Aggregate Welfare with Discrete Choice Across Places and Jobs

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Motivation

- Heterogeneous tastes are at the heart of modern discrete choice models.
- *How should we measure the aggregate value of a change to economy when tastes differ?*
- Common answers are: “expected” utility, real output, & sum of compensating variations.
- They are all seriously flawed.
- Answer matters because it summarizes outcomes and guides policy.

What We Do

- We measure aggregate welfare using approach in Baqaee-Burstein (2025).
- In response to a change, define the associated change in aggregate welfare by:

  How much we could shrink TFP s.t. everyone at least as well off as before the shock.
- If we can reduce TFP and keep everyone indifferent, then aggregate welfare increased.
- Cost-benefit in GE: how much resources left over after winners compensate losers.
- Can be characterized it in terms of supply and demand curves **only**, no cardinal utilities.
- Tractable: in perfectly competitive economies, measure obeys Hulten and coincides with multi-factor productivity growth as measured by BEA, to a first order.

Selection of Related Papers

- **Welfare in discrete choice models in partial equilibrium**

  McFadden (1981), Small & Rosen (1981), Anderson, De Palma, & Thisse (1992).

  Dagsvik & Karlstrom (2005), Bhattacharya (2015, 2021), Kim & Vogel (2020).

- **Related measures of aggregate productivity, without discrete choice**

  Allais (1978), Debreu (1951), Luenberger (1996), Baqaee & Burstein (2025).

- **Aggregation using real GDP or average utility**

  Hsieh et al. (2019), Lamadon et al. (2022), Bagga et al. (2025).

  Redding (2016), Caliendo et al. (2019), Dingel & Tintelnot (2020), Allen & Arkolakis (2022).

- **Decompositions of social welfare functions in discrete choice models**

  Donald, Fukui & Miyauchi (2023), Mongey & Waugh (2025).

Agenda

Simple Economy

General Setup

Extensions

Simple Economy: Setup

- Agent h has preferences $\succeq_h$ over location choice $l_h \in \{1, ..., R\}$ and a consumption good c_h.
- Budget constraint for h, given real wages $\{w_r\}$, TFP shifter Z, and transfer T_h:

$$c_h = \sum_{r \in R} Z w_r 1[l_h = r] + T_h.$$

- Every region produces good linearly from labor with productivity z_r, so $w_r = z_r$.
- Aggregate resource constraint:

$$\sum_h c_h = \sum_r Z z_r L_r \text{ or, equivalently, } \sum_h T_h = 0.$$

where $L_r = \sum_h 1[l_h = r]$ is the share of households that choose r.

Simple Economy: Decentralized Equilibrium

- Agents choose location $r \in R$ to maximize $u_h(w_r Z, r)$.

- Aggregate labor supply function $L_r(\boldsymbol{w}Z)$ is share of households choosing location r.

- Example: $u_h(c_h, l_h) = \bar{\varepsilon}_h \varepsilon_{hl_h} c_h$, where ε_{hr} is Fréchet with shape θ, and $\bar{\varepsilon}_h$ is h-level shifter,

$$L_r(\boldsymbol{w}Z) = \frac{w_r^\theta}{\sum_{r'} w_{r'}^\theta}.$$

  Note: $\{\bar{\varepsilon}_h\}$ is nuisance parameter with no observable/testable implications.

- Perfect competition, so equilibrium is Pareto efficient.

Simple Economy: Aggregate Measures

- Consider change in technologies from $\boldsymbol{z}(0)$ to $\boldsymbol{z}(t)$.
- What is the "aggregate effect"?
- Before showing the change in our measure, let's look at what the literature currently does:
  1. Real output (following procedures in system of national accounts)
  2. Utiltarian welfare or "expected" utility.

Simple Economy: Real Output

- In this economy, real output is just the change in the total quantity of the good:

$$Y(t) = \frac{\sum_h c_h(t)}{\sum_h c_h(0)} = \frac{\sum_r z_r(t) L_r(t)}{\sum_r z_r(0) L_r(0)}.$$

- Real output changes due to changes in real wages for stayers and for movers.
- Under Frechet,

$$\boxed{\frac{d \log Y}{d \log z_r} \approx \frac{w_r(0) L_r(0)}{\sum_k w_k(0) L_k(0)} + \theta \left[\frac{w_r(0)}{\sum_k w_k(0) L_k(0)} - 1 \right] L_r(0).}$$

- Real output falls if r has below average real wage and enough people move there.
- But everybody is weakly better off. Problem is that real output ignores amenity value.

Simple Economy: Utilitarian Welfare

- Average utility is: $W = \mathbb{E}[\max_r u_h(c_r, r)]$. This is not vNM expected utility, meta-preferences over preference parameters cannot be elicited. Agents are just heterogenous.

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- Consider Frechet example, $u_h = \bar{\varepsilon}_h \varepsilon_{hr} c_h$. Literature assumes $\bar{\varepsilon}_h = 1$, so that:

$$\frac{W(t)}{W(0)} = \left[\frac{\sum_r c_r^{\theta}(t)}{\sum_r c_r^{\theta}(0)}\right]^{\frac{1}{\theta}}.$$

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- But now we obtain a completely different nonlinear function, e.g. with $R = 2$,

$$\frac{W(t)}{W(0)} = \frac{\sum_r D(L_r(t)) c_r(t)}{\sum_r D(L_r(0)) c_r(0)} \qquad \text{where } D(x) = \int_0^x u^{-\frac{1}{\theta}} / (u^{-\frac{1}{\theta}} + (1-u)^{-\frac{1}{\theta}}) du.$$

- Different assumptions about $\bar{\varepsilon}_h$ give different rankings. No possible guidance from data.

Simple Economy: Place-Based Policies with Utilitarian Welfare

- The different choices about $\bar{\varepsilon}_h$ matter for optimal policy!
- Policymaker chooses consumption in each location subject to resource and implementability

$$\max W(c_1, c_2) \quad \text{s.t.} \quad \sum_r L_r c_r = \sum_r L_r z_r, \quad L_r = \frac{c_r^\theta}{\sum_{r'} c_{r'}^\theta}.$$

- $\bar{\varepsilon}_h = 1$ and $\bar{\varepsilon}_h = [\sum_r \varepsilon_{hr}/R]^{-1}$ imply different policies and allocations.
- These choices make equilibrium inefficient, sacrificing efficiency to pursue redistribution.
- Quantitative example later.

Aggregating Across Households in a Coherent Way

- So, real output does not respect the Pareto principle.
- Average utility is affected by arbitrary functional form choices. These influence preferences for redistribution in unintuitive ways, which result in Pareto-inefficient place-based policies.
- How can we aggregate given these issues?
- Build on ideas from Kaldor, Hicks, Debreu, Allais, etc. Loosely, we ask:

  "**If winners could compensate losers, how much stuff would be left over?**"

TFP-Equivalent Aggregate Productivity

- $A(t)$ is max reduction in Z to make everyone indifferent to status quo given some transfers.

$$A(t) \equiv \max \left\{ Z \in \mathbb{R} \ : \ \begin{array}{l} \text{there is an equilibrium } (\boldsymbol{c}, \boldsymbol{l}) \text{ given lump-sum transfers,} \\ \text{and aggregate productivity shifter } 1/Z, \\ \text{such that } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h \end{array} \right\}.$$

- If $A(t) > A(0) = 1$, there exists a potential Pareto improvement.
- e.g. If $A(t) = 1.1$, it is possible to keep everyone indifferent and discard 10% of every factor,

  No stance on how surplus should be divided.
- Unlike real output, it takes amenity into account and respects Pareto principle.
- Unlike average utility, it is invariant to monotone transformations of utility (it is empirical).

First-Order Characterization

- We characterize $A(t)$ nonlinearly. But start with a startling first-order approximation.

- **Proposition:** The elasticity of aggregate welfare with respect to productivity of r is

$$\boxed{\frac{d\log A}{d\log z_r} = \frac{w_r(0)L_r(0)}{\sum_k w_k(0)L_k(0)}.}$$

  This holds for any collection of $\succeq_h$.

- Generalizes to perfectly competitive economy with arbitrary technologies & preferences.

- To a first-order, we do not need to solve the model — spending shares suffice.

- Intuition: to a first-order, ignore relocations caused by productivity shocks. Any change in real wage from moving is offset by change in amenity value. (so Δ real output $\neq \Delta \log A$).

Relationship between $A(t)$ and Real Output

- Unlike real output, A always increases in response to a positive productivity shock.
- To a first order, the gap between real output and A is:

$$\boxed{\Delta \log Y - \Delta \log A \approx \sum_r \frac{w_r}{\sum_{r'} w_{r'} L_{r'}} \Delta L_r .}$$

  Second term is just the change in quality-adjusted labor inputs (QALI in national accounts).

- Hence, to a first-order $\Delta \log A$ coincides with multifactor productivity (e.g. computed by BEA).
- Beyond first-order, multifactor productivity suffers from similar issues as real output.

Compensated Equilibrium

- To go beyond first order, we compute $A(t)$ using a *compensated equilibrium*.

- Every h receives transfer T_h to keep them indifferent & TFP is contracted by $A(t)$.

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- Define net expenditure function as minimum transfer to be indifferent to initial equilibrium:

$$T_h^{\text{comp}}(\boldsymbol{w}) = \min_{r \in R} \{\bar{c}_{hr} - w_r\},$$

  where $\bar{c}_{hr}$ is the consumption h needs in location r to be indifferent to $u_h(0)$.
- Note that $T^{comp}(\boldsymbol{w})$ is a partial equilibrium object — can solve agent-by-agent.

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- Note that $T^{comp}(\boldsymbol{w})$ is a partial equilibrium object — can solve agent-by-agent.
- $A(t)$ is pinned down by budget balance:

$$\boxed{\int_h T_h^{\text{comp}}(\boldsymbol{z}(t)/A(t))dh = 0},$$

  using the fact that real wages are $\boldsymbol{z}(t)/A(t)$.

Exact Formula for $A(t)$ using Compensated Supply

- Let the compensated individual location choice be:

$$l_h^{\text{comp}}(\boldsymbol{w}) = \arg\min_{r \in R} \{\bar{c}_{hr} - w_r\}.$$

- Define the aggregate compensated location supply to be:

$$L_r^{comp}(\boldsymbol{w}) = \int_h 1[l_h^{\text{comp}}(\boldsymbol{w}) = r] dh.$$

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- **Proposition:** $A(t)$ solves this equation:

$$\boxed{\int_{\boldsymbol{z}(0)}^{\boldsymbol{z}(t)/A(t)} \sum_r L_r^{\text{comp}}(\boldsymbol{x}) d\boldsymbol{x} = 0.}$$

- $A(t)$ is adjustment in $\boldsymbol{z}(t)$ such that the area under the compensated supply function is zero.

Computing Compensated Labor Supply

- $L_r^{\text{comp}}(\boldsymbol{w})$ is simple to compute by simulation, but sometimes has closed form solution.

- If $u_h(c,r) = f_h(c+\varepsilon_{hr})$, then compensated and uncompensated labor supply are the same:

$$L_r^{comp}(\boldsymbol{w}) = L_r(\boldsymbol{w}).$$

  Because transfers do not alter choices.

- If ε_{hr} is type I extreme-value distribution, then

$$L_r(\boldsymbol{w}) = L_r^{\text{comp}}(\boldsymbol{w}) = \frac{e^{\theta w_r}}{\sum_{r'} e^{\theta w_{r'}}}.$$

- We can integrate labor supply analytically, so $A(t)$ solves

$$\log(\sum_r \exp(\theta z_r(t)/A(t))) = \log(\sum_r \exp(\theta z_r(0))).$$

Approximate Formulas for $A(t)$

$$\int_{\boldsymbol{z}(0)}^{\boldsymbol{z}(t)/A(t)} \sum_r L_r^{\text{comp}}(\boldsymbol{x}) d\boldsymbol{x} = 0.$$

- Log differentiate once and evaluate at $t = 0$ to get

$$\boxed{d \log A = \sum_r \frac{w_r(0) L_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} d \log z_r = \sum_r \lambda_r(0) d \log z_r.}$$

This is the formula we saw before.

Approximate Formulas for $A(t)$

- To second order, we get

$$\boxed{\Delta \log A = \sum_r \lambda_r(0) d \log z_r + \frac{1}{2} \sum_r d\lambda_r^{\text{comp}}(0) d \log z_r.}$$

  If compensated shares rise with productivity, A is convex in $\boldsymbol{z}$.

- In the paper we provide a formula for $d\lambda_r^{\text{comp}}(0)$ in terms of uncompensated supply elasticities (directly estimable, no simulations needed).

Example: Approximate Formulas for $A(t)$ for Frechet

- For Frechet supply system, we have

$$L_r(\boldsymbol{w}) = \frac{w_r^\theta}{\sum_{r'} w_{r'} L_{r'}},$$

so uncompensated cross elasticities satisfies:

$$\frac{\partial L_r}{\partial w_r'} = -\theta L_{r'} L_r \frac{1}{z_{r'}},$$

Using general formula in the paper, we get

$$\Delta \log A = \mathbb{E}_\lambda [\Delta \log \boldsymbol{z}] + \frac{1}{2}(1+\theta) Var_\lambda (\Delta \log \boldsymbol{z}),$$

the higher is θ, the more convex is $A(t)$ in productivity shocks.

Quantitative Illustration

- Calibrate 588 commuting zones with isoelastic labor supply with migration elasticity $\theta = 1.5$.
- Set productivities z_r & amenities B_r to match GDP-per-capita & shares of workers by CZ.
- Counterfactual: one percent increase in productivity in selected locations.
- Report elasticity of A, compare with real output, average utility with different $\bar{\varepsilon}_h$.

Numerical Illustration

Location	$\log A$	**Sales share**
San Francisco–Oakland, CA	0.031	0.031
New York, NY	0.057	0.057
Brownsville, TX	0.001	0.001
Daytona Beach, FL	0.001	0.001

- Hulten works well.

Numerical Illustration

Location	$\log A$	Sales share	log **Output**
San Francisco–Oakland, CA	0.031	0.031	0.050
New York, NY	0.057	0.057	0.091
Brownsville, TX	0.001	0.001	-0.000
Daytona Beach, FL	0.001	0.001	0.000

- Hulten works well.
- Real output is too high for rich locations and too low for poor locations.

Numerical Illustration

Location	$\log A$	Sales share	log Output	$\log \textbf{\textit{AvgU}}_1$
San Francisco–Oakland, CA	0.031	0.031	0.050	0.018
New York, NY	0.057	0.057	0.091	0.034
Brownsville, TX	0.001	0.001	-0.000	0.001
Daytona Beach, FL	0.001	0.001	0.000	0.002

- Hulten works well.
- Real output is too high for rich locations and too low for poor locations.
- Average utility assuming $\bar{\varepsilon}_h = 1$ is off by a similar magnitude as real output.

Numerical Illustration

Location	$\log A$	Sales share	log Output	$\log AvgU_1$	$\log \boldsymbol{AvgU_2}$
San Francisco–Oakland, CA	0.031	0.031	0.050	0.018	0.020
New York, NY	0.057	0.057	0.091	0.034	0.040
Brownsville, TX	0.001	0.001	-0.000	0.001	0.001
Daytona Beach, FL	0.001	0.001	0.000	0.002	0.002

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- Real output is too high for rich locations and too low for poor locations.
- Average utility assuming $\bar{\varepsilon}_h = 1$ is off by a similar magnitude as real output.
- Assuming same taste intensity: $\bar{\varepsilon}_h = \left(\sum_r \varepsilon_{hr}/R\right)^{-1}$ makes a difference.

Numerical Illustration

Location	$\log A$	Sales share	$\log$ Output	$\log AvgU_1$	$\log AvgU_2$	$\log \boldsymbol{AvgU_3}$
San Francisco–Oakland, CA	0.031	0.031	0.050	0.018	0.020	0.059
New York, NY	0.057	0.057	0.091	0.034	0.040	0.098
Brownsville, TX	0.001	0.001	-0.000	0.001	0.001	0.000
Daytona Beach, FL	0.001	0.001	0.000	0.002	0.002	0.014

- Hulten works well.
- Real output is too high for rich locations and too low for poor locations.
- Average utility assuming $\bar{\varepsilon}_h = 1$ is off by a similar magnitude as real output.
- Assuming same taste intensity: $\bar{\varepsilon}_h = \left(\sum_r \varepsilon_{hr}/R\right)^{-1}$ makes a difference.
- But $\bar{\varepsilon}_h$ is arbitrary. For example, setting $\bar{\varepsilon}_h = 10^8$ for households whose favorite location is in CA, NY, or FL is also consistent with any observable data.

Optimal Place-Based Policies

Location	L_r^{initial}
San Francisco–Oakland, CA	0.018
New York, NY	0.034
Brownsville, TX	0.001
Daytona Beach, FL	0.002

- Since equilibrium is efficient, A is maximized at the equilibrium allocation.

Optimal Place-Based Policies

Location	$\boldsymbol{L_r^{\text{initial}}}$	$L_r^{AvgU_1}$
San Francisco–Oakland, CA	0.018	0.013
New York, NY	0.034	0.025
Brownsville, TX	0.001	0.002
Daytona Beach, FL	0.002	0.003

- Since equilibrium is efficient, A is maximized at the equilibrium allocation.
- Policymakers that maximize average utility distort the Pareto-efficient allocation.
- "Standard" choice $\bar{\varepsilon}_h = 1$ pulls agents out of highly productive CZs for redistributive reasons.

Optimal Place-Based Policies

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San Francisco–Oakland, CA	0.018	0.013	0.014	0.023
New York, NY	0.034	0.025	0.027	0.044
Brownsville, TX	0.001	0.002	0.002	0.001
Daytona Beach, FL	0.002	0.003	0.003	0.010

- Since equilibrium is efficient, A is maximized at the equilibrium allocation.
- Policymakers that maximize average utility distort the Pareto-efficient allocation.
- “Standard” choice $\bar{\varepsilon}_h = 1$ pulls agents out of highly productive CZs for redistributive reasons.
- The arbitrarily chosen $\bar{\varepsilon}_h$ determines optimal place-based policy.

Agenda

Simple Economy

General Setup

Extensions

Generality

- Results from simple model generalize considerably.
- General model is quite flexible, allowing for arbitrary:
  - Many goods, non-traded goods, and home bias,
  - heterogeneity in preferences and skills,
  - different “types” of primary factors (e.g. by education or landlords/labor),
  - input-output linkages,
  - commuting (agents consume & work in different places), as in Monte et al (2018).
- Perfectly competitive benchmark.

Summary of Generalized Results

- Show that $A(t)$ can be computed using aggregate compensated **supply** and now **demand**.
- Compensated demand maps prices and wages into expenditures in each region.
- To a first order, we have

$$\Delta \log A \approx \sum_i \frac{sales_i}{GDP} \times \Delta \log z_i.$$

- Higher order terms depend on elasticities of compensated demand and supply.
- Intuition is familiar: more elastic supply and demand means more convexity (convexity amplifies positive shocks and mitigates negative relative to first-order).
- Compensated elasticities can be obtained from uncompensated elasticities.

Environment – Household Problem

- Choices $r \in \{1, ..., R\}$ index occupation, region, consumption/work location, etc.
- Agents have idiosyncratic skills across occupations, regions.
- Consumption prices vary across locations.
- Budget constraint is

$$\underbrace{\sum_{r \in R} p_r c_h 1[l_h = r]}_{\text{spending}} = \underbrace{Z \sum_{r \in R} a_{hr} w_r 1[l_h = r]}_{\text{labor income}} + \underbrace{T_h}_{\text{transfer}},$$

- Household-side summarized by partial equilibrium aggregate labor supply of labor type r

$$L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \int a_{hr} 1[l_h = r] dh,$$

and aggregate demand for consumption good in location r

$$E_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = w_r L_r + \int T_h 1[l_h = r] dh.$$

Environment – Firms and Equilibrium

- Producer of $i \in N$ maximizes profits taking prices as given:

$$p_i y_i - \sum_{j \in N} p_j x_{ij} - \sum_{r \in R} w_r L_{ir}, \quad s.t \quad y_i = z_i F_i \left(\{x_{ij}\}_{j \in N}, \{L_{ir}\}_{r \in R} \right).$$

- Prices clear goods market:

$$\sum_r E_r 1[i \text{ is consumption in } r] + \sum_{j \in N} p_i x_{ji} = p_i y_i$$

and wages clear factor market:

$$\sum_{i \in N} L_{ir} = Z L_r.$$

Solving for $A(t)$ using Compensated Equilibrium

- Definition of $A(t)$ is same as in simple setting, compute using compensated equilibrium.
- In this equil., every h receives a T_h to keep them indifferent and TFP is contracted by $A(t)$.
- Expenditure function $T_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p})$ same as before but depends on consumption prices too.

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- In this equil., every h receives a T_h to keep them indifferent and TFP is contracted by $A(t)$.
- Expenditure function $T_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p})$ same as before but depends on consumption prices too.
- Aggregate compensated labor supply function defined as before:

$$L_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}).$$

- Now, we need one more function:

$$E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}) = w_r L_r^{comp}(\boldsymbol{w}, \boldsymbol{p}) + \int_h T_h^{comp}(\boldsymbol{w}, \boldsymbol{p}) 1[l_h^{comp} = r] dh.$$

- L_r^{comp} and E_r^{comp} are partial equilibrium objects.

Solving for $A(t)$ using Compensated Equilibrium

- Compensated equil. conditions are standard but market clearing uses

$$L_r^{\text{comp}}(\boldsymbol{w}/A(t), \boldsymbol{p}) \qquad \text{and} \qquad E_r^{\text{comp}}(\boldsymbol{w}/A(t), \boldsymbol{p}).$$

- There is one extra unknown: $A(t)$ is pinned down by budget balance

$$\sum_r E_r^{\text{comp}}(\boldsymbol{w}/A(t), \boldsymbol{p}) = \sum_r L_r^{\text{comp}}(\boldsymbol{w}/A(t), \boldsymbol{p}) w_r / A(t).$$

- Given E_r^{comp} and L_r^{comp}, solving for $A(t)$ is straightforward.

Formulas for $A(t)$ in General Setting

- **Proposition:** For any preferences and technologies, $A(t)$ satisfies:

$$\log A(t) = \int_0^t \sum_i \lambda_i^{\text{comp}}(s) \frac{d \log z_i}{ds} ds,$$

as area under sales shares in compensated equilibrium, λ_i^{comp}.

- Generalizes simple economy to general technologies and shocks to those technologies.

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- Generalizes simple economy to general technologies and shocks to those technologies.

- Evaluating at $t = 0$ yields generalization of Hulten's Theorem:

$$\boxed{d \log A = \sum_i \lambda_i(0) d \log z_i,}$$

  holds for any perfectly competitive discrete choice economy, no need to solve model.

- Paper provides second-order approximation in terms of supply and demand elasticities.

Agenda

Simple Economy

General Setup

Extensions

Extensions

- Changes in amenities (e.g. weather in a region, health risk of occupation).

  Index location characteristic by t and apply the same definition of $A(t)$.

- Measure can also be applied to a subset of agents (e.g. by gender, or education).

  When applied to a single agent, collapses to a compensating variation.

- Measure can be defined with limited redistributive tools (e.g. place-based policies) rather than lump-sum transfers.

- Measure can be applied to distorted equilibria (e.g with agglomeration externalities).

Aggregate Productivity with Place-Based Compensations

- We defined A as factor savings after winners compensate losers using lump-sum transfers.
- Now extend definition to cases where only place-based redistributive policies are available.
- $A^{pb}(t)$ is maximum reduction in Z needed to make everyone at least indifferent to status quo given location-level consumption taxes.
- We characterize it for the simple economy.
- To a first-order approximation, $A^{pb}(t)$ and $A(t)$ agree and obey Hulten's theorem, because distortions from place-based policies are second-order around Lessaiz-faire.

Aggregate Productivity with Externalities

- Consider simple model with agglomeration externalities.
- Labor productivity in location r is $z_r L_r^\gamma$. Benchmark assumes $\gamma = 0$.
- Equilibrium has wage equal to private marginal product, $w_r = z_r L_r^\gamma$, and transfers T.
- Same definition of $A(t)$. Pinned down by budget balance in compensated equilibrium:

$$\int_h T_h^{\text{comp}}(\boldsymbol{w}(t))dh = 0 \qquad \text{where} \qquad w_r = z_r(L_r^{\text{comp}}(\boldsymbol{w}))^\gamma / A(t).$$

Only difference: wages in compensated equilibrium depend on compensated labor supply.

Broader Agenda

- Broader agenda studying this measure in different heterogeneous agent settings:
- *"Aggregate Productivity with Heterog. Agents"* (general framework & tools to solve for A)
- *"Misallocation due to Incomplete Markets"* (in closed & open economies)
- *"An Approximate General Theory of Second Best"* (optimal policy to max A with distortions)
- *"Monetary Policy without Redistributive Concerns"* (in HANK environments)

Appendix Slides

Comparison to Sum of Compensating Variations

- Traditional approach in partial equilibrium (Small&Rosen/McFadden) is to do welfare analysis by looking at the sum of compensating variations.
- Sum of CVs ignores the fact that prices are endogenous.
- So, a pure redistribution (movement along Pareto frontier) causes sum of CVs to be positive, even though there is no surplus.
- Intuition: redistributions raise prices for winners and lower them for losers — so it is feasible to compensate losers at expost prices and have money left-over.
- Positive of sum of CVs is not necessary nor sufficient for potential Pareto improvement in GE.

Relation to McFadden's Social Surplus Function

- If indirect utility function is linear in price of good, then "social surplus function" (McFadden 1981) can be used to calculate sum of compensating variations (Small & Rosen 1981).
- We show a counterpart of this result in general equilibrium.
- Define "Social Surplus Function" as $U(\boldsymbol{c}) = \mathbb{E}[\max_r u_h(c_r, r)] = \mathbb{E}[\max_r g(c_r) + \varepsilon_{hr}]$.
- If $g(c)$ is linear (symmetric second derivatives of L_r) and common consumption good, then

$$U(\boldsymbol{w}(t)/(A(t)p^c(t))) = U(\boldsymbol{w}(0)/p^c(0)),$$

where real wages are those in decentralized equilibrium with tech. $\boldsymbol{z}(t)$ and TFP $1/A(t)$.

Approximate Formulas for $A(t)$

- To second order, we get

$$\Delta \log A = \sum_r \lambda_r(0) d\log z_r + \frac{1}{2}\sum_r d\lambda_r^{\text{comp}}(0) d\log z_r.$$

- The change in the compensated spending share is

$$d\lambda_r^{\text{comp}} = \lambda_r\big(d\log(z_r L_r^{\text{comp}}) - \sum_{r'} \lambda_{r'} d\log(z_{r'} L_{r'}^{\text{comp}})\big)$$

- **Proposition:** Change in compensated supply is

$$dL_r^{\text{comp}} = \sum_{r' \neq r} dL_{r' \to r}^{\text{comp}} - \sum_{r' \neq r} dL_{r \to r'}^{\text{comp}},$$

  where $dL_{r' \to r}^{\text{comp}}$ is share of switches from r' to r in compensated equilibrium

$$dL_{r \to r'}^{\text{comp}} = \frac{\partial L_r}{\partial w_{r'}}\left(d(\frac{z_r}{A}) - d(\frac{z_{r'}}{A})\right) \times 1\left[d(\frac{z_r}{A}) \leq d(\frac{z_{r'}}{A})\right],$$

  need uncomp. cross-price aggregate supply elasticities (from regression, no simulation).