\frac{w_i}{Ap_i^c}\right)\right].$$

Note that if

$$p_i^c d\left(\frac{w_i}{Ap_i^c}\right)\leq p_j^c d\left(\frac{w_j}{Ap_j^c}\right),$$

the threshold falls and households move from i to j. The mass that moves equals κ_{ij} times the fall in the threshold. Therefore,

$$dL_{i\to j}^{\text{comp}} = \kappa_{ij} g'\left(\frac{w_j}{p_j^c}\right)\frac{1}{p_j^c}\left[p_j^c d\left(\frac{w_j}{Ap_j^c}\right) - p_i^c d\left(\frac{w_i}{Ap_i^c}\right)\right] \times \mathbf{1}\left[p_i^c d\left(\frac{w_i}{Ap_i^c}\right) \le p_j^c d\left(\frac{w_j}{Ap_j^c}\right)\right]. \tag{39}$$

Recall that the uncompensated part established that

$$\kappa_{ij} g'\left(\frac{w_j}{p_j^c}\right) = -\frac{\partial L_i}{\partial\left(w_j/p_j^c\right)}.$$

Substituting this equation into (39) gives

$$dL_{i\to j}^{\text{comp}} = \frac{\partial L_i}{\partial\left(w_j/p_j^c\right)}\frac{1}{p_j^c}\left[p_i^c d\left(\frac{w_i}{Ap_i^c}\right) - p_j^c d\left(\frac{w_j}{Ap_j^c}\right)\right] \times \mathbf{1}\left[p_i^c d\left(\frac{w_i}{Ap_i^c}\right) \le p_j^c d\left(\frac{w_j}{Ap_j^c}\right)\right],$$

which is the compensated-flow expression in Proposition 5. The partial derivative is that of the uncompensated supply system evaluated at the decentralized status quo, whereas the changes in wages, prices, and A are those in the compensated equilibrium. At $t = 0$, $d\log A$ is given by Corollary 1. Summing pairwise inflows and subtracting pairwise outflows gives

$$dL_i^{\text{comp}} = \sum_{j\neq i} dL_{j\to i}^{\text{comp}} - \sum_{j\neq i} dL_{i\to j}^{\text{comp}}.$$

□

Proof of Proposition 6. By Hulten's theorem (Hulten, 1978), to a first order we have:

$$\Delta\log Y \approx \sum_i \lambda_i(0)\Delta\log z_i + \sum_{r\in R}\lambda_r(0)\Delta\log L_r.$$

By Corollary 1,

$$\Delta\log A \approx \sum_i \lambda_i(0)\Delta\log z_i.$$

Subtracting the second equation from the first gives

$$\Delta \log Y - \Delta \log A \approx \sum_{r \in R} \lambda_r(0) \Delta \log L_r.$$

Using $\lambda_r(0) = w_r(0)L_r(0) / \sum_{r'} w_{r'}(0)L_{r'}(0)$, we have that to a first order,

$$\lambda_r(0)\Delta \log L_r \approx \frac{w_r(0)L_r(0)}{\sum_{r'} w_{r'}(0)L_{r'}(0)} \frac{\Delta L_r}{L_r(0)} = \frac{w_r(0)}{\sum_{r'} w_{r'}(0)L_{r'}(0)} \Delta L_r.$$

Summing across locations proves the proposition.

□

Proof of Proposition 7. By Hulten's theorem (Hulten, 1978)

$$\log A^{\mathrm{MFP}}(t) = \int_0^t \sum_i \lambda_i(s) \frac{d \log z_i}{ds} \, ds,$$

whereas Theorem 2 gives

$$\log A(t) = \int_0^t \sum_i \lambda_i^{\mathrm{comp}}(s) \frac{d \log z_i}{ds} \, ds.$$

Since ϵ_{hr} is the same for every household and location, and there is a common consumption good, household h ranks locations according to $(Za_{hr}w_r + T_h)/p^c$. Because Z and T_h are common across household h's location alternatives, this ranking is equivalent to ranking $a_{hr}w_r$. Consequently, location choices and labor supplies are independent of both Z and the compensating transfers. Hence,

$$l_h^{\mathrm{comp}}(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = l_h(\boldsymbol{w}, \boldsymbol{p}^c).$$

Furthermore, because there is only one common consumption good, Domar weights do not depend on the distribution of spending across locations. Hence, $\lambda_i^{\mathrm{comp}}(s) = \lambda_i(s)$, for every i and s, which implies

$$A(t) = A^{\mathrm{MFP}}(t).$$

We next show that $A^{MFP}(t) = Y(t)$. Hulten's theorem gives

$$\frac{d \log Y}{dt} = \sum_i \lambda_i(t) \frac{d \log z_i}{dt} + \sum_{r \in R} \lambda_r(t) \frac{d \log L_r}{dt}.$$

The second term is the reallocation effect. Consider a marginal mass dh of households moving from i to j. Since households choose locations to maximize nominal income, a marginal mover satisfies $a_{hi}w_i = a_{hj}w_j$. Its contribution to the reallocation effect is

$$\frac{(a_{hj}w_j - a_{hi}w_i)}{GDP}dh = 0.$$

Summing across all marginal flows implies $\sum_r \lambda_r(t) d\log L_r = 0$. Therefore,

$$\frac{d\log Y(t)}{dt} = \frac{d\log A^{\mathrm{MFP}}(t)}{dt}.$$

Since both indexes equal one at $t = 0$, integrating gives $Y(t) = A^{\mathrm{MFP}}(t)$, which proves the result.

□

Proof of Proposition 8. For a continuum of households, interpret the minimum and maximum as the endpoints of the cross-sectional support of $A_h(t)$. Thus, for any b strictly between these endpoints, a positive measure of households has $A_h(t) < b$ and a positive measure has $A_h(t) > b$. For any $b > 0$, define

$$d_h(b) \equiv \max_r u_h\left(\frac{a_{hr}w_r(t)}{bp_r^c(t)}, r\right) - u_h^0.$$

By the definition of $A_h(t)$ and monotonicity in consumption, $d_h(b) > 0$ if $A_h(t) > b$, $d_h(b) < 0$ if $A_h(t) < b$, and $d_h(b) = 0$ if $A_h(t) = b$. By definition,

$$W\left(\frac{\boldsymbol{w}(t)}{b\boldsymbol{p}^c(t)}\right) - W\left(\frac{\boldsymbol{w}(0)}{\boldsymbol{p}^c(0)}\right) = \mathbb{E}[d_h(b)].$$

If $b < \min_h A_h(t)$, then $A_h(t) > b$ for every household, so $d_h(b) > 0$ for every household. Hence, $\mathbb{E}[d_h(b)] > 0$, and average utility at t, after dividing consumption by b, exceeds average utility in the status quo. Similarly, if $b > \max_h A_h(t)$, then $d_h(b) < 0$ for every household, so $\mathbb{E}[d_h(b)] < 0$. Therefore,

$$\min_h A_h(t) \leq A^W(t) \leq \max_h A_h(t).$$

Now fix any b strictly between these bounds and let

$$D_+ \equiv \mathbb{E}[d_h(b)\mathbf{1}\{A_h(t) > b\}], \qquad D_- \equiv -\mathbb{E}[d_h(b)\mathbf{1}\{A_h(t) < b\}],$$

which are both positive. Replace each function f_h with

$$\widetilde{f}_h(x) = \begin{cases} D_- f_h(x), & A_h(t) > b, \\ D_+ f_h(x), & A_h(t) < b, \\ f_h(x), & A_h(t) = b, \end{cases} \qquad \text{for every } x.$$

These positive rescalings leave ordinal preferences and $A_h(t)$ unchanged. Under the transformed cardinal utility functions, the change in average utility when consumption at t is divided by b is

$$\begin{aligned} &D_- \mathbb{E}[d_h(b) \mathbf{1}\{A_h(t) > b\}] + D_+ \mathbb{E}[d_h(b) \mathbf{1}\{A_h(t) < b\}] \\ &\quad = D_- D_+ - D_+ D_- = 0. \end{aligned}$$

Therefore, average utility after dividing consumption at t by b equals average utility in the status quo. By the definition of $A^W(t)$, this implies that $A^W(t) = b$. Because b was arbitrary, every value strictly between the bounds can be attained.

□

Appendix B Implications of the $g(\cdot)$ function for behavior

Since utility is only pinned down up to monotone transformations, the function f_h has no observable implications. However, the shape of $g(c_h)$ has testable implications, since it controls the way income and substitution effects interact with each other. For example, if $g(c)$ is affine, then household choices are invariant to an additive increase in consumption in every location (regardless of the distribution of tastes). On the other hand, if $g(c)$ is log, then household choices are invariant to scaling consumption in every location (regardless of the distribution of tastes).

The following proposition shows that the conditional labor supply function $\boldsymbol{L}(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a})$, mapping vectors of real wages into labor supply in each location conditional on skills $\boldsymbol{a}$, uniquely pins down $g(c_h)$ up to a positive affine transformation. We impose the standard regularity and support conditions: the relevant derivatives exist, the densities of households at the relevant choice margins are positive, and labor supply is observed over an open set of real wages.

Proposition 9 (Relation between L and g)**.** *Let the efficiency units of labor supplied by workers* *given real wages $\boldsymbol{w}/\boldsymbol{p}^c$ conditional on skill type $\boldsymbol{a}$ be*

$$L_i(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a}) = \int a_i \mathbf{1}\left[g(a_i w_i/p_i^c) + \epsilon_{hi} \geq g(a_j w_j/p_j^c) + \epsilon_{hj} \quad \forall j\right] dh.$$

Given knowledge of $\boldsymbol{L}(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a})$, the function $g(c)$ is pinned down, over the relevant support of consumption, up to a positive affine transformation. A notable implication is the following. The function $\boldsymbol{L}(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a})$ has symmetric cross-derivatives in real wages if and only if $g(c)$ is affine.

That is, the functional form of $g(c_h)$ has testable implications. The affine case is noteworthy because, under this assumption, some of our calculations dramatically simplify. Intuitively, if $g(c_h)$ is an affine function, then a lump-sum transfer (in consumption units) to household h will not change household h's choice of location.[43]

Proof. Suppose that preferences take the form

$$u_h(c,r) = g(c) + \epsilon_{hr}.$$

Consider the set of taste vectors $\boldsymbol{\epsilon}$ for which the household is indifferent between locations i and j and weakly prefers them to every other location:

$$F_{ij}(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a}) = \{\boldsymbol{\epsilon} : g(a_i w_i/p_i^c) - g(a_j w_j/p_j^c) = \epsilon_{hj} - \epsilon_{hi},\, g(a_i w_i/p_i^c) - g(a_k w_k/p_k^c) \geq \epsilon_{hk} - \epsilon_{hi},\, \forall k \neq i\}.$$

Let $B_i(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a})$ denote the set of taste vectors for which location i is chosen. We can write

$$L_i(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a}) = \int a_i \mathbf{1}\left[\epsilon \in B_i(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a})\right] f(\epsilon) d\epsilon,$$

where $f(\epsilon)$ is the density of ϵ. Let $m_{ij}(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a})$ denote the density of households at the i-versus-j indifference cutoff for whom i and j are weakly preferred to every other location. Thus, m_{ij} measures the mass of households that cross between i and j per unit change in this cutoff. A marginal increase in w_j/p_j^c lowers labor supply in location i by

$$\frac{\partial L_i(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a})}{\partial(w_j/p_j^c)} = -a_i m_{ij}(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a}) a_j g'(a_j w_j/p_j^c).$$

Similarly,

$$\frac{\partial L_j(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a})}{\partial(w_i/p_i^c)} = -a_j m_{ij}(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a}) a_i g'(a_i w_i/p_i^c).$$

[43] As another example, the conditional labor supply function without transfers $\boldsymbol{L}(\boldsymbol{w},\boldsymbol{p}^c,\mathbf{0}|\boldsymbol{a})$ has symmetric cross semi-elasticities in real wages, $\partial L_r/\partial \log(w_{r'}/p_{r'}^c) = \partial L_{r'}/\partial \log(w_r/p_r^c)$, if and only if $g(c_h)$ is affine in $\log c_h$. The commonly used constant-elasticity Fréchet supply system is a special case with $g(c_h) = \log c_h$.

Hence,

$$\frac{\frac{\partial L_i(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a})}{\partial(w_j/p_j^c)}}{\frac{\partial L_j(\boldsymbol{w}/\boldsymbol{p}^c|\boldsymbol{a})}{\partial(w_i/p_i^c)}} = \frac{g'(a_j w_j/p_j^c)}{g'(a_i w_i/p_i^c)}.$$

Holding all other real wages fixed, define

$$g_{ij}(w_i/p_i^c, w_j/p_j^c) = \frac{\partial L_i(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a})/\partial(w_j/p_j^c)}{\partial L_j(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a})/\partial(w_i/p_i^c)} = \frac{g'(a_j w_j/p_j^c)}{g'(a_i w_i/p_i^c)}.$$

Although the individual derivatives depend on all real wages, their ratio depends only on the real wages in locations i and j because $m_{ij}(\boldsymbol{w}/\boldsymbol{p}^c \mid \boldsymbol{a})$ cancels. Fix any reference consumption level $\bar{c}$ in the interior of the relevant support. Setting $w_i/p_i^c = \bar{c}/a_i$ and $w_j/p_j^c = c/a_j$, we obtain

$$\frac{g'(c)}{g'(\bar{c})} = g_{ij}\left(\frac{\bar{c}}{a_i}, \frac{c}{a_j}\right).$$

By the fundamental theorem of calculus,

$$\frac{g(c) - g(\bar{c})}{g'(\bar{c})} = \int_{\bar{c}}^{c} g_{ij}\left(\frac{\bar{c}}{a_i}, \frac{s}{a_j}\right) ds.$$

Hence, g is pinned down over the relevant support up to a positive affine transformation. Moreover, the cross-derivatives are symmetric if and only if

$$\frac{g'(a_j w_j/p_j^c)}{g'(a_i w_i/p_i^c)} = 1$$

for all real wages in the relevant set. This holds if and only if g' is constant over the relevant support, or equivalently, if and only if g is affine over that support. □

Appendix C Special case with homogeneous agents

In this appendix, we consider an instructive special case where agents are homogeneous in preferences and skill levels ($\succeq_h$ does not vary by h and $a_{hr} = 1$ for every h and r). Locations may nevertheless differ in their common amenity values. For concreteness, if preferences are ordinally additively separable, then we can represent every $\succeq_h$ using:

$$u_h(c_h, l_h) = f_h\left(g(c_h) + \epsilon_{l_h}\right),$$

where ϵ_r parameterizes the common amenity value of location r. Characterizing $A(t)$ in this special case is as easy as solving the decentralized equilibrium.

Proposition 10 (Aggregate Welfare with Homogeneous Agents)**.** *Suppose $\succeq_h$ does not vary by h and $a_{hr} = 1$ for every h and r. Define $c_r(\boldsymbol{z}, Z)$ and $L_r(\boldsymbol{z}, Z)$ to be consumption per capita and labor supply in location r in a decentralized equilibrium given productivity vector $\boldsymbol{z}$ and aggregate factor-augmenting productivity Z. Suppose that at least one location is nonempty both in the status quo and in the compensated equilibrium. Then $A(t)$ satisfies*

$$c_r(\boldsymbol{z}(0), 1) = c_r(\boldsymbol{z}(t), \frac{1}{A(t)}), \tag{40}$$

for every such location r.

If, in addition, $g(c_h) = \log(c_h)$, then $A(t)$ equals the proportional change in per capita consumption (equivalently, the real wage) between the decentralized equilibria at 0 and t:

$$A(t) = \frac{c_r(\boldsymbol{z}(t), 1)}{c_r(\boldsymbol{z}(0), 1)}, \tag{41}$$

for any region r that is nonempty in both equilibria.

Before presenting the proof, we provide some intuition. Since agents are homogeneous, in a decentralized equilibrium agents are indifferent among occupied locations. If we scale aggregate factor-augmenting productivity by $1/A(t)$, every agent is affected in the same way — there are no relative winners or losers — and lump-sum transfers are not needed for compensations. If a location r is non-empty in the status quo and in the allocation that solves $A(t)$, indifference to the status quo requires that $A(t)$ satisfies

$$f_h\big(g(c_r(\boldsymbol{z}(t), 1/A(t))) + \epsilon_r\big) = f_h\big(g(c_r(\boldsymbol{z}(0), 1)) + \epsilon_r\big).$$

Because f_h is strictly increasing, it can be removed from both sides. The common amenity ϵ_r then cancels, and strict monotonicity of g implies (40). In words, $A(t)$ must be such that real consumption per capita in each location r is equated to its status quo value. This shows that computing $A(t)$ is as simple as computing the decentralized equilibrium with different productivity parameters (i.e. we do not have to worry about compensating transfers when agents are homogeneous).

In addition, if $g(c) = \log(c)$, then scaling consumption in every location by the same amount leaves the location choice problem of every agent unchanged. Since agents' location choice problems do not respond to a uniform scaling of consumption, they also do not respond to a uniform scaling of factor-augmenting productivity (for more details, see the proof in the appendix). Hence, $c_r(\boldsymbol{z}(t), 1/A(t)) = c_r(\boldsymbol{z}(t), 1)/A(t)$ and (41) follows. In words, if $g(c) = \log(c)$, then $A(t)$ can be computed purely from the growth in real consumption in any location in the decentralized equilibrium: $A(t) = c_r(\boldsymbol{z}(t), 1)/c_r(\boldsymbol{z}(0), 1)$ — one does not even need to solve a counterfactual equilibrium with different factor-augmenting productivity.

One may wonder: why not always use the growth in real consumption to measure the growth in aggregate welfare? The reason is that outside the case $g(c) = \log(c)$, the growth in real consumption in each location, $c_r(\boldsymbol{z}(t), 1)/c_r(\boldsymbol{z}(0), 1)$, can be a different number, even though all agents are homogeneous in terms of welfare.[44] Our measure of aggregate welfare, $A(t)$, in contrast provides a single number.[45]

Proof of Proposition 10. Define $u_h(\boldsymbol{z}, Z)$ to be the utility of household h in a decentralized equilibrium given productivity vector $\boldsymbol{z}$ and aggregate factor augmenting productivity Z. Because households have identical ordinal preferences, skills, and opportunity sets, they attain the same welfare change and require the same compensating transfer at any given wages and prices. Aggregate budget balance therefore implies that this common transfer is zero. Hence, to calculate $A(t)$, we simply need to solve

$$u_h(\boldsymbol{z}(t), 1/A(t)) = u_h(\boldsymbol{z}(0), 1).$$

For any location r such that $L_r(\boldsymbol{z}(t), 1/A(t)) \times L_r(\boldsymbol{z}(0), 1) > 0$ we have that

$$g(c_r(\boldsymbol{z}(t), 1/A(t))) + \epsilon_r = g(c_r(\boldsymbol{z}(0), 1)) + \epsilon_r.$$

It follows that

$$c_r(\boldsymbol{z}(t), 1/A(t)) = c_r(\boldsymbol{z}(0), 1).$$

This completes the first part of the proposition.

Consider the case where $g(c) = \log c$. We need to show that

$$c_r\big(\boldsymbol{z}(t), 1/A(t)\big) = A(t)^{-1}\, c_r\big(\boldsymbol{z}(t), 1\big).$$

[44] To see this, note that indifference between locations implies that $g(c_r) - g(c_{r'}) = \epsilon_{r'} - \epsilon_r$ must always hold. Hence, $g(c_r(\boldsymbol{z}(t), 1)) - g(c_r(\boldsymbol{z}(0), 1)) = g(c_{r'}(\boldsymbol{z}(t), 1)) - g(c_{r'}(\boldsymbol{z}(0), 1))$. This does not, in general, imply equal growth rates for real consumption per capita across locations unless $g(c) = \log(c)$.

[45] If $g(c) = \log(c)$, then one can also calculate $A(t)$ using a specific cardinalization of utility. In particular, if we use $f_h(x) = \exp(x)$ so that $u_h(c, r) = c\exp(\epsilon_r)$, then $A(t)$ is equal to the ratio of utilities in the decentralized equilibrium at t relative to the status quo. This is a common approach in spatial models with homogeneous preferences (see e.g. Allen and Arkolakis, 2014). This trick does not work if $g(c)$ is not logarithmic. However, one cannot simply assume that $g(c)$ is logarithmic without loss of generality: if ϵ_r varies by location, then the shape of $g(c)$ is determined by ordinal properties of the underlying agent-level utility functions (see Appendix B).

Let factor-augmenting productivity Z change from 1 to $1/A(t)$ while holding $\boldsymbol{z}(t)$ fixed. Conjecture an equilibrium allocation that scales all effective factor inputs, outputs, and intermediate inputs by $1/A(t)$, keeping prices and wages (per efficiency unit) unchanged. In this conjectured equilibrium, consumption in every location scales with $1/A(t)$. With logarithmic g, if consumption in every location is scaled by a common factor, then agents do not have any incentive to switch locations. Hence, labor supply in each location remains unchanged:

$$L_r\big(\boldsymbol{z}, 1/A(t)\big) = L_r\big(\boldsymbol{z}, 1\big).$$

Effective factor supply is ZL_r, so it scales by $1/A(t)$. Constant returns to scale then make it feasible for all outputs and intermediate inputs to scale by the same amount while relative prices remain unchanged. This confirms the conjectured equilibrium and completes the proof.

□

Appendix D Additional results and examples

In this appendix we present multi-industry examples in Example 8 and Example 9. We derive optimal place-based policy in Example 7. We also present an invariance result about average utility with Fréchet-distributed tastes.

D.1 Multi-industry examples

We provide two examples with multiple industries.

Example 8 (Cobb-Douglas and Logit Example). Every good is produced using a Cobb-Douglas production technology, so (5) implies that the price of each good $i \in C + N$ can be written as

$$p_i = z_i^{-1} \prod_{j \in N} p_j^{\Omega_{ij}} \prod_{r \in R} w_r^{\Omega_{ir}},$$

where Ω_{ij} and Ω_{ir} are expenditure shares of i on j and r respectively (Cobb-Douglas implies that these expenditure shares are constant). We can solve out for λ_r in the market clearing condition, (6), to get that for every factor r:

$$\lambda_r = \sum_{r'} \chi_{r'} \Psi_{c(r')r},$$

where $\Psi = (I - \Omega)^{-1}$ is the Leontief inverse (which, again, is constant due to the Cobb-Douglas assumption). This equation states that income of r must equal the dollar-weighted average factor content of final consumption $\sum_{r' \in R} \chi_{r'} \Psi_{c(r')r}$, where $\Psi_{c(r')r}$ is the total factor r content of consumption by agents in location r'.

Suppose that all consumers consume the same consumption good, so that the factor content Ψ_r is the same for every r'. The previous equation simplifies to

$$\lambda_r = \frac{w_r L_r}{\sum_{r''} w_{r''} L_{r''}} = \Psi_{0r}, \tag{42}$$

where $c(r') = 0$ is the index for the common consumption good and the right-hand side is a constant, depending only on the Cobb-Douglas share parameters. That is, we use the fact that $\sum_{r'} \chi_{r'} \Psi_{c(r')r} = \sum_{r'} \chi_{r'} \Psi_{0r} = \Psi_{0r}$.

Suppose skills are homogeneous, $a_{hr} = 1$ and normalize $Z = 1$. Utility functions are given by $u_h(c_h, l_h) = f_h(c_h + \epsilon_{hl_h})$ where ϵ_{hr} are drawn from a type-I extreme-value distribution and f_h is any strictly increasing function. The labor supply function is

$$L_r(\boldsymbol{w}, p^c, \boldsymbol{T}) = \frac{\exp(\theta w_r / p^c + B_r)}{\sum_{r'} \exp(\theta w_{r'} / p^c + B_{r'})}. \tag{43}$$

where p^c is the price of the final consumption good, and θ and B_r are parameters of the distribution of ϵ_{hr}. Given this functional form, equation (42) can be rewritten as

$$\frac{w_r \exp(\theta w_r / p^c + B_r)}{\sum_{r'} w_{r'} \exp(\theta w_{r'} / p^c + B_{r'})} = \Psi_{0r}$$

where the consumer price index satisfies

$$p^c = \prod_j z_j^{-\Psi_{0j}} \prod_r w_r^{\Psi_{0r}}.$$

These equations pin down all equilibrium prices, up to the choice of numeraire, which can then be used to pin down quantities.

Example 9 (Occupational Choice Example). Suppose that the common consumption good is a CES bundle of outputs from different industries:

$$y = \left(\sum_r \Omega_{0r}^{\frac{1}{\theta_0}} x_{0r}^{\frac{\theta_0 - 1}{\theta_0}} \right)^{\frac{\theta_0}{\theta_0 - 1}},$$

where the units of quantities are chosen so that Ω_{0r} are expenditures in the status quo. Industry r's output is

$$x_{0r} = z_r L_r.$$

Suppose that workers' utility functions can be written as

$$u_h(c_h, r) = c_h + \epsilon_{hr},$$

where ϵ_{hr} is type I extreme value. In this case, $L(\boldsymbol{w}, p^c, \boldsymbol{T})$ has the standard logit functional form. This implies that, for $r' \neq r$,

$$\frac{\partial L_r}{\partial \left[w_{r'} / p^c \right]} = -\theta L_r L_{r'}.$$

Substituting this into (21) yields

$$d \log L_i^{\text{comp}} = \theta \left(d \left[w_i / \left(A p^c \right) \right] - \mathbb{E}_{L^{\text{comp}}} \left[d \left[w / \left(A p^c \right) \right] \right] \right), \tag{44}$$

where the wages and prices are evaluated in the compensated equilibrium. We now apply Proposition 2. The sales share of industry r, in the compensated equilibrium, is given by its share of the wage bill:

$$d \log \lambda_l^{\text{comp}} = d \log w_l + d \log L_l^{\text{comp}} - \mathbb{E}_{\lambda^{\text{comp}}} \left[d \log w + d \log L^{\text{comp}} \right]. \tag{45}$$

At the same time, the sales share of industry r, in the compensated equilibrium, is also given by the share of household spending on industry r:

$$d \log \lambda_l^{\text{comp}} = (\theta_0 - 1) \left[d \log z_l - d \log w_l - \mathbb{E}_{\lambda^{\text{comp}}} \left[d \log z - d \log w \right] \right], \tag{46}$$

where we use the fact that $d \log p_r = d \log w_r - d \log z_r$. Shephard's lemma implies that the consumer price index in the compensated equilibrium is

$$d \log p^c = \sum_l \lambda_l^{\text{comp}} \left[d \log w_l - d \log z_l \right]. \tag{47}$$

Finally, from Theorem 2, the change in TFP-equivalent aggregate welfare is:

$$d\log A = \sum_i \lambda_i^{\text{comp}} d\log z_i. \tag{48}$$

Equations (44)-(48) form a system of ordinary differential equations that can be solved to obtain $A(t)$ without simulation methods. The boundary conditions are that at $t = 0$, expenditure and population shares coincide with the status quo, and $A(0) = 1$. Solving the system is simple: discretize the productivity shocks, and iterate on the linear system, updating variables each time.

D.2 Deriving optimal place-based policy in Example 7

In this example, there is a measure–1 continuum of agents, R locations, no differences in idiosyncratic skills, and a single good produced with labor in each location. Agent h has utility

$$u_h(c) = \max_{r\in R}\{c_r \bar{\epsilon}_h \epsilon_{hr}\},$$

where $\epsilon_{hr} > 0$ are i.i.d. unit Fréchet random variables with shape parameter $\theta > 1$. Given a vector of consumption by location, $\{c_r\}$, the supply of labor in location r is

$$L_r = \frac{c_r^\theta}{\sum_{r'} c_{r'}^\theta}.$$

Under the cardinalization $\bar{\epsilon}_h = 1$ for all h, the utilitarian social welfare function is

$$W = \int u_h(c)\, dh = \kappa \left[\sum_r c_r^\theta\right]^{1/\theta},$$

where $\kappa = \Gamma(1 - 1/\theta)$.

Derivation of (32)**.** Consider the problem of choosing $\{c_r^0\}$ to maximize W, subject to feasibility,

$$\sum_r L_r^0 c_r^0 = \sum_r L_r^0 z_r,$$

and household location choice. These location-choice conditions imply the labor-supply system above. Using this system, the feasibility constraint can be written as

$$\sum_r (c_r^0)^{\theta+1} = \sum_r (c_r^0)^\theta z_r.$$

Since W is increasing in $\sum_r c_r^\theta$, the Lagrangian is

$$\mathcal{L}(c,\lambda) = \sum_r c_r^\theta + \lambda \left(\sum_r c_r^{\theta+1} - \sum_r c_r^\theta z_r \right).$$

The FOC for c_r is

$$\theta c_r^{\theta-1} + \lambda \left[(\theta+1) c_r^\theta - \theta c_r^{\theta-1} z_r \right] = 0.$$

Dividing by $\theta c_r^{\theta-1} > 0$ gives

$$1 + \lambda \left[\frac{\theta+1}{\theta} c_r - z_r \right] = 0.$$

Hence $(\theta+1)c_r - \theta z_r$ is constant across locations. Denote this constant by C:

$$c_r^0 = \frac{\theta}{\theta+1} z_r + \frac{C}{\theta+1}.$$

Multiplying by L_r^0 and summing over r, using $\sum_r L_r^0 = 1$ and the resource constraint, gives

$$C = \sum_r L_r^0 c_r^0.$$

Therefore, we obtain (32).

The following proposition shows that, starting at the "optimal" allocation (32), there exists a schedule of lump-sum transfers such that every agent can be made better off. For simplicity, we consider the version of the model with two regions, but the same argument applies in an economy with R regions.

Proposition 11. *Suppose $R = 2$ and $z_1 \neq z_2$. If the status quo is the "optimal" allocation implied by* (32)*, then this allocation is Pareto inefficient and $A > 1$.*

Proof. Let $l_h^0 \in \arg\max_{r=1,2} c_r^0 \varepsilon_{hr}$ denote the initial location choice. If household h is assigned to location r, the consumption required to keep her indifferent to the status quo is

$$\bar{c}_{hr} = c_{l_h^0}^0 \frac{\varepsilon_{hl_h^0}}{\varepsilon_{hr}}.$$

Applying Theorem 1 and using $w_r = z_r$ and $p^c = 1$, aggregate welfare can be written as

$$A = \max_{l} \frac{\sum_r \int z_r \mathbf{1}[l_h = r]\, dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh}.$$

By Dinkelbach's theorem, the solution A satisfies $F(A) = 0$, where

$$F(x) = \int \max_{r=1,2} \{z_r - x\bar{c}_{hr}\}\, dh.$$

We show that $F(1) > 0$, which implies $A > 1$. Define

$$\phi_h(r) = z_r - \bar{c}_{hr}.$$

Under the initial location choice,

$$\phi_h(l_h^0) = z_{l_h^0} - c^0_{l_h^0}.$$

Therefore, using the resource constraint,

$$\int \phi_h(l_h^0)\, dh = \sum_r L_r^0 (z_r - c_r^0) = 0.$$

Since $\max_r \phi_h(r) \geq \phi_h(l_h^0)$ for every h, we have $F(1) \geq 0$. It remains to show that the inequality is strict. Without loss of generality, suppose $z_1 > z_2$. From (32),

$$c_1^0 - c_2^0 = \frac{\theta}{\theta + 1}(z_1 - z_2) > 0.$$

Consider agents initially in location 2. Initial choice of location 2 over location 1 requires

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}} \geq \frac{c_1^0}{c_2^0}.$$

For such agents,

$$\phi_h(2) = z_2 - c_2^0, \qquad \phi_h(1) = z_1 - c_2^0 \frac{\varepsilon_{h2}}{\varepsilon_{h1}}.$$

Thus $\phi_h(1) > \phi_h(2)$ if and only if

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}} < 1 + \frac{z_1 - z_2}{c_2^0}.$$

The interval

$$\left[\frac{c_1^0}{c_2^0},\ 1+\frac{z_1-z_2}{c_2^0}\right)$$

is nonempty because

$$1+\frac{z_1-z_2}{c_2^0}-\frac{c_1^0}{c_2^0}=\frac{z_1-z_2-(c_1^0-c_2^0)}{c_2^0}=\frac{z_1-z_2}{(\theta+1)c_2^0}>0.$$

Since $(\varepsilon_{h1},\varepsilon_{h2})$ has a continuous positive density, there is a positive-measure set of agents with

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}}\in\left[\frac{c_1^0}{c_2^0},\ 1+\frac{z_1-z_2}{c_2^0}\right).$$

For these agents, $l_h^0=2$ and $\phi_h(1)>\phi_h(2)=\phi_h(l_h^0)$. For all other agents, $\max_r \phi_h(r)\geq \phi_h(l_h^0)$. Therefore,

$$F(1)=\int \max_r \phi_h(r)\,dh>\int \phi_h(l_h^0)\,dh=0.$$

Since $F(x)$ is decreasing in x and satisfies $F(A)=0$, it follows that $A>1$. Hence, starting from the allocation in (32), there exists another feasible allocation, implemented with lump-sum transfers, that makes every household strictly better off. The case $z_2>z_1$ is symmetric. □

D.3 Invariance Property when Tastes are Fréchet

The Fréchet specification of utility does have an invariance property, stated below in Proposition 12. Specifically, when $\{\varepsilon_{hr}\}$ are assumed to be independently Fréchet distributed and $f_h=f$ for every h, then $A^W(t)$ is invariant to the choice of f. This result, however, follows from arbitrarily restricting the set of allowable monotone transformations of utility. For example, under the restrictions in Proposition 12, differences in the overall level of tastes across households are left unnormalized: some households have high values of ε_{hr} in every location, while others have low values in every location. For example, the cardinalizations discussed in Example 6 and Example 7 fall outside this restricted class.

Proposition 12 (Invariance to Restricted Monotone Transformations)**.** *Suppose that*

$$u_h(c_r,r)=f\big(\log c_r+\epsilon_{hr}\big),$$

where the same strictly increasing function f applies to every household and $\varepsilon_{hr}\equiv\exp(\epsilon_{hr})$ are *independent Fréchet random variables with shape parameter θ and location-specific scale parameters $\{B_r\}$:*

$$\Pr(\varepsilon_{hr} \le z) = \exp\left(-B_r^\theta z^{-\theta}\right), \qquad z > 0.$$

For any f for which average utility is finite,

$$A^W(t) = \left[\frac{\sum_r (B_r c_r(t))^\theta}{\sum_r (B_r c_r(0))^\theta}\right]^{1/\theta},$$

and hence $A^W(t)$ is invariant to the choice of f.

Proof. Let

$$v_h(\boldsymbol{c}) = \max_r\{\log c_r + \epsilon_{hr}\}.$$

Its distribution is

$$\Pr\big(v_h(\boldsymbol{c}) \le v\big) = \exp\left(-e^{-\theta v}\sum_r B_r^\theta c_r^\theta\right).$$

Hence,

$$\Pr\left(v_h\left(\frac{\boldsymbol{c}(t)}{A}\right) \le v\right) = \exp\left(-e^{-\theta v} A^{-\theta}\sum_r (B_r c_r(t))^\theta\right).$$

This distribution is identical to that of $v_h(\boldsymbol{c}(0))$ when

$$A^{-\theta}\sum_r (B_r c_r(t))^\theta = \sum_r (B_r c_r(0))^\theta.$$

The corresponding average utilities are therefore equal. Since average utility is strictly decreasing in A, this solution is unique. Solving for A gives

$$A^W(t) = \left[\frac{\sum_r (B_r c_r(t))^\theta}{\sum_r (B_r c_r(0))^\theta}\right]^{1/\theta}.$$

□

Appendix E Relation to the social surplus function

Consider the cardinalization of utility functions $f_h(x) = x$, which results in the following utilitarian social welfare function given a vector of real wages across locations $\boldsymbol{w}/\boldsymbol{p}^c$:

$$U(\boldsymbol{w}/\boldsymbol{p}^c) = \mathbb{E}[\max_{l_h} g(a_{hl_h} w_{l_h}/p^c_{l_h}) + \epsilon_{h,l_h}],$$

where $a_{hr}w_r/p^c_r$ is real consumption of agent h if location r is chosen. This social welfare function is called the "*Social Surplus Function*" in the discrete choice literature (see McFadden, 1981).[46]

The proposition below shows that under Assumption 1, the social surplus function can be used to calculate $A(t)$.

Proposition 13 (Using Social Surplus to Calculate A)**.** *Suppose Assumption 1 holds. Let $U(\boldsymbol{w}/p^c) = \mathbb{E}[\max_r a_{hr}w_r/p^c + \epsilon_{hr}]$ be the social surplus function evaluated at real wages $\boldsymbol{w}/p^c$. Then, aggregate welfare in terms of TFP-equivalents, $A(t)$, satisfies the following equation:*

$$U(\boldsymbol{w}(t)/(A(t)p^c(t))) = U(\boldsymbol{w}(0)/p^c(0)),$$

where $\boldsymbol{w}(t)/(A(t)p^c(t))$ are real wages in the decentralized equilibrium with productivities $\boldsymbol{z}(t)$ and aggregate factor-augmenting productivity $1/A(t)$ and $\boldsymbol{w}(0)/p^c(0)$ are real wages in the status quo. If, in addition, real wages do not depend on labor supplied in each region (e.g. as in the one-good economy), then we also have that $A(t) = A^U(t)$, where $A^U(t)$ is the consumption-equivalent variation defined in Equation (30) *for the utilitarian social welfare function $W = U$.*

The proof is at the end of this subsection. In words, $A(t)$ can be computed as the reduction in labor productivity in every location such that the social surplus function, with productivity shocks $\boldsymbol{z}(t)$, is equal to social surplus under the status quo (without the need to explicitly compute any compensating transfers). Assumption 1 is critical and if it does not hold, then the social surplus function U cannot be used to compute $A(t)$ (see Example 11 below).

Notably, even if Assumption 1 holds, $A(t)$ may still differ from the consumption-equivalent under the social surplus function, $A^U(t)$. This is because real wages with productivities $(\boldsymbol{z}(t), 1/A(t))$ are not the same as real wages with productivities $\boldsymbol{z}(t)$ divided by $A(t)$. Intuitively, a reduction in aggregate factor productivity can cause agents to switch locations, and if real wages are a function of labor supply, then this can cause real wages to change and $A(t)$ to be different than $A^U(t)$. The last part of the proposition points out that if, in addition to Assumption 1, real wages do not respond to changes in labor supplies, as in the one-good economy where they are pinned down by productivity parameters, then $A(t)$ coincides with $A^U(t)$.

Proposition 13 relates to a well-known property of discrete choice models where consumers choose among a discrete set of options given fixed prices and incomes. In that

[46]The social surplus function plays an important role in the literature in part because of the Williams-Daly-Zachary theorem (Williams, 1977; Daly and Zachary, 1978), which gives conditions under which the social surplus function acts like a potential function (i.e. its derivatives are equal to aggregate location choices).

literature, it is known that if the indirect utility function is linear in the price of the good, then the social surplus function can be used to calculate the sum of compensating variations (Small and Rosen, 1981).[47]

The example below illustrates Proposition 13 using a simple example.

Example 10 (Using Social Surplus to Calculate Aggregate Welfare). Consider the one-good economy example from Example 3 again. There is a single freely-traded consumption good produced linearly from labor and productivity in each location r is z_r. We impose Assumptions 1 and 2. The real wage per efficiency unit in each location is simply $w_r = z_r$. Hence, by Proposition 13, we have

$$U(\boldsymbol{z}(t)/A(t)) = \mathbb{E}[\max_r \frac{z_r(t)}{A(t)} + \epsilon_{hr}] = \mathbb{E}[\max_r z_r(0) + \epsilon_{hr}] = U(\boldsymbol{z}(0)).$$

If the ϵ_{hr} are independently drawn from Type I extreme value distributions, then

$$U(\boldsymbol{z}(t)/A(t)) = \frac{1}{\theta} \log \left[\sum_r \exp\left(\theta z_r(t)/A(t) + B_r\right) \right] + \text{constant}.$$

Therefore, since the logarithm is strictly increasing, $U(\boldsymbol{z}(t)/A(t)) = U(\boldsymbol{z}(0))$ if and only if

$$\sum_r \exp\left(\theta z_r(t)/A(t) + B_r\right) = \sum_r \exp\left(\theta z_r(0) + B_r\right),$$

which is equation (23). Furthermore, in this one-good economy, real wages in each location are equal to productivity and independent of labor supply. Hence, scaling all productivities by $1/A(t)$ is observationally equivalent to dividing all real wages by $A(t)$. This makes $A(t) = A^U(t)$.

Proposition 13 breaks down if Assumption 1 does not hold: i.e. if either $g(c)$ is nonlinear or if consumption prices vary by location. Intuitively, when Assumption 1 holds,

[47] In the absence of linearity, calculating the sum of compensating variations typically requires resorting to simulation methods. See Hortaçsu and Joo (2023), and the references therein, for a recent textbook discussion. As discussed by Hortaçsu and Joo (2023), many recent studies in the industrial organization literature ignore individual compensating variations and directly use the social surplus function as the starting point of their analysis, even if the quasi-linear justification for this approach does not hold. In this case, the social surplus function is interpreted like a social welfare function. Dubé et al. (2025) analyze the welfare properties of this social surplus function for a broad class of demand systems.

we have a transferable utility model where the sum of $g(c_h)$ across households is interpretable as an aggregate resource constraint and can be split across households freely using lump-sum transfers. However, if either $g(c)$ is nonlinear or agents in different locations consume different consumption goods, then utility is not transferable and the sum of $g(c_h)$ across households cannot be interpreted as an aggregate resource constraint.

The following example demonstrates that when Assumption 1 does not hold, as in the commonly studied Fréchet case, then $A^U(t)$ and $A(t)$ need not agree even to a first order approximation (and can even have different signs).

Example 11 (When Proposition 13 Fails). Consider Example 6 again. There is a common freely-traded consumption good produced linearly from labor, with productivity z_r in location r. Assumption 1 does not hold because $g(c)$ is logarithmic. Assume that $\exp(\epsilon)$ is distributed according to a Fréchet distribution, and workers have homogeneous skills. Utilitarian welfare, as measured by $A^U(t)$, is the solution to

$$U(\boldsymbol{z}(t)/A^U(t)) = \mathbb{E}[\max_r \log(z_r(t)/A^U(t)) + \epsilon_{hr}] = \mathbb{E}[\max_r \log(z_r(0)) + \epsilon_{hr}] = U(\boldsymbol{z}(0)).$$

One can show that $A^U(t) = \left(\sum_r (B_r z_r(t))^\theta / \sum_r (B_r z_r(0))^\theta\right)^{\frac{1}{\theta}}$.[48] To a first-order approximation, changes in $\log A^U$ are a population-weighted sum of changes in productivities:

$$\Delta \log A^U(t) \approx \sum_r L_r(0) \Delta \log z_r,$$

where $L_r(0)$ is the share of the population choosing location r at the status quo. This does not coincide, even to a first-order, with changes in aggregate welfare as measured by A, because $\Delta \log A \approx \sum_r \lambda_r(0) \Delta \log z_r$, where the Domar weights in this example are $\lambda_r(0) = L_r(0) z_r(0) / (\sum_{r'} L_{r'}(0) z_{r'}(0))$. It can easily be the case that $\Delta \log A > 0$, so that winners can compensate losers and have resources left over, and yet $\Delta \log A^U < 0$.

Proof of Proposition 13. Without loss of generality, let the common consumption good price

[48] Compare this with the right hand side of (31), which calculates $A^W(t)$ under the $f_h(x) = \exp(x)$ cardinalization. The fact that the consumption-equivalent variation of the social welfare function $U(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} \log(c_{l_h}(t)) + \epsilon_{hl_h}]$ is equal to that of the social welfare function $W(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} c_{l_h}(t) \exp(\epsilon_{hl_h})]$ is a special implication from the assumption that $\exp(\epsilon)$ is distributed according to a Fréchet distribution. See Proposition 12 in Appendix D for more details.

p^c be the numeraire. The social surplus function is

$$U(\boldsymbol{w}) = \mathbb{E}_h \left[\max_r \{a_{hr} w_r + \epsilon_{hr}\} \right],$$

where w_r is now the real wage in location r. Then,

$$\frac{\partial U}{\partial w_r} = \mathbb{E}_h \left[a_{hr} \mathbf{1} \left\{ r \in \arg\max_j (a_{hj} w_j + \epsilon_{hj}) \right\} \right] = L_r(\boldsymbol{w}).$$

Equation (35) in the proof of Theorem 2 states that with a common consumption good

$$\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A} \right] d \log \left[\frac{w_r}{A} \right] = \sum_r L_r^{\text{comp}} \left[\frac{w_r}{A} \right] d \log p^c,$$

where the wages and prices are the ones in the compensated equilibrium. Since the consumer price index is the numeraire, this is the same as:

$$\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A} \right] d \log \left[\frac{w_r}{A} \right] = 0.$$

Equivalently,

$$\sum_r L_r^{\text{comp}} d \left[\frac{w_r}{A} \right] = 0.$$

Integrating between 0 and t gives:

$$\int_{w(0)/p^c(0)}^{w(t)/(A(t))} \sum_r L_r^{\text{comp}}(\boldsymbol{x}) d\boldsymbol{x} = 0.$$

By Proposition 3, the compensated and uncompensated labor supply functions are the same. Furthermore, given productivities $\boldsymbol{z}(t)$ and aggregate factor-augmenting productivity $1/A(t)$, the compensated and uncompensated equilibrium real wages are the same. Hence, we can write

$$\int_{w(0)}^{w(t)/(A(t))} \sum_r L_r(\boldsymbol{x}) d\boldsymbol{x} = 0,$$

substituting in $\frac{\partial U}{\partial w_r}$ from above gives

$$\int_{w(0)}^{w(t)/(A(t))} \sum_r \frac{\partial U}{\partial w_r}(\boldsymbol{x}) d\boldsymbol{x} = 0,$$

which is equal to

$$U(w(t)/(A(t))) - U(w(0)) = 0,$$

by the fundamental theorem of calculus. Finally, if real wages do not depend on labor supply, real wages in the compensated equilibrium coincide with those in the decentralized equilibrium at t. The preceding equation is then precisely the equation defining $A^U(t)$, so $A(t) = A^U(t)$.

□

Appendix F Calculating $A(t)$ via differential equations

In this appendix we describe how to calculate $A(t)$ using the differential equations implied by Theorem 2 and Proposition 2, as discussed in Section 4.1. At each point along the path, the derivatives of the equilibrium variables are obtained by solving a linear system. We first present this system and then give a step-by-step algorithm.

System of equations

TFP-equivalent welfare

$$\frac{d\log A(t)}{dt} = \sum_i \lambda_i^{\text{comp}}(t)\,\frac{d\log z_i(t)}{dt}, \qquad A(0) = 1. \tag{49}$$

Input-output and prices

$$d\log\Omega_{ij} = (1-\theta_i)\begin{bmatrix} d\log p_j \mathbf{1}[j\in N] + d\log w_j \mathbf{1}[j\in R] \\ \quad - \sum_{k\in N}\Omega_{ik}\,d\log p_k - \sum_{r\in R}\Omega_{ir}\,d\log w_r \end{bmatrix}. \tag{50}$$

For entries of Ω that are always zero, including all entries in factor rows, we use the convention $d\log\Omega_{ij} = 0$.

$$d\log \boldsymbol{p} = -\Psi\, d\log \boldsymbol{z} + \Psi^F\, d\log \boldsymbol{w}, \tag{51}$$

where $\boldsymbol{p}$ and $\boldsymbol{z}$ contain the produced commodities $i \in C+N$, Ψ is the corresponding produced-commodity block of the Leontief inverse $(I-\Omega)^{-1}$, and Ψ^F is its factor-content block.

Compensated supply and demand derivatives Let

$$L_r^{\text{comp}} = L_r^{\text{comp}}\left(\tfrac{\boldsymbol{w}}{A}, \boldsymbol{p}^c, \boldsymbol{u}^0\right), \qquad \chi_r^{\text{comp}} = \chi_r^{\text{comp}}\left(\tfrac{\boldsymbol{w}}{A}, \boldsymbol{p}^c, \boldsymbol{u}^0\right),$$

and define

$$\eta_{r,s}^{L} = \frac{\partial \log L_r^{\text{comp}}}{\partial \log(w_s/A)}, \quad \eta_{r,s}^{L,p} = \frac{\partial \log L_r^{\text{comp}}}{\partial \log p_s^c},$$

$$\eta_{r,s}^{\chi} = \frac{\partial \chi_r^{\text{comp}}}{\partial \log(w_s/A)}, \quad \eta_{r,s}^{\chi,p} = \frac{\partial \chi_r^{\text{comp}}}{\partial \log p_s^c}.$$

Then

$$d\log L_r^{\text{comp}} = \sum_{s\in R} \eta_{r,s}^{L}\left(d\log w_s - d\log A\right) + \sum_{s\in R} \eta_{r,s}^{L,p}\, d\log p_s^c, \tag{52}$$

$$d\chi_r^{\text{comp}} = \sum_{s\in R} \eta_{r,s}^{\chi}\left(d\log w_s - d\log A\right) + \sum_{s\in R} \eta_{r,s}^{\chi,p}\, d\log p_s^c. \tag{53}$$

Factor Domar weights

$$d\log \lambda_r^{\text{comp}} = d\log w_r + d\log L_r^{\text{comp}} - \sum_{r'} \lambda_{r'}^{\text{comp}}\left[d\log w_{r'} + d\log L_{r'}^{\text{comp}}\right]. \tag{54}$$

Using (52), this can be written in terms of $d\log w_s$, $d\log p_s^c$, and $d\log A$.

Commodity market clearing

$$d\log \lambda_i^{\text{comp}} = (\lambda^{\text{comp}})_i^{-1}\left(\sum_{r\in R} d\chi_r^{\text{comp}}\mathbf{1}[i=c(r)] + \sum_{j\in C+N} d\lambda_j^{\text{comp}}\Omega_{ji} + \sum_{j\in C+N} \lambda_j^{\text{comp}} d\Omega_{ji}\right). \tag{55}$$

Here $d\chi_r^{\text{comp}}$ comes from (53), $d\Omega$ from (50), and $d\log\lambda_j^{\text{comp}}$ from (54)–(55). Given $d\log A$ from (49) and $d\log z_i$, the equations (50)–(51)–(52)–(53)–(54)–(55) form a linear system that determines

$$d\log p_i, \quad d\log w_r, \quad d\log \Omega_{ij}, \quad d\log \lambda_i^{\text{comp}}$$

up to the choice of a numeraire.

Algorithm

Step 0 (initialization, $t=0$)

i. Solve the decentralized equilibrium at $t=0$ to obtain

$$\boldsymbol{p}(0),\ \boldsymbol{w}(0),\ \boldsymbol{\Omega}(0),\ \boldsymbol{\lambda}(0).$$

ii. Fix the reference utilities $\boldsymbol{u}^0$ at their status quo levels and set compensated objects at $t=0$:

$$\lambda_i^{\text{comp}}(0)=\lambda_i(0),$$

and use the same $\boldsymbol{p}(0)$, $\boldsymbol{w}(0)$, $\boldsymbol{\Omega}(0)$.

iii. Set $A(0)=1$.

Step 1 (shocks and grid)

i. Specify a path $\boldsymbol{z}(t)$ and compute $d\log z_i(t)/dt$.

ii. Choose a time grid $0=t_0<\cdots<t_K=t$ with $\Delta t_k=t_{k+1}-t_k$.

Step 2 (for each $k=0,\ldots,K-1$) Given

$$\boldsymbol{p}(t_k),\ \boldsymbol{w}(t_k),\ \boldsymbol{\Omega}(t_k),\ \boldsymbol{\lambda}^{\text{comp}}(t_k),\ A(t_k),$$

proceed as follows.

2.1 Hulten step

Compute

$$\left.\frac{d\log A}{dt}\right|_{t_k}=\sum_i \lambda_i^{\text{comp}}(t_k)\ \left.\frac{d\log z_i}{dt}\right|_{t_k}.$$

2.2 Compensated demand and supply

i. Evaluate $L_r^{\text{comp}}(t_k)$ and $\chi_r^{\text{comp}}(t_k)$ at

$$\left(\frac{\boldsymbol{w}(t_k)}{A(t_k)},\boldsymbol{p}^c(t_k),\boldsymbol{u}^0\right)$$

using the compensated choice problem.

ii. Compute

$$\eta_{r,s}^{L},\ \eta_{r,s}^{L,p},\ \eta_{r,s}^{\chi},\ \eta_{r,s}^{\chi,p}$$

at that point (analytically or by finite differences).

iii. Use (52) and (53) to express $d\log L_r^{\text{comp}}$ and $d\chi_r^{\text{comp}}$ as linear functions of $d\log w_s$, $d\log p_s^c$ and the known $d\log A\big|_{t_k}$.

2.3 General equilibrium

i. Use $p_r^c = p_{c(r)}$, and plug (52), (53), (50), and (51) into (54) and (55).

ii. This yields a linear system for

$$\frac{d\log p_i}{dt}, \quad \frac{d\log w_r}{dt}, \quad \frac{d\log \Omega_{ij}}{dt}, \quad \frac{d\log \lambda_i^{\text{comp}}}{dt}\bigg|_{t_k},$$

where $\left(d\log A/dt\right)\big|_{t_k}$ and $\left(d\log z_i/dt\right)\big|_{t_k}$ are known. Solve this system.

2.4 Time update

For any variable $x \in \{p_i, w_r, \Omega_{ij}, \lambda_i^{\text{comp}}, A\}$,

$$\log x(t_{k+1}) = \log x(t_k) + \frac{d\log x}{dt}\bigg|_{t_k} \Delta t_k.$$

Repeat until t_K.

Appendix G Aggregate welfare with distortions and externalities

In the body of the paper, we consider perfectly competitive discrete choice models. In this appendix we discuss how Theorem 1 extends beyond the perfectly competitive baseline. We first introduce wedge distortions and then agglomeration externalities.

Wedge distortions

The household block of the model is unchanged. Each agent chooses consumption and location to maximize utility given prices, wages, and transfers, where transfers now include revenues from wedges (e.g. tax revenues). Aggregating individual choices gives rise to labor supply and final demand functions, $L_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})$ and $\chi_r(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})$.

Producers choose quantities to minimize costs, taking prices as given. The price of good i is now given by

$$p_i = \mu_i z_i^{-1} \mathrm{mc}_i(\boldsymbol{p}, \boldsymbol{w}), \tag{56}$$

where $\mu_i > 0$ is a tax, markup, or any other wedge between price and marginal cost for producer of good i.[49] Lump sum transfers across households must equal total revenues generated by the wedges,

$$\int T_h dh = \sum_i p_i y_i (1 - \frac{1}{\mu_i}). \tag{57}$$

In the decentralized equilibrium, transfers are distributed across households according to some rule.

To solve for a decentralized equilibrium with wedges, we define the ratio of sales, factor payments, and spending to GDP as

$$\lambda_i = \frac{p_i y_i}{GDP} \mathbf{1}[i \in C + N] + \frac{Z w_i L_i}{GDP} \mathbf{1}[i \in R], \qquad \text{and} \qquad \chi_r = \frac{Z w_r L_r + \int T_h \mathbf{1}[l_h = r] dh}{GDP},$$

where GDP (or aggregate income) is given by

$$GDP = \sum_r Z w_r L_r + \sum_i p_i y_i (1 - \frac{1}{\mu_i}).$$

Commodity market clearing continues to satisfy (6). Factor market clearing instead requires

$$\lambda_r = \frac{Z w_r L_r (Z \boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{T})}{GDP}. \tag{58}$$

Given wedges, some transfer rule, and the aggregate supply and demand functions, equilibrium prices, wages, and quantities satisfy (6), (58), (56), and the GDP identity above.

We parameterize technologies and wedges at t by $\boldsymbol{z}(t)$ and $\boldsymbol{\mu}(t)$. Define the set of feasible allocations given productivity shifters $\boldsymbol{z}(t)$, wedges $\boldsymbol{\mu}(t)$, and aggregate factor-augmenting productivity Z to be

$$\mathcal{C}(t, Z) \equiv \{\{\boldsymbol{c}_h, l_h\}_h \text{ supported via equilibrium with wedges given } \boldsymbol{z}(t), \boldsymbol{\mu}(t), Z, \text{ and some transfers.}\}$$

The change in productivity $A(t)$ is defined exactly as in Equation (11). In response to a change in technologies and wedges, we consider a hypothetical rescaling of Z by a factor $1/A(t)$, and define $A(t)$ as the largest contraction such that every agent can be made at

[49] Even though we assume that wedges are on gross output only, this is without loss of generality. This is because we can recreate buyer-seller wedges by treating firm i's purchases of an input from j as a distinct good (made linearly using j's output). A wedge on this good is then isomorphic to a buyer-seller wedge.

least as well off as in the initial equilibrium. Misallocation due to distortions in the status quo can be quantified by setting $\boldsymbol{\mu}(t) = 1$ and $\boldsymbol{z}(t) = \boldsymbol{z}(0)$.

We can calculate $A(t)$ using a compensated equilibrium with wedges, extending the results in Section 3.2. In the compensated equilibrium with wedges, every agent receives a transfer that keeps them indifferent, the sum of transfers equals the sum of wedge revenues, and TFP is contracted by $A(t)$. Compensated choices, $l_h^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ and the expenditure function, $e_h(Z\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ are obtained using Proposition 1, and these individual choices are aggregated to get location-level variables, $L_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0)$, $E_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0)$, and $\chi_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}^c, \boldsymbol{u}^0)$, exactly as in the baseline model without wedges. The generalization of Theorem 1 is:

Theorem 3 (Aggregate Welfare)**.** *TFP-equivalent aggregate welfare satisfies*

$$\sum_r E_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0) = \frac{1}{1 - \sum_i \lambda_i^{comp}(1 - \frac{1}{\mu_i})} \sum_r (w_r/A(t)) L_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0). \tag{59}$$

Given compensated labor supply $\boldsymbol{L}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0)$ and demand $\boldsymbol{\chi}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}^c, \boldsymbol{u}^0)$, the vector of commodity prices $\boldsymbol{p}$ in the compensated equilibrium satisfies (56)*, with $p_r^c = p_{c(r)}$, and commodity market clearing satisfies* (6)*. Factor shares satisfy*

$$\lambda_r^{comp} = \frac{(w_r/A(t)) L_r^{comp}}{\sum_s E_s^{comp}}. \tag{60}$$

To prove this result, we follow the same steps in the proof of Theorem 1, except for condition (57) that the sum of transfers is not equal to zero but equal to total revenues from wedges.

Introducing agglomeration externalities

We now briefly discuss how to account for agglomeration externalities. For simplicity, we only consider local externalities determined by the workers located in the location where the good is produced, but it is straightforward to also allow for spillovers between regions. Suppose that the production function of good i produced in region r is given by

$$y_i = \tilde{z}_i F_i\left(\{x_{ij}\}_{j \in N}, \{L_{ir}\}_{r \in R}\right), \tag{61}$$

where the Hicks-neutral productivity shifter is

$$\tilde{z}_i = z_i L_r^{\gamma_i}.$$

Producers take $\tilde{z}_i$ as given when minimizing costs. If $\gamma_i > 0$, then there is a positive externality from a larger mass of agents, weighted by their skills exclusive of the aggregate TFP shifter Z, in location r on the production of good i. If $\gamma_i < 0$, then there is a congestion externality from a larger mass of agents in location r on the production of good i.

For example, suppose each location produces output using labor only according to

$$y_r = Z z_r L_r^{1+\gamma_r} = Z \tilde{z}_r L_r.$$

The equilibrium wage per efficiency unit is equal to the private marginal product of an efficiency unit of labor,

$$w_r = p_r z_r L_r^{\gamma_r} = p_r \tilde{z}_r,$$

where producers take prices and $\tilde{z}_r$ as given. When solving for equilibrium and for $A(t)$, we must take into account how changes in labor supply affect productivities $\tilde{\boldsymbol{z}}$.

Theorem 1 remains unchanged, with $\tilde{\boldsymbol{z}}$ in place of $\boldsymbol{z}$, and with the additional constraint

$$\tilde{z}_i = z_i \left(L_r^{\text{comp}} \right)^{\gamma_i}. \tag{62}$$

If there are multiple compensated equilibria, choose the one that implies the highest value of $A(t)$. The solution algorithm described after Theorem 1 remains unchanged, with $\tilde{\boldsymbol{z}}$ in place of $\boldsymbol{z}$ and where each iteration must also impose condition (62).

We illustrate for the one-good economy of Example 2. TFP-equivalent aggregate welfare $A(t)$ is given by a modified version of (14):

$$A(t) = \frac{\sum_r \int \tilde{z}_r a_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh}, \tag{63}$$

where compensated location choices satisfy:

$$l_h^{\text{comp}} \in \arg\max_{r \in R} \left\{ \tilde{z}_r a_{hr} / A(t) - \bar{c}_{hr} \right\},$$

and

$$\tilde{z}_r = z_r(t) \left(L_r^{\text{comp}} \right)^{\gamma_r}, \qquad \text{where} \qquad L_r^{\text{comp}} = \int a_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh.$$

Appendix H Aggregate welfare with place-based compensation

In the body of the paper, we measure aggregate welfare by the amount of factor endowments that can be saved after winners compensate losers using lump-sum transfers. In this appendix, we briefly discuss how to extend this definition of aggregate welfare to cases where individual-level lump-sum transfers are not available, but place-based redistributive policies are.

Suppose that instead of individual-specific transfers, location-level consumption taxes, denoted by τ_r, can be levied on households that choose location r. In this case, the budget constraint of agent h is now:

$$\sum_r (1+\tau_r) p_r^c c_h \mathbf{1}\left[l_h = r\right] = Z \sum_r w_r a_{hr} \mathbf{1}\left[l_h = r\right] + T,$$

where T is a uniform lump-sum rebate (the same for all households). Budget balance requires that

$$T = \int \sum_r \tau_r p_r^c c_h \mathbf{1}\left[l_h = r\right] dh. \tag{64}$$

All other conditions defining equilibrium are the same as in Section 2. We can now define the feasible set of allocations that can be supported using these place-based tax instruments:

$$\mathcal{C}^{pb}(t, Z) \equiv \left\{ \{c_h, l_h\}_{h \in H} \text{ supported via equilibrium given } \boldsymbol{z}(t),\, Z, \text{ and some consumption taxes} \right\}.$$

Given this feasible set, the definition of TFP-equivalent aggregate welfare with place-based compensation is the same as before, with $\mathcal{C}(t, Z)$ replaced by $\mathcal{C}^{pb}(t, Z)$.

Definition 7. TFP-equivalent aggregate welfare with place-based compensation at t is

$$A^{pb}(t) = \max \left\{ Z^{-1} : \{c_h, l_h\}_{h \in H} \in \mathcal{C}^{pb}(t, Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h \right\} \tag{65}$$

where $c_h(0)$ and $l_h(0)$ are the consumption and location of h in the status quo.

To see how this works in practice, consider the one-good economy of Example 2. We focus on the one-good economy for simplicity, but the first-order result in Proposition 14 extends to economies with multiple consumption goods.

Example 12 (One-Good Economy with Place-Based Policies). Consider the one-good economy again: there is a single freely traded consumption good produced linearly from labor, and productivity in each location r is $z_r(t)$. Production in location r is

$$y_r = Zz_r(t)L_r = Zz_r(t) \int a_{hr}\mathbf{1}[l_h = r]\, dh.$$

The following shows that, in the one-good economy, $A^{pb}(t)$ is bounded above by $A(t)$ but that, to a first-order approximation, the two are the same.

Proposition 14 (Hulten's Theorem for One-Good Economy). *In the one-good economy, we have*

$$A^{pb}(t) \leq A(t) \quad \textit{for all } t.$$

Moreover, around the status quo $t = 0$, the two measures coincide to a first-order approximation and both obey Hulten's theorem:

$$\Delta \log A^{pb} \approx \Delta \log A \approx \sum_r \lambda_r(0)\, \Delta \log z_r.$$

The first-order result extends to economies with multiple consumption goods. The no-reallocation allocation provides a similar lower bound. The upper bound must instead use a version of $A(t)$ defined over all technologically feasible allocations, because the second welfare theorem need not hold and $A(t)$ considers only allocations that can be supported as equilibria with transfers. Both bounds obey Hulten's theorem to a first order around the laissez-faire status quo, so the same sandwich argument applies.

Before providing a formal proof (below), we provide a heuristic discussion of the non-linear problem. For this discussion, we assume homogeneous skills, $a_{hr} = 1$ for every h and r, so that all households choosing a given location have the same consumption. To calculate A^{pb}, rearrange the budget-balance condition, (64), as

$$Z^{-1} = \frac{\sum_r w_r L_r}{\sum_r p^c c_r L_r} = \frac{\sum_r z_r L_r}{\sum_r c_r L_r}, \tag{66}$$

where c_r denotes per-capita consumption in location r, and we use the fact that the real wage in location r is equal to $z_r(t)$ in this one-good economy. Using equation (65) and (66), we can write

$$A^{pb}(t) = \max_{\boldsymbol{c}} \left\{ \frac{\sum_r z_r(t) L_r(\boldsymbol{c})}{\sum_r c_r L_r(\boldsymbol{c})} : u_h(c_r, l_h(\boldsymbol{c})) \geq u_h^0 \text{ for every } h \right\},$$

where $L_r(\boldsymbol{c})$ is labor supply in location r as a function of the vector of location-level per capita consumption $\boldsymbol{c}$.

If, in the solution to the problem above, for every location $r \in R$, some agent $h \in H$ stays in the same location as in the status-quo and all locations are non-empty, then compensating the stayers requires ensuring that consumption per capita in each location is at least as high as in the status quo. Hence, we can replace the H inequalities above with just R inequalities:

$$A^{pb}(t) = \max_{\boldsymbol{c}} \left\{ \frac{\sum_r z_r(t) L_r(\boldsymbol{c})}{\sum_r c_r L_r(\boldsymbol{c})} : c_r \geq c_r^0 \text{ for every } r \right\}.$$

This is a relatively simple constrained maximization problem given the supply function.

To make it more concrete, suppose that $\boldsymbol{L}(\boldsymbol{c})$ is the isoelastic labor supply function:

$$L_r(\boldsymbol{c}) = \frac{c_r^\theta}{\sum_{r'} c_{r'}^\theta}.$$

Then, it is straightforward to show that the solution to $A^{pb}(t)$ satisfies:

$$c_r^{pb}(t) = z_r(0)\mathbf{1}\left[\frac{z_r(t)/z_r(0)}{A^{pb}(t)} \leq \frac{\theta+1}{\theta}\right] + \frac{\theta}{\theta+1}\frac{z_r(t)}{A^{pb}(t)}\mathbf{1}\left[\frac{z_r(t)/z_r(0)}{A^{pb}(t)} > \frac{\theta+1}{\theta}\right].$$

If the shocks are sufficiently small, $\frac{z_r(t)/z_r(0)}{A^{pb}(t)} \leq \frac{\theta+1}{\theta}$ for every r, then the solution sets consumption in each location equal to its status quo value $c_r^{pb}(t) = z_r(0)$. This is achieved by scaling aggregate factor productivity by $1/A^{pb}(t)$ and setting consumption taxes appropriately. Since consumption is equal to its status-quo value in every location, no agent switches locations, and hence every agent is exactly indifferent to the status-quo.

However, if region r experiences much faster productivity growth than average — in the sense that $\frac{z_r(t)/z_r(0)}{A^{pb}(t)} > \frac{\theta+1}{\theta}$ — then consumption in region r must exceed its status-quo level to incentivize additional workers to move there. In this case, stayers in region r are strictly better off than in the status quo. Given the optimal choices $\boldsymbol{c}^{pb}(t)$ and the implied labor supplies $L_r(\boldsymbol{c}^{pb}(t))$, the place-based aggregate welfare measure $A^{pb}(t)$ is obtained from (66).

Table 5 reports A^{pb} for the same calibrated economy as in Example 5. For a 1% productivity increase, A, A^{pb}, and the shocked location's sales share are nearly identical, consistent with the first-order result in Proposition 14. For a larger 10% increase, nonlinearities become more visible: the elasticity of A exceeds the sales share, as discussed in Example 4. Moreover, A^{pb} is slightly lower than A, as implied by the inequality in

Proposition 14.

Table 5: Aggregate welfare with lump-sum versus place-based compensations

Location of shock	Sales share λ_r	1 percent increase $\Delta \log A$	1 percent increase $\Delta \log A^{pb}$	10 percent increase $\Delta \log A$	10 percent increase $\Delta \log A^{pb}$
San Francisco–Oakland, CA	0.031	0.031	0.031	0.035	0.032
New York, NY	0.057	0.057	0.057	0.064	0.059
Seattle–Tacoma, WA	0.024	0.024	0.024	0.027	0.025
Brownsville, TX	0.001	0.001	0.001	0.001	0.001
McAllen, TX	0.002	0.002	0.002	0.003	0.002
Daytona Beach, FL	0.001	0.001	0.001	0.001	0.001

Notes: Entries in the $\Delta \log A$ and $\Delta \log A^{pb}$ columns are elasticities, computed as the log change in the welfare measure divided by the size of the productivity shock, 0.01 or 0.10. A^{pb} restricts compensation to place-based taxes and a common rebate. Sales share is λ_r in the initial equilibrium. Results use $\theta = 1.5$ and 400,000 simulated households.

The proof of Proposition 14 makes use of the following lemma:

Lemma 1. *In the one-good economy, for any time-invariant skills a_{hr} and preferences $u_h(c_h, l) = f_h(g(c_h) + \epsilon_{hl})$, the problem where aggregate welfare is calculated by choosing feasible locations agent by agent,*

$$\tilde{A} = \max_{\boldsymbol{l}} \frac{\sum_r \int z_r a_{hr} \mathbf{1}[l_h = r]\, dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh}, \tag{67}$$

yields the same value as aggregate welfare with household-specific lump-sum transfers: $\tilde{A} = A$.

Proof of Lemma 1. Let $\tilde{A}$ be the value of the maximization problem in (67), and denote by $\tilde{\boldsymbol{l}}$ the corresponding optimal location profile. By Dinkelbach (1967), $\tilde{\boldsymbol{l}}$ also solves the alternative problem

$$\max_{\boldsymbol{l}} \left[\sum_r \int \frac{z_r a_{hr}}{\tilde{A}} \mathbf{1}[l_h = r]\, dh - \sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh \right],$$

or equivalently

$$\max_{\boldsymbol{l}} \sum_r \int \left(\frac{z_r a_{hr}}{\tilde{A}} - \bar{c}_{hr} \right) \mathbf{1}[l_h = r]\ dh.$$

Given $\tilde{A}$, this problem is separable across h, so it can be solved household by household:

$$\tilde{l}_h \in \arg\max_{r \in R} \left\{ \frac{z_r a_{hr}}{\tilde{A}} - \bar{c}_{hr} \right\}.$$

By Proposition 1, the compensated location choice of household h when aggregate factor productivity is $1/\tilde{A}$ is

$$l_h^{\text{comp}} \in \arg\max_{r \in R} \left\{ \frac{z_r a_{hr}}{\tilde{A}} - \bar{c}_{hr} \right\},$$

so $\tilde{l}_h = l_h^{\text{comp}}$ for all h. By definition, the value of A that supports this compensated allocation is precisely $A = \tilde{A}$, as claimed. □

Proof of Proposition 14. The first part is a consequence of Lemma 1. Consider any place-based compensated equilibrium with aggregate factor productivity $Z = 1/A^{pb}(t)$ and location profile $\boldsymbol{l}$. Since every household is at least as well off as in the status quo, its consumption in its chosen location is at least $\bar{c}_{hr}$. Resource feasibility therefore implies

$$\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh \leq \frac{1}{A^{pb}(t)} \sum_r \int z_r(t) a_{hr} \mathbf{1}[l_h = r]\, dh.$$

Hence, $A^{pb}(t)$ cannot exceed the value of the feasible agent-by-agent location problem in (67). By Lemma 1, that value is $A(t)$. Therefore,

$$A^{pb}(t) \leq A(t) \quad \text{for all } t.$$

For the second part, we construct an explicit feasible place-based policy that yields a lower bound for $A^{pb}(t)$ and then apply a squeezing argument. Let

$$L_r^0 = \int a_{hr} \mathbf{1}[l_h(0) = r]\, dh$$

denote the efficiency units of labor supplied in location r in the status quo. Define

$$A^{lb}(t) \equiv \frac{\sum_r z_r(t) L_r^0}{\sum_r z_r(0) L_r^0}. \tag{68}$$

Normalize the common consumption price to one. Consider the following policy. Set the common rebate to zero, scale aggregate factor productivity by $1/A^{lb}(t)$, and choose the consumption tax in each location so that

$$1 + \tau_r^{lb} = \frac{z_r(t)/A^{lb}(t)}{z_r(0)} \quad \text{for every } r. \tag{69}$$

Under this policy, the consumption of household h in location r is

$$c_{hr}^{lb}(t) = \frac{a_{hr} z_r(t) / A^{lb}(t)}{1 + \tau_r^{lb}} = a_{hr} z_r(0) = c_{hr}(0).$$

Thus, every household faces exactly its status-quo consumption in every location. The household's entire menu is unchanged, so every household is indifferent to the status quo and location choices remain at their status-quo values. Moreover, aggregate tax revenue is

$$\sum_r \tau_r^{lb} z_r(0) L_r^0 = \frac{\sum_r z_r(t) L_r^0}{A^{lb}(t)} - \sum_r z_r(0) L_r^0 = 0,$$

where the last equality follows from (68). Hence, the policy satisfies budget balance with a zero common rebate.

Since this policy keeps everyone indifferent to the status quo and is implementable with place-based consumption taxes, it is a feasible solution for the problem that defines $A^{pb}(t)$. Therefore,

$$A^{pb}(t) \geq A^{lb}(t) \quad \text{for all } t.$$

For the no-reallocation allocation, the efficiency units $\{L_r^0\}$ are fixed, so the lower bound is the standard aggregate productivity index $A^{lb}(t) = \sum_r z_r(t) L_r^0 / \sum_r z_r(0) L_r^0$. Its derivative at the status quo satisfies

$$\frac{d}{dt} \log A^{lb}(t)\Big|_{t=0} = \sum_r \lambda_r(0) \frac{d}{dt} \log z_r(t)\Big|_{t=0}, \tag{70}$$

where $\lambda_r(0) = z_r(0) L_r^0 / \sum_{r'} z_{r'}(0) L_{r'}^0$. On the other hand, the full-transfer measure $A(t)$ also satisfies Hulten's theorem at the status quo (by Corollary 1). So,

$$\frac{d}{dt} \log A(t)\Big|_{t=0} = \sum_r \lambda_r(0) \frac{d}{dt} \log z_r(t)\Big|_{t=0}. \tag{71}$$

Combining $A^{lb}(t) \leq A^{pb}(t) \leq A(t)$ for all t with $A^{lb}(0) = A^{pb}(0) = A(0) = 1$, we have for $t > 0$,

$$\frac{\log A^{lb}(t) - \log A^{lb}(0)}{t} \leq \frac{\log A^{pb}(t) - \log A^{pb}(0)}{t} \leq \frac{\log A(t) - \log A(0)}{t}.$$

Using (70) and (71), the left and right terms converge to the same limit as $t \downarrow 0$. By the squeeze theorem, the middle term must converge to the same limit. Applying the same argument for $t < 0$, with the inequalities reversed after division by t, establishes the corresponding left derivative. Hence $\log A^{pb}(t)$ is differentiable at $t = 0$ and

$$\frac{d}{dt} \log A^{pb}(t)\Big|_{t=0} = \sum_r \lambda_r(0) \frac{d}{dt} \log z_r(t)\Big|_{t=0}.$$

Rewriting this in terms of finite changes yields the first-order approximation in the statement of the proposition. $\square$

Appendix I Further Discussion of Sum of Compensating Variations and Kaldor-Hicks efficiency

In this appendix, we discuss aggregate welfare as measured by the sum of compensating variations. First, we provide a worked-out example showing that the sum of compensating variations can be strictly positive in response to pure transfers. Second, we discuss an alternative but very closely related definition of Kaldor-Hicks efficiency that coincides with $A(t)$ when prices and wages do not respond to transfers.

I.1 Sum of Compensating Variations Rises in Response to Transfers

The following example demonstrates that a pure transfer, which moves the allocation along the Pareto frontier, can nevertheless cause $S(t)$ to rise.

Example 13 (Redistribution raises Kaldor-Hicks efficiency). Assume that consumption in each location is a Cobb-Douglas aggregator of inputs from different locations:

$$y_{c(r)} = \int c_h \mathbf{1}[l_h = r] dh = x_{rr}^{\alpha} \prod_{r' \in R} x_{rr'}^{(1-\alpha)/R},$$

where $0 \leq \alpha < 1$ controls the degree of home bias in consumption. Output in location r is produced one-for-one from labor:

$$\sum_{r'} x_{r'r} = L_r.$$

Workers' utility functions are

$$u_h(c_h, l_h) = f_h(c_h + \epsilon_{hl_h}),$$

where f_h is any strictly increasing function. Consider an economy with two symmetric locations and suppose that $\epsilon_{h1} = 1$ and the ϵ_{h2} are i.i.d. uniform random variables on the interval $[1-d, 1+d]$, where d controls the elasticity of location choices to changes in real consumption. This elasticity falls to zero as d rises because as d rises, taste dispersion grows, and the measure of households that are close enough to indifferent between locations for a given wage change goes to zero.

The status quo is a symmetric equilibrium without transfers. Consider a lump-sum transfer from households in location 2 to location 1. That is, agents that chose location 1 in the status quo, $l_h(0) = 1$, receive a lump-sum transfer of T_1. The transfer is financed by a lump-sum tax on agents that chose location two in the status quo, $l_h(0) = 2$. Budget balance requires the transfer received by these agents to be $T_2 = -L_1(0)/L_2(0)T_1$.

We solve for the post-shock equilibrium at t and calculate the Kaldor-Hicks efficiency measure, $S(t)$. Figure 1 reports $S(t)$ (with nominal GDP as the numeraire) for different levels of worker mobility (by varying the parameter d). We measure mobility by the mass of agents that start in location 2 (for whom $l_h(0) = 2$) and move to location 1 after the redistribution (for whom $l_h(t) = 1$).[50]

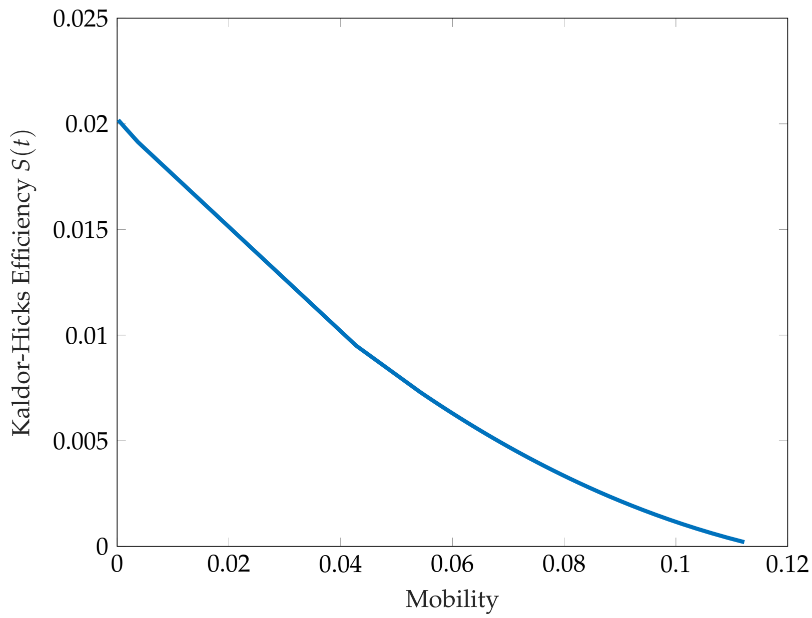
Figure 1 shows that Kaldor-Hicks efficiency, $S(t)$, rises in response to a pure redistribution. In contrast, our measure of aggregate welfare in terms of TFP equivalents is unchanged, $A(t) = A(0) = 1$ for every value of mobility and preference parameters. Pure transfers move the economy along the Pareto frontier; by definition of $A(t)$, they do not change the factor-saving amount needed to keep everyone indifferent.

The reason $S(t)$ is non-zero for pure redistributions is because redistributions change relative wages and prices in general equilibrium. Since $S(t)$ holds wages and prices constant in the compensation, undoing the transfers does not necessarily result in $S(t) = 0$. The magnitude of $S(t)$ depends on parameters — two important parameters are the extent of mobility and home bias. For example, if households do not have heterogeneous tastes, then real wages are equated in the two locations by arbitrage. Since productivities are symmetric, this implies that relative wages and prices are also equated in equilibrium. In this limit, the transfer does not alter relative wages and prices, and so $S(t)$ tends to 0 as we increase mobility.

Similarly, in the absence of home bias ($\alpha = 0$), $S(t) = 0$. Household h's consump-

[50]In our numerical example, we set the transfer to be 10% of wages in the status quo $T_1 = 0.1w_1(0)$.

Figure 1: Kaldor-Hicks Efficiency as a function of mobility for a pure transfer



tion in location r is $(w_r + T_h)/p_r^c$, and its location choice depends on $(w_r + T_h)/p_r^c + \epsilon_{hr}$. Without home-bias, the price of consumption in the two locations is the same: $p_1^c = p_2^c$. Hence, because Assumption 1 holds, the transfer T_h does not change h's location choice. That is, $\boldsymbol{L}(t) = \boldsymbol{L}(0)$. Furthermore, since there is no home-bias, agents in both locations spend their income in the same proportion, so that $w_i L_i = 1/2$ (recall that GDP is the numeraire). Since location choices do not respond to the transfers, wages also do not respond to the transfer. Since wages do not respond to the transfers, prices do not respond to the transfers. Since prices and wages do not respond to the redistribution, $S(t) = 0$ in this limit because undoing the redistribution at the new prices is equivalent to undoing it at the old prices. Hence, in this limit, the compensating-variation based $S(t)$ correctly detects that a pure transfer does not alter efficiency.

In Example 13, the pure redistribution is due to lump-sum transfers. However, the same logic applies when redistributions are caused by productivity shocks.[51] These ex-

[51]For example, to generate a pure redistribution using productivity shocks, suppose productivity rises for some agents and falls for others and the labor of these agents is perfectly substitutable in production. Hence, the consumption possibility set of the economy does not change, but some agents gain and some agents lose. In this case, if agents consume different goods, the productivity shocks will alter relative prices and the sum of compensating variations is positive. This is because the winners can compensate the losers at post-shock prices and have money left over. Intuitively, relative prices fall for goods more valued by losers, so they can be compensated more cheaply using post-shock prices.

amples illustrate that Kaldor-Hicks efficiency is a poor measure of efficiency in environments where prices and wages are endogenous to the distribution of income and expenditures.

I.2 Alternative Definition of Kaldor-Hicks Efficiency

In Section 6.3 we defined Kaldor-Hicks efficiency as

$$S(t) \equiv -\int_h e_h(\boldsymbol{w}(t), \boldsymbol{p}^c(t), u_h^0)\, dh$$

where

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = \min \left\{ T_h : u_h(c_h, l_h) \geq u_h^0, \sum_r p_r^c c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h \right\}.$$

The following lemma shows that $S(t)$ is equal to the maximum difference between total income and the sum of compensating incomes (minimum income to attain indifference with the status quo), evaluated at t equilibrium wages and prices.

Lemma 2 (Equivalent definition of $S(t)$)**.**

$$S(t) = \max_{\{(c_h, l_h)\}_{h \in H}} \left\{ \int_h \left[w_{l_h}(t)\, a_{hl_h} - p_{l_h}^c(t)\, c_h \right] dh : \ u_h(c_h, l_h) = u_h^0 \ \forall h \right\}.$$

Proof. The minimizer in the definition of $e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0)$ must satisfy $u_h(c_h, l_h) = u_h^0$. Thus

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = \min_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \left\{ T_h : \sum_r p_r^c c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h \right\}.$$

For any (c_h, l_h), the budget constraint implies $T_h \geq p_{l_h}^c c_h - w_{l_h} a_{hl_h}$, so at the minimum $T_h = p_{l_h}^c c_h - w_{l_h} a_{hl_h}$. Hence

$$e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = \min_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \{ p_{l_h}^c c_h - w_{l_h} a_{hl_h} \},$$

and therefore

$$-e_h(\boldsymbol{w}, \boldsymbol{p}^c, u_h^0) = \max_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \{ w_{l_h} a_{hl_h} - p_{l_h}^c c_h \}.$$

Summing over h,

$$S(t) = \int_h \max_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \left\{ w_{l_h}(t) a_{hl_h} - p_{l_h}^c(t) c_h \right\} dh.$$

Because choices and payoffs are additively separable across households, maximizing the integral over all feasible profiles $\{(c_h, l_h)\}_{h \in H}$ (i.e. those that satisfy $u_h(c_h, l_h) = u_h^0 \ \forall h$) is equivalent to choosing, for each h, a maximizer of the integrand, giving the stated result. □

Consider now an alternative definition of Kaldor-Hicks efficiency as the maximum *ratio* — rather than the difference in terms of the numeraire — between aggregate income and the sum of compensating incomes, at t wages and prices:

$$A^{KH}(t) \equiv \max_{\{(c_h, l_h)\}_{h \in H}} \left\{ \frac{\int_h w_{l_h}(t)\, a_{hl_h}\, dh}{\int_h p_{l_h}^c(t)\, c_h\, dh} : \ u_h(c_h, l_h) = u_h^0 \ \forall h \right\}.$$

Note that, in general, the allocations that maximize $S(t)$ do not coincide with those that maximize $A^{KH}(t)$. However, the following proposition shows that Kaldor-Hicks efficiency rises (i.e. $S(t) > 0$) if and only if $A^{KH}(t) > 1$. Therefore, $A^{KH}(t)$ inherits the same undesirable properties in general equilibrium as $S(t)$. For example, as discussed in Section 6.3, pure transfers cause Kaldor-Hicks efficiency to rise.

Proposition 15 (Equivalence in sign between $S(t)$ and $A^{KH}(t)$.)**.**

$$\operatorname{sign}\big(A^{KH}(t) - 1\big) = \operatorname{sign}\big(S(t)\big).$$

Proof. We can write

$$\begin{aligned}
\int_h \left[w_{l_h}(t)\, a_{hl_h} - p_{l_h}^c(t)\, c_h \right] dh &= \int_h w_{l_h}(t)\, a_{hl_h}\, dh - \int_h p_{l_h}^c(t)\, c_h\, dh \\
&= \left(\int_h p_{l_h}^c(t)\, c_h\, dh \right) \left(\frac{\int_h w_{l_h}(t)\, a_{hl_h}\, dh}{\int_h p_{l_h}^c(t)\, c_h\, dh} - 1 \right).
\end{aligned}$$

Because prices and compensating consumption expenditures are positive, $\int_h p_{l_h}^c(t)\, c_h\, dh > 0$ for every admissible profile. Thus, the sign of the left-hand side is the same as the sign of the term in parentheses. For every admissible profile, the sign of

$$\int_h \left[w_{l_h}(t)\, a_{hl_h} - p_{l_h}^c(t)\, c_h \right] dh$$

coincides with the sign of

$$\frac{\int_h w_{l_h}(t)\, a_{hl_h}\, dh}{\int_h p^c_{l_h}(t)\, c_h\, dh} - 1.$$

By Lemma 2, $S(t)$ is the maximum of the former over all feasible allocations, while $A^{KH}(t)$ is the maximum of the latter over all feasible allocations. Since the sign equivalence holds allocation by allocation and both $S(t)$ and $A^{KH}(t)$ are maxima over the same feasible set, it follows that

$$S(t) > 0 \iff A^{KH}(t) > 1, \quad S(t) = 0 \iff A^{KH}(t) = 1, \quad S(t) < 0 \iff A^{KH}(t) < 1,$$

which is equivalent to $\mathrm{sign}(A^{KH}(t) - 1) = \mathrm{sign}(S(t))$. $\square$

The virtue of defining Kaldor-Hicks as a ratio, rather than a difference, is that when real wages are pinned by technologies — i.e., they do not depend on labor supply $\boldsymbol{L}$ or on demand $\boldsymbol{\chi}$ — then $A^{KH}(t) = A(t)$. The assumption that real wages are independent of $\boldsymbol{L}$ and $\boldsymbol{\chi}$, that holds in the one-good economy of Example 2, is very strong. It implies that the real marginal product of labor is constant, which is to say that labor in each location can be converted into consumption goods in that location using a linear technology.

Proposition 16 (Equivalence between $A(t)$ and $A^{KH}(t)$)**.** *Suppose that real wages and relative prices in the decentralized equilibrium are independent of labor supply $\{L_r\}$ and demand $\{\chi_r\}$. Then $A^{KH}(t) = A(t)$.*

Proof. By Dinkelbach's theorem (Dinkelbach, 1967), the allocation that attains $A^{KH}(t)$ also maximizes

$$\int_h \left[\frac{w_{l_h}(t) a_{hl_h}}{A^{KH}(t)} - p^c_{l_h}(t) c_h \right] dh$$

subject to $u_h(c_h, l_h) = u^0_h$ for every h, and the maximized value is zero. By separability and Proposition 1, this allocation consists of the compensated choices evaluated at wages $\boldsymbol{w}(t)/A^{KH}(t)$. Therefore,

$$A^{KH}(t) = \frac{\sum_r w_r(t)\, L^{\mathrm{comp}}_r\Big(\frac{\boldsymbol{w}(t)}{A^{KH}(t)}, \boldsymbol{p}^c(t), \boldsymbol{u}^0\Big)}{\sum_r E^{\mathrm{comp}}_r\Big(\frac{\boldsymbol{w}(t)}{A^{KH}(t)}, \boldsymbol{p}^c(t), \boldsymbol{u}^0\Big)}. \tag{72}$$

This representation has the same form as Equation (13) in Theorem 1: there, $A(t)$ is characterized as the maximum proportional reduction in aggregate factor-augmenting productivity such that winners can compensate losers, with wages and prices evaluated in the compensated equilibrium. Here, $A^{KH}(t)$ is the maximum proportional reduction in wages in all locations such that it is possible for winners to compensate losers (based on prices and wages at t) and there is no money left over (sum of net transfers equals zero).

By assumption, equilibrium real wages and relative consumption prices do not depend on labor supply or demand. Hence the real wages and relative consumption prices in the compensated equilibrium coincide, up to a common nominal normalization, with those in the decentralized equilibrium without transfers. Under this condition, Equation (13) reduces exactly to (72). Since (72) characterizes $A^{KH}(t)$ and Equation (13) characterizes $A(t)$, it follows that $A^{KH}(t) = A(t)$. $\square$