Aggregate Welfare with Discrete Choice Across Places and Jobs

David R. Baqaee
UCLA

Ariel Burstein
UCLA*

June 10, 2026

Abstract

Discrete-choice general-equilibrium models are widely used to study how people sort across places, sectors, and jobs, but they lack a canonical aggregate welfare statistic. This paper proposes a TFP-equivalent welfare measure: the largest uniform reduction in factor productivity such that it is possible to keep every household at least as well off as in the status quo. Equivalently, it measures the productive surplus left after winners compensate losers, allowing prices, wages, and choices to adjust in general equilibrium. We characterize this measure using compensated supply and demand. A main result is a version of Hulten's theorem for discrete-choice economies: under perfect competition, the first-order welfare effect of a productivity shock to producer i is producer i's sales share, regardless of the distribution of preferences and technologies. Beyond first order, we provide approximations in terms of observable income and expenditure shares and uncompensated supply and demand elasticities. We compare this measure with common alternatives. Real GDP can fall even when every household is better off; average utility depends on arbitrary cardinalizations of individual utility; and the sum of compensating variations can rise after pure redistributions because it holds general-equilibrium prices fixed during compensation. The measure avoids these problems while preserving the logic of cost-benefit analysis in general equilibrium.

*We thank Rodrigo Adao, John Asker, Pablo Fajgelbaum, Hannes Malmberg, Yuhei Miyauchi, Will Rafey, Jon Vogel, Pierre-Olivier Weill for their comments. We are especially grateful to Natalie Bau for her insights.

1 Introduction

General-equilibrium models in which agents make discrete decisions—where to live, which sector to work in, or what job to take—are now standard in trade, spatial, labor, and macroeconomics. These models are popular because they capture rich heterogeneity in preferences, skills, and individual outcomes. Yet they make it surprisingly hard to answer a basic question: what is the aggregate welfare value of a change in the economy when agents are affected differently by that change? A meaningful answer is important for summarizing the overall welfare consequences of economic change and for guiding policy.

A useful aggregate welfare statistic for these environments should satisfy two requirements. First, it should be ordinal, relying only on revealed preference rather than arbitrary cardinalizations of utility. It should respect Pareto improvements: when every household is made better off, aggregate welfare should rise. These requirements are minimal, but they already create problems for common aggregate measures in discrete-choice settings. Real output, average utility, and cost-benefit measures each provide a natural benchmark, but each can give misleading answers in these environments:

i. **Chain-weighted real output**: This is how real output and consumption are measured in the national accounts. In economies with a single final good, this is simply the change in the quantity of that good. However, as is well-known, these measures can be an inappropriate gauge for welfare because they ignore amenity value (Roback, 1982). For example, if an agent moves from a high-real-wage to low-real-wage location/job, then real output and consumption fall, even though that agent may be better off once we take into account her location preferences. Indeed, it is simple to construct examples where a shock makes every agent better off, but causes real output to decline (e.g. if a shock improves productivity in a low-productivity but high-amenity location and causes reallocation of workers toward those locations, real output can fall).[1]

ii. **Utilitarian welfare or "expected" utility.** This measure is the cross-sectional average of individual utilities. A fundamental issue with this measure is that it is not invariant to monotone transformations of individual utility functions. This implies that replacing a utility function with a monotone transformation that generates exactly the same individual choice behavior can nonetheless alter the social welfare

[1]This measure is nevertheless popular in the literature, partly because it is consistent with objects in national income accounting. Some recent examples that report real output in models with occupation choice include Hsieh et al. (2019), Lamadon et al. (2022), and Bagga et al. (2025).

ranking. For instance, in the spatial literature, a standard utility specification is $c\varepsilon_{hr}$, where c is consumption and ε_{hr} is h's idiosyncratic taste for choosing r.[2] This specification is observationally equivalent to one in which each agent's tastes are normalized by that individual's average taste intensity, $\mathbb{E}[\varepsilon_{hr} \mid h]$, so that individual h's utility function is $c\varepsilon_{hr}/\mathbb{E}[\varepsilon_{hr} \mid h]$. While both specifications predict identical individual choices, they imply different rankings of social allocations according to average utility and give rise to different Pareto-inefficient place-based policies when average utility is used as the policymaker's objective. The issue is that there is no canonical basis for choosing between these normalizations. Normalizing average taste intensity to one is just as plausible as leaving it unscaled. Since the data cannot distinguish between these cardinalizations, the resulting aggregate welfare measure depends on untestable and nonintuitive assumptions about the scale of utility.[3]

iii. **Sum of compensating variations**: This measure is also called Kaldor-Hicks efficiency or cost-benefit efficiency, and has been analyzed by Small and Rosen (1981) for settings with discrete choice. It measures the amount of money left over after winners compensate the losers holding fixed post-shock equilibrium prices and wages.[4,5] While this measure does not have the issues of the previous two, it can nevertheless be unreliable in general equilibrium. In particular, it may not be technologically feasible for winners to compensate the losers even though the sum of compensating variations is positive (since post-shock prices do not stay constant when winners try to compensate losers). We show, for example, that in response to pure transfers that move the economy along the Pareto-efficient frontier, the sum of compensating variations can rise even though winners cannot be better off after

[2]Some examples that use average utility (or a consumption-equivalent variation applied to average utility) to measure aggregate welfare in the spatial literature include Redding (2016), Caliendo et al. (2019), Dingel and Tintelnot (2020), and Allen and Arkolakis (2022).

[3]See Section 6.2 for a discussion and worked-out example. Although this measure is sometimes called "expected utility," average utility is not a von Neumann–Morgenstern expected utility function. Expected utility functions represent ordinal preference relations over lotteries; they are not expectations of utility values across households with different preferences.

[4]The pioneers in the discrete choice literature, like McFadden (1981) and Anderson et al. (1992), emphasize that consumer welfare in discrete choice models should be measured using compensating or equivalent variations of the individual agents, as in standard consumer theory. Dagsvik and Karlström (2005) show how to calculate the distribution of compensating variation across agents using compensated choice probabilities. Bhattacharya (2015) and Bhattacharya (2021) study non-parametric identification of the distribution of compensating and equivalent variation.

[5]While this approach is common in the industrial organization literature, it is less common in general equilibrium spatial and occupational choice models. One exception is Kim and Vogel (2020), who study a model where workers choose among a discrete number of sectors including non-employment. They show that the elasticity of the sum of compensating variations with respect to wages is given by the income share of each sector.

compensating losers.

These shortcomings point toward a general-equilibrium version of compensating variation. We follow Harberger (1971) and Small and Rosen (1981) in using willingness-to-pay as the basis for welfare measurement, but we do not add up willingness-to-pay at fixed prices. Instead, following Baqaee and Burstein (2025) and inspired by Debreu (1951), we ask: "*What is the maximum reduction in total factor productivity that is consistent with an equilibrium in which no household is worse off than in the status quo?*" We measure the increase in aggregate welfare by the amount of primary factors that can be saved while keeping every household at least indifferent.

The sum of compensating variations measures welfare gains by the amount of aggregate income that could be taken away — holding prices and wages fixed — while leaving everyone at least as well-off as under the status quo. We use a general equilibrium counterpart of this idea, measuring welfare gains by the amount of primary factors that could be removed — holding technologies fixed — while still allowing an equilibrium in which no household is worse off than in the status quo.

This measure has several useful features. First, the answer has interpretable units expressed in terms of total factor productivity (or equivalently, in units of every good, since scaling total factor productivity also scales the consumption possibility set). Second, and unlike real output, it respects the Pareto principle — welfare rises if every agent can be made better off. Third, and unlike average utility, this measure is defined in terms of observables and is invariant to monotone transformations of utility (because it depends only on revealed preference comparisons, not on utility levels). Fourth, unlike the sum of compensating variations, which calculates compensation at fixed prices and wages, the compensation exercise that accompanies the TFP-equivalent variation is internally consistent in general equilibrium. This implies, for example, that the metric is unaffected by pure redistributions that move the economy along the Pareto frontier. Fifth, it does not take a stance on distributional considerations or embed normative judgements about interpersonal utility comparisons beyond indifference to the status quo. Finally, it is relatively easy to communicate its meaning to non-experts and policymakers.

A main payoff is that this aggregate welfare measure is simple to characterize. In perfectly competitive economies, the first-order effect of a productivity shock to producer i is equal to producer i's sales divided by total income. Thus, despite the heterogeneity of preferences and the non-convexity created by discrete choices, aggregate welfare obeys a Hulten theorem just as in continuous-choice economies. To a first order, one does not need to solve for how households switch locations or occupations in response to the shock. Intuitively, when a household switches choices, the pecuniary gain from switching is exactly offset at the margin by the amenity loss required to make the household willing to switch.

Our baseline model abstracts from distortions and externalities (see, e.g., Fajgelbaum and Gaubert, 2020). Focusing on a perfectly competitive, and hence Pareto-efficient, benchmark helps clarify how our approach differs from standard practice in the literature. We sketch how to extend our approach beyond perfectly competitive economies in Appendix G, allowing for externalities and wedges. Similarly, when calculating aggregate welfare, we assume that compensations can be implemented through individual-specific lump-sum transfers. We abstract from limitations on transfers arising, for example, from private information as discussed in Schulz et al. (2023). Appendix H discusses how to adjust our measure of aggregate welfare when individual-specific lump-sum transfers are not available and compensations must instead be contingent on location choices (i.e. place-based policies).

The structure of the paper is as follows. We set up the preferences and technologies and define perfectly competitive equilibrium in Section 2. The economy is static, and agents choose consumption and the location and/or industry in which they live and work. Agents can differ flexibly in their skills and tastes over locations and occupations. Production uses different types of primary factors and intermediate inputs, and accommodates input-output networks and costly trade.

In Section 3, we define our notion of aggregate welfare. We show that it can be characterized using compensated supply and demand functions. Compensated supply functions map relative wages and prices to labor supply across locations or occupations, holding each agent on a fixed indifference curve. Compensated demand maps wages and prices to total spending on different consumption goods, holding each agent on a fixed indifference curve. We show how to compute these compensated supply and demand functions using Monte Carlo methods. In this section, we also introduce a running example of a one-good economy that we return to in every section to illustrate results.

In Section 4, we provide some analytical (as opposed to simulation-based) characterizations of our measure. First, we show that our measure of TFP-equivalent aggregate welfare, to a first-order approximation, obeys Hulten's theorem in perfectly competitive economies. That is, the elasticity of aggregate welfare to a productivity shock to producer i is equal to the sales of i divided by total income. Hence, for our notion of aggregate welfare, there is nothing special about discrete choice: perfectly competitive discrete choice economies satisfy Hulten's theorem by exactly the same logic as continuous choice economies.[6] In particular, one does not need to know anything about the underlying

[6]Baqaee and Burstein (2025) show that the same aggregate welfare metric obeys Hulten's theorem in

preferences or technologies. Moreover, one does not need to take into account changes in behavior in response to the shock (to a first order). Intuitively, any changes in real wages experienced by agents that switch their choice in response to a shock are exactly offset by changes in the amenity value of their choices. This is in contrast to utilitarian welfare and real GDP, both of which require solving a general equilibrium model, even to a first order.

In this section, we also discuss a special case where compensated and uncompensated supply functions are the same because compensating transfers do not alter location choices. This allows us to nonlinearly solve for aggregate welfare given knowledge of the uncompensated supply system and without the use of simulations. Under these assumptions, aggregate welfare can be computed using the "Social Surplus Function" (McFadden, 1981). However, this special case excludes spatial models with home bias, in which consumption prices vary across regions and transfers can therefore affect location choices.

In Section 5, we investigate nonlinearities in cases where compensations do affect location choices. We provide a second-order approximation of changes in aggregate welfare. We show that the first-order approximation understates positive shocks and overstates negative shocks if compensated sales shares rise for producers with positive shocks. The reverse is true if compensated sales shares fall in response to positive shocks. We express the elasticities of compensated sales shares in terms of observables in the initial equilibrium: income and expenditure shares, and price elasticities of uncompensated (market-level) supply and demand systems. Given these statistics, the distribution of tastes and the functional forms of utility functions do not matter.

In Section 6, we compare our approach to the three popular alternative measures of aggregate efficiency/welfare discussed above

i. **Chain-weighted real output**: Real output differs from our measure of TFP-equivalent aggregate welfare even to a first-order (despite the fact that the equilibrium is Pareto-efficient). Whereas our measure differs from real output, it does coincide, to a first-order approximation, with measures of multi-factor productivity growth (or the Solow residual) computed using a quality-adjusted labor input. That is, our analysis provides a theoretical justification for those statistics in efficient economies with heterogeneous tastes and discrete choice.[7]

ii. **Utilitarian welfare or "expected" utility**: We show that the utilitarian approach

perfectly competitive economies with continuous choice. The results of that paper cannot be applied directly in this context because consumption possibility sets, at the individual level, are non-convex due to the discreteness of choices. In particular, this means that results like the second welfare theorem may not apply.

[7]See, e.g., chapter 3 of the OECD's manual on measuring productivity for quality-adjusted labor input.

typically does not coincide with our measure, even to a first-order. [8] Using a model calibrated to regional U.S. data, we show how different arbitrary but equally valid monotone transformations of individual utility functions have very different implications for optimal policy.

iii. **Sum of compensating variations**: Our measure does not generally coincide with the sum of compensating variations (Kaldor-Hicks/cost-benefit efficiency) because our measure accounts for the fact that compensating transfers can alter equilibrium prices and wages. We provide an example where redistribution with lump-sum transfers, which do not generate a change in efficiency according to our measure because they only move the allocation along the Pareto frontier, results in an increase in Kaldor-Hicks efficiency. Intuitively, using post-transfer prices and wages, the winners can compensate losers and still come out ahead. Since this is technologically unfeasible, this example shows that summing up compensating variations can give misleading answers in discrete choice general equilibrium economies.

Our paper complements the analysis of social welfare functions in spatial models by Donald et al. (2023). They consider a class of social welfare functions (defined over social surplus functions of groups of agents) and then study changes in social welfare as the equilibrium allocation changes.[9] They decompose changes in social welfare into different components reflecting different mechanisms. The most important difference between our papers is that we ask a different question. Instead of decomposing changes in the value of a social welfare function, we define and characterize a general equilibrium counterpart to cost-benefit analysis.

This difference is important because, as with cost-benefit analysis, our measure does not take a stance on optimal redistribution. In contrast, social welfare functions encode distributional preferences (see the discussion by Fajgelbaum and Gaubert, 2025). Therefore, policymakers that maximize a social welfare function have incentives to intervene in Pareto-efficient allocations in pursuit of redistributive motives. Which interventions are desirable (for example, whether workers should be induced to move from

[8]This is also implied by the decomposition of social welfare functions in Donald et al. (2023). In their terminology, even in the absence of any externalities and distortions, the change in social welfare need not obey Hulten's theorem because of a "marginal utility dispersion" term that can be non-zero. In contrast, our measure obeys Hulten's theorem to a first-order in perfectly competitive economies.

[9]More concretely, Donald et al. (2023) consider a model with a set of types $\theta \in \{\theta_1, \ldots, \theta_S\}$. Within each type, agent h has idiosyncratic tastes ϵ^{θ}_{hr} for location r. The social surplus function for type θ is: $U^{\theta}(\boldsymbol{c}^{\theta}) = \mathbb{E}\left[\max_r g(c^{\theta}_r) + \epsilon^{\theta}_{hr}\right]$, where c^{θ}_r is consumption and g is an increasing function. They study the class of social welfare functions that can be written as: $W = \mathcal{W}\left[U^{\theta_1}(\boldsymbol{c}^{\theta_1}), \ldots, U^{\theta_S}(\boldsymbol{c}^{\theta_S})\right)$. That is, whereas $\mathcal{W}$ is a flexible function, U^{θ} embeds a specific cardinal assumption about the representation of individual utility functions (within θ).

high-productivity to low-productivity locations) then depend on arbitrary assumptions about the monotone transformation used to represent individual utility (see Example 7 in Section 6).[10] In contrast, our measure is always maximized if the status quo is Pareto-efficient. Moreover, if a policy yields a higher value of TFP-equivalent aggregate welfare, then it must be that this policy can make every household weakly better off than in the status quo given appropriate transfers.

2 Environment and Equilibrium

In this section we define the environment and equilibrium. We consider perfectly competitive economies without externalities (e.g. positive or negative spillovers).

Commodity space. There is a set C of consumption bundles, a set N of intermediates, and a set R of primary factors. Commodities are indexed by a comprehensive set of characteristics that define their uniqueness including region (e.g. haircuts in LA and New York can be treated as different commodities).[11]

Primary factors can be arbitrarily finely differentiated in the set R. For example, the set R can distinguish factors by region, occupation, skill-level, and type (e.g. high-skilled accountants in LA and commercial real estate in New York are different factors). Agents have different ability to provide the different factor services. For example, some agents (low-skilled workers) can only supply low-skilled labor, or landowners in one region can only supply land to firms in their region but not outside. Each agent h makes a discrete choice $l_h \in R$ that we refer to as the *location* of h, with the understanding that it may index location in space as well as occupation, industry, and so on.[12]

For simplicity, the set of consumption bundles C has the same size as the set of primary factors R and there is a one-to-one correspondence between consumption bundles and

[10] Mongey and Waugh (2024) provide an alternative interpretation where the social surplus function is the expected utility of individuals who are ex-ante identical (before their taste shocks are known) and lack access to insurance markets for their idiosyncratic tastes. However, if such an ex-ante state does not exist, then the results of this type of analysis rest on an untestable cardinalizing assumption. That is, if households cannot make choices in the ex-ante stage that reveal their risk preferences about how they rank one set of taste parameters against another, then choice data identifies utility up to a monotone transformation so that the ex-ante expected utility function cannot be recovered from such data. See also the discussion of this issue by Davis and Gregory (2021).

[11] Similarly, if there are trade costs, then traded goods are indexed by both origin and destination. For example, oranges produced in California and consumed in New York are different to oranges produced in California and consumed in Illinois.

[12] This setup is general enough to capture settings that feature commuting, as in Monte et al. (2018), where agents consume and work in different locations. In this case, we let r index pairs of locations instead.

locations. Thus, for each $r \in R$ there is a primary factor ("labor in location r") and an associated consumption bundle $c(r) \in C$ ("the consumption bundle in location r"). By allowing consumption bundles to differ by location, we allow for the possibility that some consumption goods are non-tradeable (e.g. barbers in Los Angeles consume a different bundle of goods than waiters in New York).

Households' problem. There is a set H of agents. Each agent $h \in H$ chooses a location $l_h \in R$ (e.g. being a barber in Los Angeles or a waiter in New York, etc.) and consumes a scalar quantity of a homothetic bundle c_h (composed of goods and services) in that location.

Agent h has preferences $\succeq_h$ over the location choice l_h and the quantity of the consumption bundle consumed. These preferences are represented by a well-behaved utility function $u_h(c_h, l_h)$. Almost all the results in the paper hold without putting any additional functional form requirements on $u_h(c_h, l_h)$. However, for concreteness, we suppose preferences are *ordinally additively separable* between location choices and consumption.[13] This means that the utility function can be written as

$$u_h(c_h, l_h) = f_h(g(c_h) + \epsilon_{hl_h}),$$

where g is a strictly increasing function, ϵ_{hl_h} is a taste parameter for household h's preferences for location l_h, and f_h is any strictly increasing (potentially household-specific) function. Since utility is only pinned down up to monotone transformations, the function f_h has no observable implications and can vary at the level of each h in arbitrary ways. However, the shape of f_h does affect utilitarian welfare, and including it allows us to contrast our approach to the utilitarian approach in Section 6.2. In contrast to f_h, the shape of the function g has testable implications and can be recovered from ordinal choice data.[14, 15]

[13] To be explicit, only Propositions 3, 5 and 11 require this ordinally additive specification of preferences. The rest of the results can be proven without relying on this assumption.

[14] The shape of $g(c_h)$ has testable implications because it controls the way income and substitution effects interact with each other. In particular, $g(\cdot)$ affects how households switch choices in response to lump-sum transfers. For example, if $g(\cdot)$ is an affine function, then households do not switch their choices if we add a constant amount to their consumption in every location. By contrast, if $g(\cdot)$ is log, then households do not switch their choices if we multiply their consumption by a constant in every location. These statements are true regardless of the distribution of tastes. See Appendix B for more information on how the function $g(\cdot)$ can be identified from behavior (see also Allen and Rehbeck, 2019).

[15] Since location choices are discrete, agents' preferences are nonconvex. This means that theorems that require convexity, like the second welfare theorem, cannot be used. The nonconvexity arises because the convex combination of two location choices is undefined. To convexify preferences, we could consider lotteries over location choices (e.g. with probability π_r consumption c_r and location r is realized). If we augment the

Each agent maximizes utility choosing consumption c_h and location l_h. Recall that the choice of location encompasses both spatial location and choice of occupation. The household's budget constraint is that consumption expenditures are equal to income plus transfers:

$$\sum_{r\in R} p_r c_h \mathbf{1}[l_h = r] = \sum_{r\in R} a_{hr} Z w_r \mathbf{1}[l_h = r] + T_h,$$

where p_r is the price of the composite consumption bundle in location r, $a_{hr}Z$ is agent h's efficiency units of factor type r, w_r is the wage per efficiency unit of factor r, and T_h is a lump-sum transfer to agent h. In writing the budget constraint, we slightly abuse notation and denote the consumer price index in location $r \in R$ as p_r.[16] We also anticipate that firms earn zero profits in equilibrium. The scalar Z is an aggregate factor-augmenting productivity (or total factor productivity) shifter that uniformly scales the efficiency of factors in every location for all agents.[17] We use Z to define the TFP-equivalent variation later.

Each agent h chooses the location l_h to maximize $g((Za_{hr}w_r + T_h)/p_r) + \epsilon_{hr}$ over $r \in R$. Denote the optimal *location choice* by $l_h(Z\boldsymbol{w}, \boldsymbol{p}, T_h)$. We refer to the quantity of factor type r supplied by h as *labor* supplied by h, with the understanding that this is just a shorthand and factors could in principle include non-labor inputs like land. Given this shorthand, efficiency units of labor supplied in location r (not including the aggregate shifter Z) are

$$L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \int a_{hr}[l_h(Z\boldsymbol{w}, \boldsymbol{p}, T_h) = r]dh.$$

We call the function $L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ the aggregate *labor supply function*. Efficiency units of labor supplied in location r, and available to firms, are $Z \times L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$. Finally, let $\boldsymbol{\chi}$ denote the vector of final consumption spending shares in each location by

$$\chi_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \frac{Zw_r L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) + \int T_h[l_h(Z\boldsymbol{w}, \boldsymbol{p}, T_h) = r]dh}{\sum_{r'} Zw_{r'} L_{r'}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})}.$$

model with lotteries, and impose the von-Neumann & Morgenstern assumptions, then the function $f_h(\cdot)$ is identifiable because it disciplines h's degree of risk-aversion. However, if such lotteries are not observed, then f_h cannot be elicited from observed choices. Furthermore, if we augment the model with lotteries so that f_h is pinned down by choices, then h's utility for lottery $\boldsymbol{\pi}$ is $u_h(\boldsymbol{\pi}) = v_h(\mathbb{E}_\pi[f_h(g(c_h) + \epsilon_{hr})|h])$ where $v_h(\cdot)$ is any strictly increasing function. That is, in models with lotteries, utilitarian social welfare functions still depend on cardinal properties of the utility function and are not invariant to monotone transformations that do not alter individual choices even though $f_h(\cdot)$ is pinned down.

[16]Technically, consumption goods in location r are indexed by some $c(r) \in C$, where $c(r)$ is the label of the consumption bundle in the set of all consumption bundles C corresponding to location r. Since there is no ambiguity, instead of writing $p_{c(r)}$, we simply just write p_r.

[17]Equivalently, we can model Z as a shifter that scales value added productivity of all producers.

We call the function $\boldsymbol{\chi}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ the *final demand* function. Note that these functions are partial equilibrium objects that can be evaluated for any vectors of wages, prices, and lump-sum transfers.

Producers' problem. The set of commodities is the union of C and N, denoted by $C + N$. This includes consumption bundles in different locations, as well as other goods used as intermediate inputs. Each commodity i is produced by perfectly competitive firms that maximize profits

$$p_i y_i - \sum_{j \in N} p_j x_{ij} - \sum_{r \in R} w_r L_{ir},$$

subject to a constant-returns technology

$$y_i = z_i F_i \left(\{x_{ij}\}_{j \in N}, \{L_{ir}\}_{r \in R} \right), \tag{1}$$

where z_i is a Hicks-neutral productivity shifter for producer i, F_i is a constant returns production technology, x_{ij} are intermediate inputs from $j \in N$, and L_{ir} are efficiency units of factor services from r.

Resource Constraints. The resource constraint requires that consumption and intermediate input usage be equal to production. Recall that commodities are split into pure consumption goods ($i \in C$) and pure intermediates ($i \in N$). Hence, if i is a consumption bundle, then i is not used as an intermediate $x_{ji} = 0$. On the other hand, if $i \in N$, then i is not directly consumed by any household. The resource constraints are therefore:

$$\int c_h \mathbf{1}[l_h = r] dh = y_{c(r)} \text{ if } c(r) \in C \qquad \text{and} \qquad \sum_{j \in N} x_{ji} = y_i \text{ if } i \in N. \tag{2}$$

The resource constraint for factor $r \in R$ requires that total factor demand from producers is equal to total factor supply from households:

$$\sum_{i \in N} L_{ir} = Z L_r. \tag{3}$$

Finally, lump-sum transfers add up to zero:

$$\int T_h dh = 0. \tag{4}$$

Definition 1 (Equilibrium with Discrete Choice)**.** An equilibrium is a collection of consumptions, c_h, location choices, l_h, outputs, y_i, intermediate input choices, $\{x_{ij}, L_{ir}\}$, prices,

p_i and wages, w_r, such that each agent chooses consumption and location to maximize utility subject to their budget constraint, producers maximize profits subject to technology taking prices as given, transfers add up to zero, and all resource constraints are satisfied.

We focus on decentralized equilibria where transfers are equal to zero: $\boldsymbol{T} = 0$.

Solving for Equilibrium. To solve for the equilibrium, we introduce some useful accounting variables. Let $\boldsymbol{\lambda}$ be the vector of Domar (1961) weights. The ith component is the sales of i divided by total income:

$$\lambda_i = \frac{p_i y_i}{\sum_r Z w_r L_r} \mathbf{1}[i \in C + N] + \frac{Z w_i L_i}{\sum_r Z w_r L_r} \mathbf{1}[i \in R].$$

Let Ω be the $|C + N + R| \times |C + N + R|$ input-output matrix that results from profit maximization. The first $|C|$ rows and columns are the consumption bundles, the next $|N|$ are the intermediates, and the final $|R|$ are the primary factors. The ijth element of Ω is the expenditure share of i on inputs from j relative to total revenues:

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i} \mathbf{1}[i \in C + N] + \frac{w_j L_{ij}}{p_i y_i} \mathbf{1}[j \in R].$$

Using this notation, we can now describe the equilibrium conditions. Perfect competition and cost-minimization by producer i implies that p_i is equal to its marginal cost:

$$p_i = z_i^{-1} \mathrm{mc}_i(\boldsymbol{p}, \boldsymbol{w}), \tag{5}$$

which depends on input prices, factor wages, and productivity shifters z_i.

Market clearing for $i \in C + N + R$ requires that total sales of good i equal total spending on good i by households and other producers:

$$\lambda_i = \sum_{r \in R} \chi_r \Omega_{c(r)i} + \sum_{j \in N} \lambda_j \Omega_{ji}, \tag{6}$$

where $c(r)$ is the consumption bundle associated with location r, and χ_r is spending in location r. The factor market clearing condition is

$$\lambda_r = \frac{Z w_r L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})}{\sum_{r'} Z w_{r'} L_{r'}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})}. \tag{7}$$

In an equilibrium without lump-sum transfers, $\boldsymbol{T} = 0$, and the share of spending in each location is equal to the share of income earned in that location $\chi_r = \lambda_r$. Hence, given the aggregate supply function $L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$, one can solve for equilibrium prices, wages, and quantities using (5), (6), and (7).

The example below provides a concrete illustration of the structure of the model using Cobb-Douglas and logit functional forms.

Example 1 (Cobb-Douglas with Logit). Suppose each location produces a single good using labor in that location. The consumption bundle in every location is the same Cobb-Douglas aggregator with a common price. Equation (5) implies the (common) consumer price index is

$$p^c = \prod_r \left(\frac{w_r}{z_r} \right)^{\Omega_{cr}}, \tag{8}$$

where Ω_{cr} are constant parameters that do not depend on productivity.

Suppose there is a unit mass of agents with homogeneous skills ($a_{hr} = 1$ for every $h \in H$ and $r \in R$), and preferences take the form $u_h(c_h, l_h) = f_h(c_h + \epsilon_{hr}\mathbf{1}[l_h = r])$ where ϵ_{hr} are drawn from type I extreme value distribution and f_h is any strictly increasing function. Following McFadden (1973), the labor supply function is:

$$L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \frac{\exp(\theta Z w_r / p^c + B_r)}{\sum_{r'} \exp(\theta Z w_{r'} / p^c + B_{r'})}, \tag{9}$$

where θ and B_r are parameters of the distribution of ϵ_{hr}.

The labor market clearing condition (6) specialized to factor $r \in R$, is $\lambda_r = \sum_{r' \in R} \chi_{r'} \Omega_{c(r')r} + \sum_{j \in N} \lambda_j \Omega_{jr} = \Omega_{cr}$.[18] Equating labor supply with labor demand yields

$$\frac{w_r \exp(\theta Z w_r / p^c + B_r)}{\sum_{r'} w_{r'} \exp(\theta Z w'_r / p^c + B'_r)} = \Omega_{cr}, \tag{10}$$

where Ω_{cr} is a constant. Equations (8) and (10) jointly pin down wages and the consumption price index in the decentralized equilibrium. Given prices and wages, quantities are then straightforward to calculate. Example 9 in the appendix provides a more involved example with intermediate inputs.

---

[18]The second equality follows from the fact that there are no intermediates: $\sum_{j \in N} \lambda_j \Omega_{jr} = 0$; there is a common consumption good: $\Omega_{c(r')r} = \Omega_{cr}$; and spending shares add up to one: $\sum_{r'} \chi_{r'} = 1$.

3 Defining and Characterizing Aggregate Welfare

In this section, we define our measure of aggregate welfare and then characterize it. For convenience, we parameterize technologies by a scalar t and write them as $\boldsymbol{z}(t)$. We assume that $t = 0$ corresponds to an initial equilibrium, which we call the *status quo*. We calibrate the status quo to the allocation observed in the data. Counterfactual technologies of interest are indexed by $t > 0$. As t increases, productivity parameters change. Without loss of generality, we normalize Z to be constant and equal to one as a function of t.[19]

3.1 Definition of TFP-Equivalent Aggregate Welfare

Define the set of feasible allocations given productivity shifters $\boldsymbol{z}(t)$ and aggregate factor-augmenting productivity Z to be

$$\mathcal{C}(t, Z) \equiv \left\{ \{\boldsymbol{c}_h, l_h\}_{h \in H} \text{ supported via competitive equilibrium given } \boldsymbol{z}(t), Z, \text{ and some transfers} \right\}.$$

Feasible here means allocations that can be implemented as competitive equilibrium allocations given some transfers.

Inspired by Debreu (1951), we measure changes in aggregate welfare using equivalent changes in factor-augmenting productivity Z.

Definition 2 (TFP-Equivalent Aggregate Welfare)**.** Aggregate welfare in TFP-equivalent terms at t is

$$A(t) = \max \left\{ Z^{-1} : \{c_h, l_h\}_{h \in H} \in \mathcal{C}(t, Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h \right\}, \quad (11)$$

where $c_h(0)$ and $l_h(0)$ are the consumption and location of h in the status quo.

In words, to measure aggregate welfare, we consider a hypothetical rescaling of Z by a factor $1/A(t)$, and define $A(t)$ as the largest contraction such that every agent can be made at least as well off as in the status quo.[20] Intuitively, if $A(t) = 1.01$, then this

[19] To model changes in aggregate factor-augmenting productivity, we can introduce intermediaries that sell primary factor services to other producers and instead scale their productivities.

[20] This is not exactly the definition in Debreu (1951) because we restrict the set of feasible allocations to be the ones that can be reached using individual-specific lump-sum transfers as opposed to any technologically feasible allocation. This distinction matters because, in this economy, the second welfare theorem need not hold. In Appendix H, we discuss how to further restrict the set of feasible allocations to be the ones that can be supported using place-based, rather than individual-specific, transfers. We show that aggregate welfare using place-based compensations is weakly less than aggregate welfare using individual-specific lump-sum transfers, since compensations using place-based policy are distortionary, but these distortions are second-order around lasseiz-faire.

means that we can shrink the productivity of factors by roughly 1% (more precisely, $1 - 1.01^{-1}$) and still keep every household indifferent to the initial equilibrium. Equivalently, because Z uniformly scales the production possibility set, every household can be kept indifferent to the status quo with roughly 1% less of every good — that is, there is a 1% TFP-equivalent surplus.

Throughout, we assume that at the solution to (11), every agent can be made exactly indifferent to the status quo. This rules out pathological cases where agents are completely disconnected in goods and factor markets.[21]

Remark (Changes in amenities). Although we focus on changes in productivity parameters $\boldsymbol{z}$, the same definition of aggregate welfare and many of the results we present can be extended to cover changes in other parameters as well. For example, we can allow for changes in location characteristics that affect amenity values (e.g. changes in the weather in a region or in the health risk associated with particular occupations). In that case, we index the value of the relevant characteristic by t and apply Equation (11) without any other modification. If the amenity value of choosing location l_h can be written as $u_h(b_{l_h} c_h, l_h)$, where b_{l_h} is a location-specific quality shifter, then an increase in b_{l_h} has the same effect on aggregate welfare $A(t)$ as the same proportional increase in the productivity of the consumption good in that location, $z_{c(l_h)}$.

Remark (Welfare for a subset of agents). Definition (11) can also be applied to measure collective welfare for a subset of agents (e.g. by gender, race, or age). In this case, we contract the endowments of agents in the chosen subset subject to indifference conditions and balanced-budget transfers among only those agents. If the subset contains only a single agent, then $A(t)$ collapses to a compensating variation for that agent.[22]

Remark (Imperfect competition and externalities). Although in the body of the paper we restrict attention to perfectly competitive economies, in Appendix G we show how to calculate aggregate welfare in the presence of distortions and externalities. This extension can be used to study the effects of policies (modelled as implicit or explicit tax-like wedges) or to quantify misallocation.

[21] For example, all households are immobile across locations and only consume goods produced by their own location (i.e. autarky in both goods and factor markets). In such a case, lump-sum transfers between locations have no allocative effect.

[22] Technically, this is a compensating variation defined in ratios rather than differences. Formally, since any individual agent is too small to affect prices, $A(t)$ is the proportional reduction in t factor income that solves $\max_r u_h\left(a_{hr} w_r(t)/(A(t) p_r(t)), r\right) = u_h^0$. This is very similar to the conventional definition of compensating variation but in proportional terms rather than differences. For a comparison of these two measures, see Appendix I.

3.2 Exact Characterization of $A(t)$

We characterize $A(t)$ using the aggregate *compensated* supply and demand functions.[23] The compensated supply and demand functions map wages and prices into labor supplied and spending in each location, holding each agent on a given indifference curve using lump-sum transfers.

3.2.1 Expenditure Function and Compensated Choices

Begin by defining the expenditure function of each consumer.

Definition 3 (Expenditure function)**.** Define $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ to be the (net) expenditure function for agent h given wages $\boldsymbol{w}$, prices $\boldsymbol{p}$, and status quo utility $u_h^0 = u_h(c_h(0), l_h(0))$:

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min\{T_h : u_h(c_h, l_h) \geq u_h^0, \text{ and } \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

We call the location choice $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ associated with this optimization problem *the compensated choice* of agent h.

In words, $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ is the minimum lump-sum transfer agent h requires to be made at least indifferent to some reference utility level u_h^0, given prices and wages. The location choice that the agent makes, given that transfer, is what we call the compensated choice. This definition parallels the classical definition of expenditure functions and compensated demand in consumer theory.[24]

To characterize the compensated choice of each agent, we first define the consumption-equivalent variation.

Definition 4 (Consumption-equivalent variation)**.** Define $\bar{c}_{hl_h}$ as the solution to

$$u_h(\bar{c}_{hl_h}, l_h) = u_h(c_h^0, l_h^0),$$

where c_h^0 is the consumption and l_h^0 is the location of h in the status quo. Hence, $\bar{c}_{hl_h}$ is the consumption agent h must have in location l_h to be indifferent to the status quo.

[23] In Appendix C, we consider a special case in which agents are homogeneous in both preferences and skill levels. In this case, lump-sum transfers are not needed to evaluate the compensated supply and demand functions, since all agents are affected symmetrically by shocks. When $g(c) = \log(c)$, $A(t)$ equals the growth rate of real consumption per capita, which is common across locations. Outside the log case, however, changes in real consumption may vary by location. We show that computing $A(t)$ is as simple as solving for the decentralized equilibrium under different productivity parameters.

[24] See also Small and Rosen (1981) for a related analysis. Whereas they consider discrete consumption choices, given fixed income, here the level of income depends also on the discrete choice (via the wage).

Given the consumption-equivalent variation, the following proposition shows how to compute the compensated choices and the expenditure function at the agent level.

Proposition 1 (Compensated Choices and Expenditure Function)**.** *The compensated choice* $l_h^{comp}(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ *satisfies*

$$l_h^{comp}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) \in \arg\max_{l \in R} \left[\sum_r \left[a_{hr} w_r - p_r \bar{c}_{hr}\right] \mathbf{1}\left[l = r\right] \right].$$

The expenditure function, or transfer needed to ensure indifference to u_h^0*, is*

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = p_{l_h^{comp}} \bar{c}_{hl_h^{comp}} - a_{hl_h^{comp}} w_{l_h^{comp}}.$$

In words, $[a_{hr} w_r - p_r \bar{c}_{hr}]$ is the surplus, in dollars, agent h receives from being sent to location r. The compensated choice maximizes this surplus, and the expenditure function is equal to the negative of the surplus. Proposition 1 gives a straightforward way to calculate the compensated choice and the compensating transfer for each agent given any vector of prices $\boldsymbol{p}$ and wages $\boldsymbol{w}$.

We aggregate agent-level location choices and expenditure functions, given by Proposition 1, to get location-level variables. The compensated labor supply function, the efficiency of units of labor in location r (not including the aggregate shifter Z) given compensating transfers, is:

$$L_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = \int a_{hr} \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = r] dh.$$

Similarly, the compensated spending by households that choose location r is the sum of their labor income and net transfer payments:

$$E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = w_r L_r^{\text{comp}} + \int e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = r] dh,$$

where $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ and $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ are given by Proposition 1. We can also denote the compensated share of total spending by households in location r to be

$$\chi_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = \frac{E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)}{\sum_{r'} E_{r'}^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)}.$$

By duality, compensated and uncompensated labor supply and spending are connected by the identities

$$L_r^{\text{comp}}(\boldsymbol{w},\boldsymbol{p},\boldsymbol{u}^0) = L_r(\boldsymbol{w},\boldsymbol{p},\boldsymbol{e}(\boldsymbol{w},\boldsymbol{p},\boldsymbol{u}^0)), \qquad \text{and} \qquad E_r^{\text{comp}}(\boldsymbol{w},\boldsymbol{p},\boldsymbol{u}^0) = E_r(\boldsymbol{w},\boldsymbol{p},\boldsymbol{e}(\boldsymbol{w},\boldsymbol{p},\boldsymbol{u}^0)).$$

Proposition 1 provides a way to evaluate the compensated share function $\boldsymbol{L}^{\text{comp}}$ and expenditures $\boldsymbol{E}^{\text{comp}}$ (and hence also $\boldsymbol{\chi}^{\text{comp}}$) given any arbitrary vector of wages, prices, and utility levels.

3.2.2 Calculating $A(t)$ using Compensated Choices and Spending

The next result uses the compensated supply and demand functions to solve for $A(t)$.

Theorem 1 (Aggregate Welfare using Compensated Equilibrium)**.** *TFP-equivalent aggregate welfare satisfies*

$$\sum_r E_r^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0) = \sum_r (w_r/A(t)) L_r^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0), \tag{12}$$

The vector of prices $\boldsymbol{p}$ equal marginal costs, (5), *given wages $\boldsymbol{w}$ and productivities $\boldsymbol{z}(t)$. The vector of wages $\boldsymbol{w}$ satisfy market clearing conditions* (6) *and* (7), *given compensated labor supplied $\boldsymbol{L}^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0)$ and demand $\boldsymbol{\chi}^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0)$. The wages, prices, labor supplies, and quantities that satisfy these conditions are not the same as the ones in the decentralized equilibrium. We call them variables in the* compensated *equilibrium instead.*

In the compensated equilibrium, every agent h receives a transfer of $e_h(\boldsymbol{w}/A(t),\boldsymbol{p},u_h^0)$, to keep them indifferent and TFP is contracted by $A(t)$. The value of $A(t)$ is pinned down, according to Equation (12), by the condition that total spending equals income in the compensated equilibrium or, equivalently, that lump-sum transfers add up to zero.

Solving for $A(t)$ using Theorem 1 is relatively straightforward. To see the logic, consider the following iterative procedure. First, conjecture a vector of wages in the compensated equilibrium, $\boldsymbol{w}$, and aggregate TFP shifter $1/A$. (For example, $A = 1$ and $\boldsymbol{w}$ equal to its status quo values.) Use the wages and the productivities, $\boldsymbol{z}(t)$, to solve for prices by setting prices equal to marginal costs, as in (5). Then draw a population of agents with different ϵ's and apply Proposition 1 to get compensated supplies $\boldsymbol{L}^{\text{comp}}(\boldsymbol{w}/A,\boldsymbol{p},\boldsymbol{u}^0)$ and spending $\boldsymbol{E}^{\text{comp}}(\boldsymbol{w}/A,\boldsymbol{p},\boldsymbol{u}^0)$ using conjectured prices, wages, and aggregate TFP shifter. Given $\boldsymbol{L}^{\text{comp}}$ and $\boldsymbol{\chi}^{\text{comp}} \equiv \boldsymbol{E}^{\text{comp}}/\sum_r \boldsymbol{E}_r^{\text{comp}}$, use equations (6) and (7) to update wages, and given $\boldsymbol{L}^{\text{comp}}$ and $\boldsymbol{E}^{\text{comp}}$ use equation (12) to update the TFP shifter A. In a compensated equilibrium, these new wages and TFP shifter coincide with the initial conjectures.

We evaluate the compensated supply and demand functions at the scaled waged vector $\boldsymbol{w}/A(t)$ because in the compensated equilibrium with total factor productivity $1/A(t)$, households earn factor income $a_{hr}w_r/A(t)$, so their compensated choices in (12) depend on $w/A(t)$.

We illustrate Theorem 1 using a simple example.

Example 2 (One-Good Economy). Suppose there is a single consumption good produced linearly from labor. The production technology in location r is

$$y_r = z_r(t)L_r = z_r(t)\int a_{hr}\mathbf{1}[l_h = r]dh,$$

where z_r is the productivity and L_r is the supply of labor in location r. Consumption goods produced in each location are perfectly substitutable and freely traded. The aggregate resource constraint is

$$\int c_h dh = \sum_r z_r(t)L_r.$$

The feasible set is the set of equilibrium allocations given transfers:

$$\mathcal{C}(t,Z) = \left\{\{\boldsymbol{c}_h, \boldsymbol{l}_h\} : \int T_h dh = 0,\, c_h = Za_{hl_h}z_{l_h}(t) + T_h,\, l_h \in \arg\max u_h(c_h, l_h)\right\}.$$

The first condition is that transfers must be feasible, the second are households' budget constraints, and the last states that households choose their locations to maximize utility. Applying Theorem 1 is simple in this economy since the real wage (per efficiency unit) in each location in the compensated equilibrium is equal to exogenous parameters: $z_r(t)$. Hence, (12) is

$$A(t) = \frac{\sum_r \int z_r(t)a_{hr}\mathbf{1}[l_h^{\text{comp}} = r]dh}{\sum_r \int \bar{c}_{hr}\mathbf{1}[l_h^{\text{comp}} = r]dh}, \tag{13}$$

which is the ratio of production (given the original technologies but compensated locations) and the amount of consumption necessary to keep everyone indifferent at those locations. By Proposition 1, the compensated choices satisfy:

$$l_h^{\text{comp}} \in \arg\max_{r\in R}\left\{z_r(t)a_{hr}/A(t) - \bar{c}_{hr}\right\}.$$

This is a simple numerical problem to solve agent-by-agent through simulation.

---

4 Analytical Results for Aggregate Welfare

In this section, we provide a more analytical characterization of aggregate welfare using compensated supply and demand curves (e.g. their elasticities and integrals). Section 4.1 provides a differential exact-hat algebra approach to computing $A(t)$. This has the virtue of being computationally simpler, since it does not require solving a system of nonlinear equations, and it also provides us with some analytical results, including a version of Hulten's theorem. Section 4.2 focuses on a special case where the uncompensated supply system coincides with the compensated one, considerably simplifying analysis.

Throughout, for any variable x, we use $x(t)$ and $x^{\text{comp}}(t)$ to refer to its value in the decentralized equilibrium (where there no transfers and aggregate factor-augmenting productivity $Z = 1$) and the compensated equilibrium (where there are compensating transfers and $Z = 1/A(t)$). When it is clear from context, we suppress the comp superscript to make the notation less dense.

4.1 Sufficient Statistics for Aggregate Welfare

We begin with the following result showing that aggregate welfare is exactly the chained sum of microeconomic productivity shocks, weighted by Domar weights in the compensated equilibrium.

Theorem 2 (Welfare as Area Under Compensated Sales Shares)**.** *The change in TFP-equivalent aggregate welfare satisfies*

$$\log A(t) = \int_0^t \sum_i \lambda_i^{comp}(s) \frac{d \log z_i}{ds} ds, \tag{14}$$

where $\lambda_i^{comp}(s)$ are i's sales divided by GDP in the compensated equilibrium with productivities $\boldsymbol{z}(s)$. Equivalently, for any $t \geq 0$,

$$\frac{d \log A(t)}{dt} = \sum_i \lambda_i^{comp}(t) \frac{d \log z_i}{dt}. \tag{15}$$

with boundary condition $\log A(0) = 0$.

As shown by Baqaee and Burstein (2025), the exact same formula also holds in economies with continuous choice. In this sense, there is nothing conceptually different about measuring aggregate welfare when choices are discrete.

Theorem 2 characterizes $A(t)$ as the solution to a differential equation. Solving $A(t)$ nonlinearly requires knowledge of compensated sales shares, $\lambda_i^{\text{comp}}(s)$, for every $s \in [0, t]$.

We characterize $\lambda_i^{\text{comp}}(s)$ below, but first we make an important observation. In the status quo, $t = 0$, compensated Domar weights coincide with decentralized Domar weights $\lambda_i^{\text{comp}}(0) = \lambda_i(0)$.[25] This allows us to establish the following useful corollary of (15).

Corollary 1 (First-Order Approximation)**.** *To a first-order approximation, at $t = 0$, the change in TFP-equivalent aggregate welfare satisfies*

$$\Delta \log A \approx \sum_i \lambda_i(0) \Delta \log z_i,$$

where $\lambda_i(0)$ is the sales of i divided by GDP in the decentralized equilibrium at the status quo.

Corollary 1 is a generalization of Hulten's theorem to economies with discrete choice. Using Theorem 2 beyond first-order approximations requires knowledge of sales shares, $\lambda_i^{\text{comp}}(t)$, in the compensated equilibrium away from status quo $t > 0$. This is easy to do, using standard methods, if we know the compensated location choices, $\boldsymbol{L}^{\text{comp}}$ and compensated final demand $\boldsymbol{\chi}^{\text{comp}}$.

Given these, computing $\lambda^{\text{comp}}(s)$ follows from the results in Baqaee and Farhi (2019). For example, assume that all production and consumption functions are CES, with the ith producer in the input-output matrix Ω corresponding to a CES aggregator with elasticity of substitution θ_i. This structure is flexible enough to capture any nested-CES production and consumption functions through relabeling.[26] Then the following proposition determines sales shares in both the compensated and uncompensated equilibrium.

Proposition 2 (Prices and Expenditures in Equilibrium)**.** *For a given path of labor supplies $\boldsymbol{L}(t)$ and final demand $\boldsymbol{\chi}(t)$, the following differential equations determine the evolution of sales shares:*

$$d \log \Omega_{ij} = (1 - \theta_i) \left[d \log p_j - \sum_{k \in N} \Omega_{ik} d \log p_k - \sum_{r \in R} \Omega_{ir} d \log w_r \right], \tag{16}$$

$$d \log p_i = -d \log z_i + \sum_j \Omega_{ij} d \log p_j + \sum_{r \in R} \Omega_{ir} d \log w_r, \tag{17}$$

$$d \log \lambda_r = d \log w_r + d \log L_r - \sum_{r'} \lambda_{r'} \left[d \log w_{r'} + d \log L_{r'} \right] \textit{ for } r \in R \tag{18}$$

$$d \log \lambda_i = \lambda_i^{-1} \left(\sum_r d\chi_r \Omega_{c(r)i} + \sum_r \chi_r d\Omega_{c(r)i} + \sum_{j \in N} d \left[\lambda_j \Omega_{ji} \right] \right), \tag{19}$$

[25]This is because the decentralized equilibrium is Pareto efficient (the first welfare theorem holds), and hence, $A(0) = 1$ and the decentralized equilibrium supports the compensated allocation with no transfers needed.

[26]One can extend to non-nested CES along similar lines to Baqaee and Farhi (2019).

where all the variables are indexed by t. If we swap $\boldsymbol{L}$ and $\boldsymbol{\chi}$ for $\boldsymbol{L}^{comp}$ and $\boldsymbol{\chi}^{comp}$, we get the evolution of sales shares in the compensated equilibrium instead. The boundary conditions are the same for both the compensated and decentralized equilibria — spending shares are equal to their values in status quo, and the initial level of prices and wages is irrelevant.

Proposition 2 can be used to calculate either the decentralized equilibrium, given $d\log\chi_r$ and $d\log L_r$, or the compensated equilibrium, given $d\log\chi_r^{\text{comp}}$ and $d\log L_r^{\text{comp}}$. The derivatives of $d\log\chi_r^{\text{comp}}$ and $d\log L_r^{\text{comp}}$ can be computed numerically using Proposition 1. This approach is useful both computationally and analytically. Computationally, it obviates the need to solve a system of nonlinear equations to compute $A(t)$ (i.e. as in Theorem 1). Instead, it requires repeatedly solving a system of linear equations and updating variables (Appendix F provides additional information). Analytically, it allows us to prove some useful results.

Before using Proposition 2, we briefly describe the intuition for equations (16)-(19). The expression in (16) is log-linearized input demand by i for input j, which depends on the elasticity of substitution, θ_i, and on how the price of j changes relative to the marginal cost of i. The expression in (17) is Shephard's lemma, relating the marginal cost of i to the expenditure-share weighted change in input prices and the productivity shock to i. Equation (18) is log-linearizing the definition of λ_r and (19) is the log-linearized versions of the market clearing conditions (6).

A simple corollary of Theorem 2 and Proposition 2 is the following exact characterization of $A(t)$.

Corollary 2 (Cobb-Douglas Economies with Common Consumption)**.** *Suppose that all production functions are Cobb-Douglas and that there is a common consumption good in every location. Then, for every t,*

$$\log A(t) = \sum_i \lambda_i(0)[\log z_i(t) - \log z_i(0)],$$

regardless of the shape of the $g(c)$ function and distribution of taste and productivity shifters.

If all production functions are Cobb-Douglas and there is a common consumption good, then the right-hand sides of both (16) and (19) are zero.[27] That is, changes in location choices and spending shares do not affect the sales shares. In fact, sales shares are constant: $\lambda_i^{\text{comp}}(s) = \lambda_i^{\text{comp}}(0)$. Since compensated variables are equal to decentralized

[27]When $\theta_i = 1$ for every i, it follows from (16) that $d\Omega = 0$. From (19), we know that $d\lambda_i = \left(\sum_r d\chi_r \Omega_{c(r)i} + \sum_j d\lambda_j \Omega_{ji}\right) = \sum_j d\lambda_j \Omega_{ji}$. The last equality follows from the common consumption good assumption, which implies $\Omega_{c(r)i} = \Omega_{c(r')i}$ for every r and i. Hence, the only solution is $d\lambda = 0$.

variables in the status quo, $\lambda_i^{\text{comp}}(0) = \lambda_i(0)$, the integral in Theorem 2 can be solved easily.

4.2 Equivalence of Compensated and Uncompensated Supply

Theorem 2 shows that computing $A(t)$ nonlinearly requires knowledge of compensated Domar weights. In this section, we consider a tractable special case where compensated Domar weights are simple to solve for, because the compensated and uncompensated supply systems coincide.

Assumption 1 (Common Consumption Good and Affine g)**.** Suppose that the consumption aggregator in every location is the same as every other location, so that the consumption price index p_r is the same in every location r and can be denoted by p^c. Suppose that preferences take the form $u_h(c_h, l) = f_h(c_h + \epsilon_{hl})$ for some arbitrary increasing function f_h. (i.e. the function $g(c)$ is affine).

Under this assumption, lump-sum transfers do not change agents' location choices. To see this, note that if preferences are $f_h(c_h + \epsilon_{hl})$ then a constant increase in consumption in every location does not alter household h's location choice. Moreover, if the price of consumption is the same in every location, then a lump-sum transfer gives the same amount of consumption in every location. That is, under Assumption 1, agent h chooses the location with the highest $(Za_{hr}w_r + T_h)/p^c + \epsilon_{hr}$ — and this choice is independent of the value of T_h. This implies the following.

Proposition 3 (Characterization Using Differential Equations)**.** *If Assumption 1 holds, then the compensated supply function coincides with the uncompensated supply function:*

$$L^{comp}(\boldsymbol{w}/A, p^c, \boldsymbol{u}) = L(\boldsymbol{w}/A, p^c) = L(\boldsymbol{w}/(Ap^c), 1)$$

So, for any $t > 0$, the total derivative of L^{comp} with respect to t is just:

$$dL_r^{comp} = \sum_{r'} \frac{\partial L_r}{\partial \left[w_{r'}/(Ap^c)\right]} d(\frac{w_{r'}}{Ap^c}), \tag{20}$$

where prices and wages are evaluated in the compensated equilibrium at t. Moreover, because there is a common consumption good, equation (19) *does not depend on compensated demand χ^{comp}. Hence, equations* (15) *to* (20) *fully pin down $A(t)$,*

Under Assumption 1, the compensated and uncompensated supply functions coincide. Furthermore, the distribution of spending χ^{comp} is irrelevant for determining compensated sales shares λ^{comp} because budget shares of households are the same in every location r. In particular, the distribution of spending χ^{comp} only shows up in (19) and has no effect (since $\Omega_{c(r)i} = \Omega_{ci}$ for every $r \in R$). Therefore, Proposition 3 allows us to compute $A(t)$ as the solution to a system of ordinary differential equations.

The following example solves these differential equations in the one-good economy and relates $A(t)$ to the area under compensated supply curves.

Example 3 (One-Good Economy under Assumption 1). Consider Example 2 again: there is one common and freely-traded consumption good made linearly from labor in each location. In this economy, since $\sum_r \lambda_r = 1$, we can rewrite (15) as

$$\sum_r \lambda_r^{\text{comp}}(t) \frac{[d \log z_r / A(t)]}{dt} = 0.$$

By definition of λ_r^{comp}, and since $w_r(t) = z_r(t)$, this is

$$\sum_r L_r^{\text{comp}}(t) \frac{d\,[z_r(t)/A(t)]}{dt} = 0.$$

Since this equation holds for every $t > 0$, we can integrate both sides to get

$$\int_0^t \sum_r L_r^{\text{comp}}(s) \frac{d\,[z_r(s)/A(s)]}{ds} ds = 0. \tag{21}$$

In words, (21) shows that $A(t)$ is the reduction in TFP such that the area under the compensated labor supply curve is zero. Suppose that Assumption 1 holds, so that we can use the uncompensated supply function in place of the compensated supply function, and hence

$$\int_0^t \sum_r L_r\left(\frac{\boldsymbol{z}(s)}{A(s)}, 1\right) \frac{d\,[z_r(s)/A(s)]}{ds} ds = 0.$$

If we know the functional form for the uncompensated supply function, then the equation above is a single nonlinear equation that pins down $A(t)$. For example, suppose that workers have homogeneous skills ($a_{hr} = 1$ for every $h \in H$ and $r \in R$), and that the uncompensated supply system is logit. In this case, the uncompensated supply is

$$L_r\left(\frac{\boldsymbol{z}(s)}{A(s)}, 1\right) = \frac{\exp(\theta \frac{z_r(s)}{A(s)} + B_r)}{\sum_{r'} \exp(\theta \frac{z_{r'}(s)}{A(s)} + B_{r'})},$$

where B_r is an exogenous location-level taste shifter or amenity. With this functional form, we can evaluate the integral in closed-form as:

$$\log\sum_r \exp(\theta\frac{z_r(t)}{A(t)} + B_r) - \log\sum_r \exp(\theta z_r(0) + B_r) = 0. \tag{22}$$

This nonlinear equation pins down $A(t)$.

The general equilibrium structure in this example is simple since real wages must equal labor productivity in each location (i.e. labor demand is infinitely elastic at a fixed real wage). Example 10 in the appendix provides a slightly richer multi-industry example where all households consume the same CES consumption bundle, but choose which industry to work in based on their preferences.

Relation to social surplus function. In Appendix E we show that under Assumption 1, $A(t)$ can be computed using the utilitarian social welfare function $U(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} g(c_{l_h}) + \epsilon_{h,l_h}]$, which is called the "*Social Surplus Function*" in the discrete choice literature (see McFadden, 1981). The reason is that, by the Williams-Daly-Zachary theorem, the social surplus function is the integral of uncompensated labor supply. When Assumption 1 holds, compensated and uncompensated labor supply coincide, so the social surplus function is also the integral of compensated supply. Hence, by a similar logic to (21), $U(\boldsymbol{c})$ can be used to determine $A(t)$. However, if Assumption 1 does not hold, then the social surplus function can no longer be used to compute $A(t)$ because compensated and uncompensated labor supply are no longer the same.

5 Second-Order Approximation of Aggregate Welfare

In this section, we investigate nonlinearities in cases where compensations do affect location choices (i.e. Assumption 1 does not hold). We do this by relating derivatives of compensated labor supply and demand to derivatives of their uncompensated counterparts. This allows us to characterize changes in aggregate welfare to a second-order approximation in terms of uncompensated elasticities and expenditure shares. Throughout this section, we explicitly use the fact that preferences are ordinally additively separable. We also assume that idiosyncratic tastes $\boldsymbol{\epsilon}$ have an absolutely continuous density $f(\boldsymbol{\epsilon})$.

The previous section shows that to a first-order approximation, $A(t)$ is a Domar-weighted sum of microeconomic productivity shocks. Differentiating (15) once more and applying a second-order Taylor expansion at $t = 0$ yields the following approximation for $A(t)$.

Corollary 3 (Second-Order Approximation). *To a second-order approximation, the change in TFP-equivalent aggregate welfare satisfies*

$$\Delta \log A \approx \sum_i \lambda_i(0) \Delta \log z_i + \frac{1}{2} \sum_i \Delta \lambda_i^{comp}(0) \Delta \log z_i,$$

where $\Delta \lambda_i^{comp}$ *is the (first-order) change in the compensated sales of i divided by GDP at the status quo.*

The intuition is very similar to the formulas derived by Baqaee and Farhi (2019) for an economy without discrete choice. If sales shares in the compensated equilibrium rise in response to positive productivity shocks, then aggregate welfare rises more quickly than its first-order approximation would suggest. If sales shares fall, then the reverse happens.

Calculating $d\lambda_i^{\text{comp}}(0)$ is a straightforward application of Proposition 2 if we know $dL^{\text{comp}}(0)$ and $d\chi^{\text{comp}}(0)$. When Assumption 1 holds, this is simple to do using Proposition 3, since dL^{comp} can be computed using the uncompensated supply system and $d\chi^{\text{comp}}$ is irrelevant. In this section, we focus on characterizing dL^{comp} and $d\chi^{\text{comp}}$ at the status quo, without imposing Assumption 1, and we compare these to their decentralized uncompensated counterparts.

To simplify analysis, we focus on the case with only preference heterogeneity and abstract from skill-heterogeneity. This implies that agents make different choices only due to differences in tastes.[28]

Assumption 2 (Homogeneous Idiosyncratic Skills). Agents have homogeneous idiosyncratic skill, which we normalize to one: for every agent $h \in H$ and location $r \in R$, we have $a_{hr} = 1$.

We now proceed to characterizing $d\chi^{\text{comp}}$ (in Proposition 4) and dL^{comp} (in Proposition 5) at the status quo. Given these derivatives, we can compute $d\lambda^{\text{comp}}$ using Proposition 2, and hence, we can approximate $\log A(t)$ to a second-order using Corollary 3.

Proposition 4 (First-Order Change in Spending Shares). *At* $t = 0$*, in the decentralized equilibrium, the spending share in location r satisfies*

$$d \log \chi_r = d \log w_r - \mathbb{E}_\chi \left[d \log w \right] + d \log L_r - \mathbb{E}_\chi \left[d \log L \right],$$

[28] Assumption 2 can be slightly relaxed to allow for the possibility that some agents h have skill $a_{hr} = 0$ for some locations r. This is because such an extension is isomorphic to giving those agents ϵ_{hr} values that are so low that those agents would never choose r in equilibrium. Since we do not impose strong assumptions on the distribution of ϵ's, we model such scenarios by picking an appropriate distribution of ϵ.

where the changes in wages $\boldsymbol{w}$ and locations $\boldsymbol{L}$ are in the decentralized equilibrium and $\mathbb{E}_\chi[d\log L] = \sum_r \chi_r(0) d\log L_r$. At $t = 0$, in the compensated equilibrium, we instead have

$$d\log \chi_r^{comp} = d\log p_r - \mathbb{E}_\chi\left[d\log p\right] + d\log L_r^{comp} - \mathbb{E}_\chi\left[d\log L^{comp}\right],$$

where the changes in prices $\boldsymbol{p}$ and locations $\boldsymbol{L}^{comp}$ are in the compensated equilibrium and $\mathbb{E}_\chi[d\log p] = \sum_r \chi_r(0) d\log p_r$.

The intuition is the following. In the decentralized equilibrium, the share of spending by each location must move in line with the share of income earned by households in that location. This is the product of wages w_r and population shares L_r. A location's share of spending rises if wages in that location increase quickly or if households flock to that location. On the other hand, in the compensated equilibrium, the share of spending in each location depends on the rate at which the cost of consumption in location r has increased and the population shares in the compensated equilibrium. If consumption prices rise quickly in a location, or households flock to a location, in the compensated equilibrium, then that location's share of total spending rises.

The next proposition provides a general characterization of dL^{comp} in terms of the derivatives of the uncompensated labor supply function at the status quo $\partial L_r/\partial(w_r'/p_r')$. This is important because the derivatives of the uncompensated supply system are estimable using exogenous variation in the decentralized equilibrium.[29]

Proposition 5 (First-Order Changes in Location Choices)**.** *The change in the share of households in location r can be decomposed into inflows minus outflows:*

$$dL_r = \sum_{r' \neq r} dL_{r' \to r} - \sum_{r' \neq r} dL_{r \to r'}, \tag{23}$$

where $dL_{r' \to r}$ is the share of switchers from r' to r. To a first-order, the flows satisfy:

$$dL_{r \to r'} = \left[\frac{\partial L_{r'}}{\partial(w_r/p_r)} d(\frac{w_r}{p_r}) - \frac{\partial L_r}{\partial(w_{r'}/p_{r'})} d(\frac{w_{r'}}{p_{r'}})\right] \mathbf{1}\left[\frac{\partial L_r}{\partial(w_{r'}/p_{r'})} d(\frac{w_{r'}}{p_{r'}}) \leq \frac{\partial L_{r'}}{\partial(w_r/p_r)} d(\frac{w_r}{p_r})\right], \tag{24}$$

where the changes in wages and prices are the ones in the decentralized equilibrium.

[29] Formally, in the decentralized equilibrium, where transfers are zero, the supply function $L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ is a function only of the vector of real wages per efficiency unit in each location w_r/p_r. Hence, $L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{0}) = L_r(\boldsymbol{w}/\boldsymbol{p}, 1, \boldsymbol{0})$, where with some abuse of notation, $\boldsymbol{w}/\boldsymbol{p}$ is the vector of real wages. This allows us to define $\partial L_r/\partial(w_r'/p_r')$, which can be estimated using the observed changes in market-level labor supply in response to exogenous variation in real wages.

The compensated supply function instead satisfies:

$$dL_r^{comp} = \sum_{r' \neq r} dL_{r' \to r}^{comp} - \sum_{r' \neq r} dL_{r \to r'}^{comp},$$

where $dL_{r' \to r}^{comp}$ is the share of switches from r' to r in the compensated equilibrium. At $t = 0$, this satisfies:

$$dL_{r \to r'}^{comp} = \frac{\partial L_r}{\partial \left[w_{r'}/p_{r'}\right]} \frac{1}{p_{r'}} \left(d(\frac{w_r}{Ap_r})p_r - d(\frac{w_{r'}}{Ap_{r'}})p_{r'} \right) \times \mathbf{1} \left[d(\frac{w_r}{Ap_r})p_r \leq d(\frac{w_{r'}}{Ap_{r'}})p_{r'} \right], \quad (25)$$

where the changes in wages and prices are the ones in the compensated equilibrium and $d \log A$, at $t = 0$, is given by Corollary 1.

Equation (24) states that in the decentralized equilibrium, households switch from r to r' if the change in L_r due to real wage changes in r' are outpaced by the change in $L_{r'}$ due to real wage changes in r. The rate at which households switch depends on cross-derivatives between r and r'. The overall change in the share of households choosing r, in (23), depends on the sum of all these pairwise switchings.

Equation (25) is the counterpart to (24) in the compensated equilibrium. Here, households are given individualized transfers and aggregate factor productivity is adjusted to ensure every household can be made exactly indifferent to the status quo. In this case, households switch from r to r' if the compensated real wage in dollar terms rises faster in r' than in r, and the rate at which households switch depends on the (uncompensated) cross-derivative of L_r with respect to the real wage in r'. Crucially, the partial derivatives in (25) are those of the uncompensated supply system evaluated at the status quo in the decentralized economy.

We illustrate Corollary 3 and Proposition 5 with a simple example.

Example 4 (Second-Order Approximation of Aggregate Welfare with Fréchet). Consider again the one-good economy described in Example 2. Applying Corollary 3 to the one-good economy we get that, to a second-order,

$$\Delta \log A \approx \mathbb{E}_\lambda[\Delta \log z] + \frac{1}{2} \left[Var_\lambda \left(\Delta \log z \right) + Cov_\lambda(\Delta \log L^{\text{comp}}, \Delta \log z) \right], \quad (26)$$

where the variance and covariance operators use λ as the probability weights and we use the fact that $d\lambda_r^{\text{comp}} = \lambda_r(d\log(z_r L_r^{\text{comp}}) - \mathbb{E}_\lambda[d\log(zL^{\text{comp}})])$. In words, the nonlinear change in aggregate welfare depends on the variance of the shocks and the covariance of compensated migrations with the shocks. The variance term reflects the fact that, even if households are not mobile, an increase in productivity in one location raises that location's sales share, which magnifies the value of positive productivity growth in that location. The covariance reflects the fact that if, in the compensated equilibrium, households move to locations experiencing more rapid productivity growth, then this further boosts the compensated sales shares of producers in that location, and raises aggregate welfare.

To solve out for $\Delta \log \boldsymbol{L}^{\text{comp}}$, suppose that $u_h(c_h, l) = \log(c_h) + \epsilon_{hl}$ and $\exp(\epsilon_{hl})$ is distributed according to a Fréchet distribution with shape parameter θ and scale parameters $\boldsymbol{B}$ (i.e. ϵ_{hl} is a type I extreme value distribution). Under this assumption, the share of households that choose location r given a vector of real wages per efficiency unit $\boldsymbol{z}$ is

$$L_r(z) = \frac{B_r z_r^{\theta}}{\sum_{r'} B_{r'} z_{r'}^{\theta}}, \tag{27}$$

for some parameters $B_r > 0$ (see, e.g. Redding and Rossi-Hansberg, 2017). Since in this one-good economy, the real wage equals productivity, the uncompensated cross-derivatives of supply are:

$$\frac{\partial L_{r'}}{\partial w_r} = -\theta L_{r'} L_r \frac{1}{z_r}.$$

For simplicity, suppose that initial productivities, $z_r(0)$, are the same in every location. Then applying Proposition 5 yields

$$\Delta \log L_r^{\text{comp}} = \theta \left(\Delta \log z_r - \mathbb{E}_{\lambda} \left[\Delta \log z \right] \right).$$

In this example, changes in population shares in the compensated equilibrium, $d \log L^{\text{comp}}$, are the same as the ones in the decentralized equilibrium, $d \log L$, up to a first-order approximation. (If $z_r(0)$ are not the same, then this equation changes).

Substituting this into the approximation for $\Delta \log A$ in (26) yields:

$$\Delta \log A \approx \mathbb{E}_{\lambda}[\Delta \log z] + \frac{1}{2}(1 + \theta) Var_{\lambda} \left(\Delta \log z \right).$$

Hence, the higher is θ, the more convex is aggregate welfare.[30]

[30] If instead the share function were logit (i.e. $g(c_h)$ is linear and ϵ_{hr} are type I extreme value with parameter θ^{logit}), then $\Delta \log A \approx \mathbb{E}_{\lambda}[\Delta \log z] + \frac{1}{2}(1 + \theta^{\text{logit}} z(0)) Var_{\lambda} \left(\Delta \log z \right)$, where we abuse notation and treat $z(0)$ as a scalar because productivity in every location is the same in the status quo. Calibrating the derivative of L_r with respect to real wages in r' to be the same as in the Fréchet model implies $\theta^{\text{logit}} = \theta / z(0)$. Hence, the implications for $\Delta \log A$ in the two models are the same up to a second-order approximation as long as we match the same cross-derivatives of supply.

6 Comparison to Other Aggregate Measures

In this final section, we compare our measure of aggregate welfare with three alternative aggregate measures that are popular in the literature. In Section 6.1, we relate our measure to real output (GDP) and multi-factor productivity growth as measured in the national accounts. In Section 6.2, we contrast our measure with the popular utilitarian or average utility measure. Finally, in Section 6.3, we compare our results to the sum of compensating variations (i.e. Kaldor-Hicks efficiency) previously analyzed by Small and Rosen (1981) in a discrete-choice context. Throughout the section, we quantify the differences between the different measures using the one-good economy calibrated to U.S. data.

6.1 Real Output

We contrast changes in aggregate welfare in TFP equivalent terms, defined via $A(t)$, with changes in a chain-weighted (Divisia) index of real output, as measured in the national accounts.

Definition 5 (Real Output)**.** The change in real output is the share-weighted sum of changes in final consumption quantities:

$$\log Y(t) = \int_0^t \sum_{r \in C} \frac{p_r(s) c_r(s)}{\sum_{j \in C} p_j(s) c_j(s)} \frac{d \log c_r}{ds} ds,$$

where C is the set of consumption bundles and $c_r(t) = \int c_h(t) \mathbf{1}[l_h(t) = r] dh$ is total consumption of final good r at t.

Using Hulten (1978), we can write real output as a Domar-weighted sum of technology and labor supply changes:

$$\log Y(t) = \int_0^t \left[\sum_{i \in N} \lambda_i(s) \frac{d \log z_i}{ds} + \sum_{r \in R} \lambda_r(s) \frac{d \log L_r}{ds} \right] ds.$$

To a first-order approximation in t, evaluated at $t = 0$, we have

$$\Delta \log Y \approx \sum_i \lambda_i(0) \Delta \log z_i + \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r.$$

Real output responds positively to productivity growth, and — holding productivities fixed — benefits from relocations of labor towards higher-wage locations. These labor relocations, ΔL_r, are endogenous and require solving the general equilibrium model. Real output can fall in response to a positive productivity shock that induces large flows out of very high-wage regions into the still-low-wage region.[31] Intuitively, because agents care about amenities, they may move to a lower-wage region and this reduces production even though every agent can be made weakly better off.[32]

In contrast, aggregate welfare, $A(t)$, does not respond to relocations (see Corollary 1). If households choose to move from low to high wage locations, this has no first-order effect on aggregate welfare. The increase in wages experienced by movers is exactly offset by the reduction in the amenity value of the move. Hence, whereas real output changes due to the changes in location choices, aggregate welfare does not. We summarize this in the proposition below.

Proposition 6 (First-Order Difference between Aggregate Output and Welfare)**.** *To a first-order approximation, the difference between aggregate output and aggregate welfare in terms of TFP equivalents is given by the change in income caused by relocation:*

$$\Delta \log Y - \Delta \log A \approx \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r.$$

Hence, real output and aggregate welfare can differ, even in sign, to a first-order approximation. The following example quantifies these differences.

Example 5 (Quantitative Difference Between Output and Aggregate Welfare). We compute the quantitative difference between the response of aggregate welfare and real output to productivity shocks using a calibrated one-good economy with isoelastic labor supply, as in Equation (27), with elasticity parameter $\theta = 1.5$. We consider an economy with 588 locations calibrated to US commuting zones.[33] We choose productivities z_r and

[31] In the one-good economy with isoelastic labor supply, as in Example 4, the elasticity of real output with respect to a productivity shock is

$$\frac{\Delta \log Y}{\Delta \log z_r} \approx \frac{w_r(0) L_r(0)}{\sum_k w_k(0) L_k(0)} + \theta \left[\frac{w_r(0)}{\sum_k w_k(0) L_k(0)} - 1 \right] L_r(0).$$

This elasticity can be negative if θ is high and w_r relatively low.

[32] There is a strong analogy to economies with labor-leisure choice. In such economies, real output can fall even as welfare rises because of the endogenous response of leisure. This happens because GDP does not include the value of leisure. In contrast, because of the envelope theorem, in an economy with continuous labor-leisure choice, $\Delta \log A$ still obeys Hulten's theorem (as does $\Delta \log A^{MFP}$).

[33] We define locations using the USDA Economic Research Service commuting-zone classification based on county boundaries (U.S. Department of Agriculture, Economic Research Service, 2026). The ERS crosswalk assigns each county or county-equivalent to exactly one commuting zone; commuting zones may contain multiple counties. After excluding Puerto Rico, this leaves 588 commuting zones in our sample. County GDP is from BEA's GDP by County series (U.S. Bureau of Economic Analysis, 2026), and county labor force is from the BLS Local Area Unemployment Statistics program (U.S. Bureau of Labor Statistics,

amenities B_r in each region to match the observed GDP share and labor-force share of each commuting zone in 2024.

Table 1 displays the change in aggregate welfare and aggregate output in response to a 1% rise in productivity in selected locations.[34] The gap between real output and aggregate welfare can be substantial. In response to a productivity shock in a high-productivity location like New York, the increase in real output is almost twice as large as aggregate welfare, consistent with the logic in Proposition 6. In response to a shock in a low-productivity location such as Brownsville, TX, real output falls even though every agent is weakly better off in the new equilibrium.

Table 1: Aggregate effects of a 1 percent productivity increase in selected locations

Location of shock	$\Delta \log A / \Delta \log z_r$	Sales share λ_r	$\Delta \log Y / \Delta \log z_r$
San Francisco–Oakland, CA	0.031	0.031	0.050
New York, NY	0.057	0.057	0.091
Seattle–Tacoma, WA	0.024	0.024	0.034
Brownsville, TX	0.001	0.001	-0.000
McAllen, TX	0.002	0.002	0.000
Daytona Beach, FL	0.001	0.001	0.000

Notes: Entries are elasticities with respect to the regional productivity increase. Sales share is λ_r in the initial equilibrium. Results use $\theta = 1.5$ and 400,000 simulated households.

If we define multi-factor productivity as output growth minus the growth in quality-adjusted labor:

$$\Delta \log A^{MFP} \approx \Delta \log Y - \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r,$$

then $\Delta \log A^{MFP} \approx \Delta \log A$ to a first-order.[35] The intuition here is that if we separate changes in output due to technology shocks from changes in output due to relocation, then multi-factor productivity, as measured by a national income accountant, coincides to a first-order with our measure of aggregate welfare.

We note an important and tractable special case. Suppose there is no amenity value associated with different locations, and that there is a single consumption good. Agents can have idiosyncratic skills by location, so that a_{hr} can vary as a function of both h and r.

2026). For each commuting zone r, we sum GDP and labor force across all counties assigned to that commuting zone.

[34]The change in aggregate welfare is very well approximated by the shocked region's sales share (also displayed in the table), which shows that the first-order Hulten approximation in Corollary 1 is very accurate for a 1% shock.

[35]The nonlinear definition of $A^{MFP}(t)$ is $\log A^{MFP}(t) = \log Y(t) - \int_0^t \sum_r \lambda_r(s)(d \log L_r / ds) ds$.

This leads agents to make different choices in equilibrium. Nevertheless, in this case, real output, $Y(t)$, and our measure of aggregate welfare, $A(t)$, coincide.

Proposition 7 (Coincidence of $Y(t)$ and $A(t)$)**.** *Suppose that ϵ_{hr} does not vary by h and r (i.e. $\epsilon_{hr} = \epsilon$), and that there is a common consumption good. Then*

$$A(t) = A^{MFP}(t) = Y(t).$$

This follows from the fact that under these assumptions, $\boldsymbol{L} = \boldsymbol{L}^{\text{comp}}$, and changes in location spending shares, χ^{comp} have no effect on relative prices since all households spend income on the same consumption good (e.g. $d\chi$ drops out of equation (19)). Hence, $\lambda_i^{\text{comp}}(s) = \lambda_i(s)$. This implies that $A(t) = A^{MFP}(t)$. Furthermore, since every household chooses location to maximize nominal income, households that move from one location to another do not experience a change in their nominal wage, which implies that $d\log Y = d\log A^{MFP}$ — since this holds for any $s > 0$, by integration, it follows that $Y(t) = A^{MFP}(t)$.

The assumptions of Proposition 7 hold in many models of skill-based occupational (but not spatial) choice. A typical assumption in these models is that households vary in their efficiency of different occupations, but have the same preferences across occupations, and consume the same consumption good regardless of their choice of occupation. In such models, our measure of welfare $A(t)$ coincides with real output $Y(t)$.

6.2 Utilitarian Social Welfare

In economies where households have heterogeneous preferences, by far the most common way to evaluate aggregate welfare is the utilitarian social welfare function (sometimes called "ex-ante expected utility").

Definition 6. Define utilitarian social welfare function to be

$$W(\boldsymbol{c}(t)) = \mathbb{E}[\max_{l_h} u_h(c_{l_h}(t), l_h)], \tag{28}$$

where $\mathbb{E}$ is a population-weighted cross-sectional average and $c_{l_h}(t)$ is consumption in location l_h at t. Although the function in (28) is sometimes called "expected utility," the expectation is over preference parameters not lotteries, so this is a misleading label.[36]

[36]Expected utility is formally defined to be a representation of an ordinal preference relation over lotteries of allocations (see, e.g., chapter 6, of Mas-Colell et al., 1995, and Footnote 15 for a definition in the context of our model) not an expectation of utility values across different agents. In this paper, each household has

The consumption-equivalent variation for the utilitarian social welfare function W is a scalar $A^W(t)$ that solves:

$$W(\boldsymbol{c}(t)/A^W(t)) = W(\boldsymbol{c}(0)),$$

Unlike $A(t)$ and $Y(t)$, the average-utility-based measure $A^W(t)$ is not invariant to monotone transformations of individual utility functions. In particular, its value is not pinned down by any observables. To see this, recall that $u_h(c_h, l) = f_h(g(c_h) + \epsilon_{hl})$, where f_h is an arbitrary strictly increasing function. The function f_h has no testable implications in terms of households' choices. In particular, any choice of f_h represents the same underlying preference relation $\succeq_h$. However, altering f_h has important implications for the value of A^W. We illustrate this fact using a simple example below.

Example 6 (Average utility not pinned down by observables). Consider a two-region example, $R = 2$, and suppose agent h has utility function:

$$u_h(c_r, r) = f_h(g(c_r) + \epsilon_{hr}) = \exp\left(\log c_r + \epsilon_{hr} + \log \bar{\epsilon}_h\right) = \bar{\epsilon}_h \varepsilon_{hr} c_r,$$

where $\varepsilon_{hr} = \exp(\epsilon_{hr})$ are independent Fréchet random variables with common shape parameter θ and location-specific scale parameters B_r, which capture amenities. Suppose that skills are homogeneous $a_{hr} = 1$ for every r and h. As in Example 4, the labor supply in location r is

$$L_r = \frac{B_r c_r^\theta}{B_1 c_1^\theta + B_2 c_2^\theta}.$$

Because monotone transformations do not alter choices, the value of $\bar{\epsilon}_h$ is not pinned down by observables (for example, it has no bearing on L_r).

However, the average utility metric $A^W(t)$ depends on the choice of $\bar{\epsilon}_h$. In particular, using the law of total expectation, we can write

$$A^W(t) = \frac{\mathbb{E}\left[\bar{\epsilon}_h \mathbb{E}\left[\max_{l_h}\{c_{l_h}(t)\varepsilon_{hl_h}\} \middle| \bar{\epsilon}_h\right]\right]}{\mathbb{E}\left[\bar{\epsilon}_h \mathbb{E}\left[\max_{l_h}\{c_{l_h}(0)\varepsilon_{hl_h}\} \middle| \bar{\epsilon}_h\right]\right]}, \tag{29}$$

If we further assume that the household-level shifter $\bar{\epsilon}_h$ is independent of the taste shifters $\{\varepsilon_{hr}\}$ (e.g. $\bar{\epsilon}_h$ is constant), then, as shown by Redding and Rossi-Hansberg (2017), $A^W(t)$

fixed preferences u_h and there is no lottery across household tastes. This is to say, no household ever makes a choice about the parameters of their utility function, which means preferences over these parameters cannot be revealed by choices.

can be written in closed-form as[37]

$$A^W(t) = \left[\frac{B_1 c_1^\theta(t) + B_2 c_2^\theta(t)}{B_1 c_1^\theta(0) + B_2 c_2^\theta(0)}\right]^{\frac{1}{\theta}}. \tag{30}$$

This is a very popular measure of aggregate welfare in spatial models.

However, inspection of Equation (29) shows that the untestable assumption about the joint independence of $\bar{\epsilon}_h$ and ε_{hr} is important. Other assumptions will lead to other social welfare functions that are consistent with the same observables. For example, suppose that instead of assuming $\bar{\epsilon}_h$ is independent of ε_{hr}, we assume that $\bar{\epsilon}_h$ is equal to $1/\mathbb{E}[\varepsilon_{hr}|h]$. That is, we normalize the utility of agent h by the average value of ε_{hr} across locations so that, on average, taste shocks for every household are equal to one (households cannot have higher taste parameters in every location).

Under this assumption, (29) is instead,

$$A^W(t) = \frac{D_1(L_1(t))c_1(t) + D_2(L_2(t))c_2(t)}{D_1(L_1(0))c_1(0) + D_2(L_2(0))c_2(0)},$$

where $D_r(x) = \int_0^x B_r^{\frac{1}{\theta}} u^{-\frac{1}{\theta}} / (B_r^{\frac{1}{\theta}} u^{-\frac{1}{\theta}} + B_{-r}^{\frac{1}{\theta}} (1-u)^{-\frac{1}{\theta}}) du$ and $L_r(t) = B_r c_r(t)^\theta / \sum_{r'} B_{r'} c_{r'}(t)^\theta$. Hence, with this alternative untestable and equally plausible assumption about the average level of taste shocks by household, we arrive at different results about $A^W(t)$. Since these two assumptions are observationally equivalent, there is no conceivable choice data that can distinguish between these two assumptions.

Table 2: Welfare effects of a 1 percent productivity increase in selected commuting zones

Location of shock	$\Delta \log A$	$\Delta \log A_1^W$	$\Delta \log A_2^W$	$\Delta \log A_3^W$	$\Delta \log A_4^W$
San Francisco–Oakland, CA	0.031	0.018	0.020	0.024	0.059
New York, NY	0.057	0.034	0.040	0.050	0.098
Seattle–Tacoma, WA	0.024	0.017	0.019	0.023	0.003
Brownsville, TX	0.001	0.001	0.001	0.001	0.000
McAllen, TX	0.002	0.004	0.004	0.004	0.000
Daytona Beach, FL	0.001	0.002	0.002	0.001	0.014

Notes: Entries are elasticities with respect to the regional productivity increase. The columns $\Delta \log A_i^W$ report utilitarian welfare using different normalizations of household-level taste shifters: A_1^W sets $\bar{\epsilon}_h = 1$, A_2^W sets $\bar{\epsilon}_h = 1/\mathbb{E}[\varepsilon_{hr}|h]$, A_3^W sets $\bar{\epsilon}_h = 1/\max_r\{\varepsilon_{hr}\}$, and A_4^W sets $\bar{\epsilon}_h = 10^8$ for households whose largest idiosyncratic taste is in a commuting zone containing a CA, NY, or FL county, and $\bar{\epsilon}_h = 1$ otherwise.

[37]Technically, if $\theta < 1$, then $\mathbb{E}[u_h(c_h(t), l_h(t))]$ diverges under the assumption that $\bar{\epsilon}_h$ is independent of ε_{hr}. However, $A^W(t)$ is well-defined even for $\theta < 1$ in the limit as the number of households goes to infinity.

Table 2 quantifies the difference between A^W and A using the same calibrated economy as in Example 5. We display the consumption-equivalent for average utility, A^W, for four alternative choices of household-level taste shifters $\bar{\epsilon}_h$. Different normalizations of $\bar{\epsilon}_h$ matter quantitatively and result in consumption-equivalents for average utility that can be higher or lower than TFP-equivalent aggregate welfare A (which is invariant to $\bar{\epsilon}_h$).

The following example considers optimal place-based policies chosen by a policymaker who maximizes utilitarian welfare. The policymaker intervenes in a Pareto-efficient equilibrium and chooses Pareto-inefficient policies. The direction and magnitude of these policies depend on the chosen cardinalization of individual utility.

Example 7 (Optimal policy for average utility). Consider again the one-good economy in Example 6. Suppose that the policymaker chooses place-based consumption levels to maximize the utilitarian social welfare function (28) subject to feasibility,

$$\sum_r L_r c_r = \sum_r L_r z_r,$$

and individual rationality,

$$l_h \in \arg\max_r [\varepsilon_{hr} c_r].$$

The individual rationality constraints imply aggregate labor supply curves

$$L_r = \frac{B_r c_r^\theta}{\sum_{r'} B_{r'} c_{r'}^\theta}.$$

Under the common assumption that $\bar{\epsilon}_h$ is independent of ε_{hr}, the optimal c_r is a convex combination of productivity in region r and average consumption:

$$c_r = \frac{\theta}{\theta+1} z_r + \frac{1}{\theta+1}\left(\sum_k L_k c_k\right). \tag{31}$$

See the appendix for the derivation. This policy redistributes consumption from high-productivity to low-productivity regions. Other observationally equivalent choices of $\bar{\epsilon}_h$, such as $\bar{\epsilon}_h = 1/\mathbb{E}[\varepsilon_{hr}|h]$, generate the same mapping from wages to labor allocations but imply different optimal place-based policies, as shown below.

This intervention is not an efficiency improvement. By the first welfare theorem, the decentralized equilibrium is Pareto efficient, so a policymaker maximizing aggregate welfare A would leave the allocation unchanged. In contrast, the allocation in (31) is Pareto inefficient: the marginal revenue product of labor is not equal to the wage in each location.

Equivalently, starting from this allocation, $A(t) > 1$: there exists another feasible allocation, implemented with lump-sum transfers, that makes every household strictly better off. Thus, the utilitarian planner sacrifices aggregate efficiency to pursue redistributive goals that depend on an arbitrary cardinalization of utility.

To show how much the choice of $\bar{e}_h$ matters, we use the calibrated economy from Examples 5 and 6. Table 3 displays the labor allocation induced by the optimal place-based policy under four choices of $\bar{e}_h$. Except for the fourth, deliberately extreme cardinalization, the policies move workers out of high-productivity and into low-productivity regions. The optimal allocations vary substantially across cardinalizations, even though all four are consistent with the same individual choice behavior.

Table 3: Labor-supply allocation implied by optimal place-based policies

Commuting zone	L_r^{initial}	$L_r^{W,1}$	$L_r^{W,2}$	$L_r^{W,3}$	$L_r^{W,4}$
San Francisco–Oakland, CA	0.018	0.013	0.014	0.015	0.023
New York, NY	0.034	0.025	0.027	0.032	0.044
Seattle–Tacoma, WA	0.017	0.014	0.014	0.016	0.011
Brownsville, TX	0.001	0.002	0.002	0.001	0.001
McAllen, TX	0.004	0.005	0.005	0.005	0.003
Daytona Beach, FL	0.002	0.003	0.003	0.002	0.010

Notes: The first column reports the initial labor share. The next columns report the labor shares that maximize the corresponding utilitarian social welfare functions defined in the notes to Table 2.

6.3 Sum of Compensating Variations / Kaldor-Hicks Efficiency

In partial equilibrium contexts, where there is an outside good, like money, a common measure of aggregate efficiency is the sum of compensating variations (Small and Rosen, 1981). This measure, also known as the Kaldor-Hicks measure of efficiency, can be defined as follows:

$$S(t) = -\int e_h(\boldsymbol{w}(t), \boldsymbol{p}(t), u_h^0) dh,$$

where e_h is the net expenditure function — the transfer h needs to attain u_h^0, given prices and wages. Note that if h prefers $(\boldsymbol{w}(t), \boldsymbol{p}(t))$ to status quo prices and wages, then e_h is a negative number. So, the scalar $S(t)$ measures the amount of money left, in terms of the numeraire, after winners exactly compensate the losers, holding prices and wages constant at t.[38]

[38] An unimportant difference between $S(t)$ and $A(t)$ is that $S(t)$ is measured in dollars, whereas $A(t)$ is unit-free, being defined as a ratio. In Appendix I we use an alternative, ratio-based version of Kaldor-Hicks

While it is an intuitive measure in partial equilibrium, it has undesirable properties in general equilibrium, when wages and prices endogenously respond to redistribution. The following example demonstrates that a pure transfer, which moves the allocations along the Pareto frontier, can nevertheless cause Kaldor-Hicks efficiency to rise.[39] By continuity, this implies that the consumption possibility set can strictly shrink and yet $S(t)$ may rise.

Example 8 (Redistribution raises Kaldor-Hicks efficiency). Consumption in each location is a Cobb-Douglas aggregator of inputs from different locations:

$$\int c_h \mathbf{1}[l_h = r] dh = x_{rr}^{\alpha} \prod_{r' \in R} x_{rr'}^{(1-\alpha)/R},$$

where $\alpha \geq 0$ controls the degree of home bias in consumption. Output in location r is produced one-for-one from labor:

$$\sum_{r'} x_{r'r} = L_r.$$

Workers' utility functions are

$$u_h(c_h, l_h) = f_h(c_h + \epsilon_{hl_h}),$$

where f_h is any monotone increasing function. Consider an economy with two symmetric locations and suppose that $\epsilon_{h1} = 1$ and ϵ_{h2} is an i.i.d. uniform random variable in the interval $[1-d, 1+d]$, where d controls the elasticity of location choices to changes in real consumption. This elasticity falls to zero as d rises because as d rises, taste dispersion grows, and the measure of households that are close enough to indifferent between locations for a given wage change goes to zero.

The status quo is a symmetric equilibrium without transfers. Consider a lump-sum transfer from households in location 2 to location 1. That is, agents that chose location 1 in the status quo, $l_h(0) = 1$, receive a lump-sum transfer of T_1. The transfer is financed by a lump-sum tax on agents that chose location two in the status quo, $l_h(0) = 2$. Budget balance requires the lump-sum tax T_2 to be $T_2 = L_1(0)/L_2(0)T_1$.

We solve for the post-shock equilibrium at t and calculate the Kaldor-Hicks efficiency measure, $S(t)$. Figure 1 reports $S(t)$ (with nominal GDP as the numeraire) for different

efficiency — defined as total income at t relative to total income required to compensate households at those prices — and show that the conclusions of this section are unchanged.

[39]This phenomenon also occurs in general equilibrium models with continuous choice and is known as the Boadway (1974) paradox. See Baqaee and Burstein (2025) for more information in a continuous choice setting.

levels of worker mobility (by varying the parameter d). We measure mobility by the mass of agents that start in location 2 (for whom $l_h(0) = 2$) and move to location 1 after the redistribution (for whom $l_h(t) = 1$).[40]

Figure 1: Kaldor-Hicks Efficiency as a function of mobility for a pure transfer

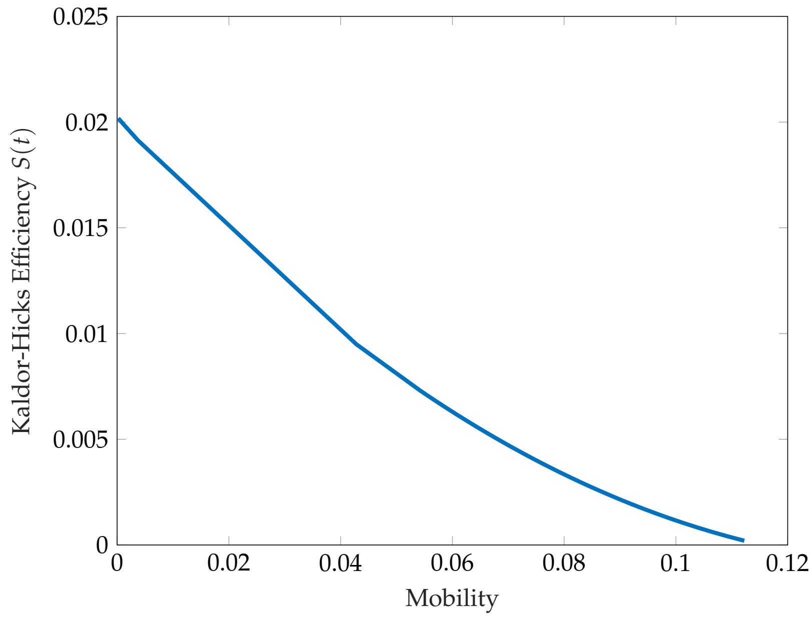

Figure 1 shows that Kaldor-Hicks efficiency, $S(t)$, rises in response to a pure redistribution. In contrast, our measure of aggregate welfare in terms of TFP equivalents is unchanged, $A(t) = A(0) = 1$ for every value of mobility and preference parameters. Pure transfers move the economy along the Pareto frontier; by definition of $A(t)$, they do not change the factor-saving amount needed to keep everyone indifferent.

The reason $S(t)$ is non-zero for pure redistributions is because redistributions change relative wages and prices in general equilibrium. Since $S(t)$ holds wages and prices constant in the compensation, undoing the transfers does not necessarily result in $S(t) = 0$. The magnitude of $S(t)$ depends on parameters — two important parameters are the extent of mobility and home bias. For example, if households do not have heterogeneous tastes, then real wages are equated in the two locations by arbitrage. Since productivities are symmetric, this implies that relative wages and prices are also equated in equilibrium. In this limit, the transfer does not alter relative wages and prices, and so $S(t)$ tends to 0 as we increase mobility.

Similarly, in the absence of home bias ($\alpha = 0$), then $S(t) = 0$. The utility agent h gets in location r is $(w_r + T_h)/p_r$. Without home-bias, the price of consumption in the

[40]In our numerical example, we set the transfer to be 10% of wages in the status quo $T_1 = 0.1w_1(0)$.

two locations is the same: $p_1 = p_2$. Hence, because Assumption 1 holds, the transfer T_h does not change h's location choice. That is, $\boldsymbol{L}(t) = \boldsymbol{L}(0)$. Furthermore, since there is no home-bias, agents in both locations spend their income in the same proportion, so that $w_i L_i = 1/2$ (recall that GDP is the numeraire). Since location choices do not respond to the transfers, wages also do not respond to the transfer. Since wages do not respond to the transfers, prices do not respond to the transfers. Since prices and wages do not respond to the redistribution, $S(t) = 0$ in this limit because undoing the redistribution at the new prices is equivalent to undoing it at the old prices. Hence, in this limit, the compensating-variation based $S(t)$ correctly detects that a pure transfer does not alter efficiency.

In Example 8, we engineer a pure redistribution between agents using lump-sum transfers. However, the same logic applies when redistributions are caused by productivity shocks.[41] These examples illustrate that Kaldor-Hicks efficiency is a poor measure of efficiency in environments where prices and wages are endogenous to the distribution of income and expenditures.

7 Conclusion

We generalize the cost-benefit approach of Harberger (1971) and Small and Rosen (1981) to measure aggregate welfare in general equilibrium environments with discrete choice. Our measure converts shocks into a Debreu-style welfare-equivalent change in total factor productivity.

We show that, to a first-order approximation, aggregate welfare in discrete-choice economies obeys a Hulten-type Domar-weighted formula, and we provide second-order corrections expressed in terms of uncompensated supply and demand elasticities. We contrast our welfare measure with chain-weighted real output, average utility, and the sum of compensating variations (Kaldor-Hicks efficiency), and show that these popular alternatives have serious flaws. Real output can fall when every household is better off. Rankings based on average utility hinge on arbitrary cardinalizations of individual utilities and can recommend Pareto-inefficient place-based policies that move the economy

[41] For example, to generate a pure redistribution using productivity shocks, suppose productivity rises for some agents and falls for others and the labor of these agents are perfect substitutes in production. Hence, the consumption possibility set of the economy does not change, but some agents gain and some agents lose. In this case, if agents consume different goods, the productivity shocks will alter relative prices and the sum of compensating variations is positive. This is because the winners can compensate the losers at post-shock prices and have money left over. Intuitively, relative prices fall for goods more valued by losers, so they can be compensated more cheaply using post-shock prices.

away from efficiency. Kaldor-Hicks efficiency can rise in response to redistributions that only move the economy along the Pareto frontier, attributing "efficiency gains" to changes that are purely distributional. In contrast, our measure only rises when Pareto improvements are possible, is invariant to monotone transformations of utility, and stays constant in response to pure redistributions.

Our benchmark environment is perfectly competitive, Pareto-efficient, and features lump-sum transfers that can be used for compensation. The appendices generalize the basic approach to settings with distortions, externalities, and more restricted redistributive tools. A natural next step is to use our measure of aggregate welfare A as the objective function in optimal policy problems in these settings.

References

Allen, R. and J. Rehbeck (2019). Identification with additively separable heterogeneity. *Econometrica 87*(3), 1021–1054.

Allen, T. and C. Arkolakis (2022). The welfare effects of transportation infrastructure improvements. *The Review of Economic Studies 89*(6), 2911–2957.

Anderson, S. P., A. De Palma, and J. F. Thisse (1992). *Discrete choice theory of product differentiation*.

Bagga, S., L. F. Mann, A. Şahin, and G. L. Violante (2025). Job amenity shocks and labor reallocation. Technical report, National Bureau of Economic Research.

Baqaee, D. and A. Burstein (2025). Aggregate productivity with heterogeneous agents. Technical report.

Baqaee, D. R. and E. Farhi (2019). The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem. *Econometrica 87*(4), 1155–1203.

Bhattacharya, D. (2015). Nonparametric welfare analysis for discrete choice. *Econometrica 83*(2), 617–649.

Bhattacharya, D. (2021). The empirical content of binary choice models. *Econometrica 89*(1), 457–474.

Boadway, R. W. (1974). The welfare foundations of cost-benefit analysis. *The Economic Journal 84*(336), 926–939.

Caliendo, L., M. Dvorkin, and F. Parro (2019). Trade and labor market dynamics: General equilibrium analysis of the china trade shock. *Econometrica 87*(3), 741–835.

Dagsvik, J. K. and A. Karlström (2005). Compensating variation and hicksian choice probabilities in random utility models that are nonlinear in income. *The Review of Economic Studies 72*(1), 57–76.

Daly, A. J. and S. Zachary (1978). Improved multiple choice models. In D. A. Hensher and M. Q. Dalvi (Eds.), *Determinants of Travel Choice*, pp. 335–357. Farnborough, UK: Saxon House.

Davis, M. and J. M. Gregory (2021). Place-based redistribution in location choice models. Technical report, National Bureau of Economic Research.

Debreu, G. (1951). The coefficient of resource utilization. *Econometrica: Journal of the Econometric Society*, 273–292.

Dingel, J. I. and F. Tintelnot (2020). Spatial economics for granular settings. Technical report, National Bureau of Economic Research.

Dinkelbach, W. (1967). On nonlinear fractional programming. *Management science 13*(7), 492–498.

Domar, E. D. (1961). On the measurement of technological change. *The Economic Journal 71*(284), 709–729.

Donald, E., M. Fukui, and Y. Miyauchi (2023). Unpacking aggregate welfare in a spatial economy. Technical report, Tech. rep.

Dubé, J.-P., J. Joo, and K. Kim (2025). Discrete/continuous choice models and representative consumer theory. *Journal of Economic Theory*, 106012.

Fajgelbaum, P. D. and C. Gaubert (2020). Optimal spatial policies, geography, and sorting. *The Quarterly Journal of Economics 135*(2), 959–1036.

Fajgelbaum, P. D. and C. Gaubert (2025). Optimal spatial policies. Technical report, National Bureau of Economic Research.

Harberger, A. C. (1971). Three basic postulates for applied welfare economics: an interpretive essay. *Journal of Economic literature 9*(3), 785–797.

Hortaçsu, A. and J. Joo (2023). Structural econometric modeling in industrial organization and quantitative marketing: theory and applications.

Hsieh, C.-T., E. Hurst, C. I. Jones, and P. J. Klenow (2019). The allocation of talent and us economic growth. *Econometrica 87*(5), 1439–1474.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *The Review of Economic Studies*, 511–518.

Kim, R. and J. Vogel (2020). Trade and welfare (across local labor markets). Technical report, National Bureau of Economic Research.

Lamadon, T., M. Mogstad, and B. Setzler (2022). Imperfect competition, compensating differentials, and rent sharing in the us labor market. *American Economic Review 112*(1), 169–212.

Mas-Colell, A., M. D. Whinston, J. R. Green, et al. (1995). *Microeconomic theory*, Volume 1. Oxford university press New York.

McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. In P. Zarembka (Ed.), *Frontiers in Econometrics*, pp. 105–142. New York: Academic Press.

McFadden, D. (1981). Econometric models of probabilistic choice. *Structural analysis of discrete data with econometric applications 198272*.

Milgrom, P. and I. Segal (2002). Envelope theorems for arbitrary choice sets. *Econometrica 70*(2), 583–601.

Mongey, S. and M. E. Waugh (2024). Discrete choice, complete markets, and equilibrium. Technical report, National Bureau of Economic Research.

Monte, F., S. J. Redding, and E. Rossi-Hansberg (2018). Commuting, migration, and local employment elasticities. *American Economic Review 108*(12), 3855–3890.

Redding, S. J. (2016). Goods trade, factor mobility and welfare. *Journal of International Economics 101*, 148–167.

Redding, S. J. and E. Rossi-Hansberg (2017). Quantitative spatial economics. *Annual Review of Economics 9*(1), 21–58.

Roback, J. (1982). Wages, rents, and the quality of life. *Journal of political Economy 90*(6), 1257–1278.

Schulz, K., A. Tsyvinski, and N. Werquin (2023). Generalized compensation principle. *Theoretical Economics 18*(4), 1665–1710.

Small, K. A. and H. S. Rosen (1981). Applied welfare economics with discrete choice models. *Econometrica: Journal of the Econometric Society*, 105–130.

Solow, R. M. (1957). Technical change and the aggregate production function. *The review of Economics and Statistics*, 312–320.

U.S. Bureau of Economic Analysis (2026, February). Gross domestic product by county and personal income by county, 2024.

U.S. Bureau of Labor Statistics (2026). Local area unemployment statistics.

U.S. Department of Agriculture, Economic Research Service (2026, January). 2020 commuting zones.

Williams, H. C. (1977). On the formation of travel demand models and economic evaluation measures of user benefit. *Environment and planning A 9*(3), 285–344.

Appendix

Appendix A Proofs of propositions and theorems in the body of the paper

Proof of Proposition 1. The expenditure function is

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{T_h, c_h, l_h} \{T_h : u_h(c_h, l_h) \geq u_h^0, \text{ and } \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

Since utility is increasing in consumption, the weak inequalities above have to be equalities. The first constraint implies that:

$$u_h(c_h, l_h) = g(\bar{c}_{hl_h}) + \epsilon_{hl_h} = u_h^0$$

Hence,

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{T_h, l_h} \{T_h : \sum_r p_r \bar{c}_{hr} \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

The weak inequality above is strict at an optimum point, so substituting out for T_h yields

$$e_h(\boldsymbol{w},\boldsymbol{p},u_h^0) = -\max_{l_h}\{\sum_r a_{hr}w_r\mathbf{1}[l_h=r] - \sum_r p_r\bar{c}_{hl_h}\mathbf{1}[l_h=r]\}.$$

The maximizer of this problem is the compensated choice $l_h^{\text{comp}}(\boldsymbol{w},\boldsymbol{p},u_h^0)$. □

Proof of Theorem 1. Recall that each decentralized equilibrium is associated with a vector of lump-sum transfers to each agent, $\boldsymbol{T}$. TFP-equivalent welfare solves

$$A(t) = \max_{\boldsymbol{T},Z}\left\{Z^{-1} : \{c_h,l_h\}_{h\in H}\in\mathcal{C}(t,Z) \text{ and } (c_h,l_h)\succeq_h (c_h(0),l_h(0)) \text{ for every } h\right\},$$

where the choice of transfers determines the equilibrium allocation. Let $\boldsymbol{T}^*$ be a maximizer of the problem above. The indifference condition of each household implies that:

$$u_h(c_h(\boldsymbol{T}^*),l_h(\boldsymbol{T}^*)) = u_h^0.$$

Hence,

$$c_h(\boldsymbol{T}^*) = u_h^{-1}\Big(u_h^0;\, l_h(\boldsymbol{T}^*)\Big) \equiv \bar{c}_{hl_h(\boldsymbol{T}^*)},$$

where $u_h^{-1}(\cdot;\, l_h)$ denotes the inverse of $u_h(c,l_h)$ with respect to its first argument, holding l_h fixed.[42] In the compensated equilibrium with wages $\boldsymbol{w}$, prices $\boldsymbol{p}$, and $Z=1/A$,

$$\frac{\frac{a_{hl_h(\boldsymbol{T}^*)}w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{A}+T_h^*}{p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)} = \bar{c}_{hl_h(\boldsymbol{T}^*)}.$$

Rearrange this to get that the optimal compensating transfers satisfies:

$$T_h^* = p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)\bar{c}_{hl_h(\boldsymbol{T}^*)} - \frac{a_{hl_h(\boldsymbol{T}^*)}w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{A}$$

The sum of these transfers must add up to zero, which implies that

$$\sum_h\left[p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)\bar{c}_{hl_h(\boldsymbol{T}^*)} - \frac{a_{hl_h(\boldsymbol{T}^*)}w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{A}\right]=0.$$

[42] If we impose that preferences are ordinally additively separable (not required for this proof),

$$g(c_h(\boldsymbol{T}^*))+\epsilon_{hl_h(\boldsymbol{T}^*)} = u_h^0,$$

so

$$c_h(\boldsymbol{T}^*) = g^{-1}\left(u_h^0-\epsilon_{hl_h(\boldsymbol{T}^*)}\right) \equiv \bar{c}_{hl_h(\boldsymbol{T}^*)}.$$

Rearranging this gives an equation that A has to satisfy given the optimal $\boldsymbol{T}^*$:

$$A = \frac{\sum_h a_{hl_h(\boldsymbol{T}^*)} w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{\sum_h p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)\bar{c}_{hl_h(\boldsymbol{T}^*)}}.$$

We know by definition that

$$l_h(\boldsymbol{T}^*) = l_h^{\text{comp}}\left(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0\right)$$

and that

$$T_h^* = e_h(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0).$$

Using the fact that

$$L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0) = \sum_h a_{hr} \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0) = r],$$

and the fact that

$$\begin{aligned}\sum_h p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)\bar{c}_{hl_h(\boldsymbol{T}^*)} &= \frac{w_r}{A} L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0) + \sum_h e_h(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0) \mathbf{1}\left[l_h^{\text{comp}} = r\right] \\ &= E_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0),\end{aligned}$$

we can write

$$E_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0) = \sum_r \frac{w_r(\boldsymbol{T}^*)}{A} L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0)$$

where $\boldsymbol{w}(\boldsymbol{T}^*)$ and $\boldsymbol{p}(\boldsymbol{T}^*)$ are prices and wages in the compensated equilibrium given compensated location choices $L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0)$ and compensated final demand $E_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0)$ and total factor productivity $1/A$. □

Proof of Theorem 2. Define the expenditure function to be

$$\begin{aligned}e_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h) &= \min_{T_h}\left\{T_h : \max_r g\left(\frac{\frac{w_r a_{hr}}{A} + T_h}{p_r}\right) + \epsilon_{hr} \geq u_h^0\right\} \\ &= T_h + \mu_h\left[\max_r\left\{g\left(\frac{\frac{w_r a_{hr}}{A} + T_h}{p_r}\right) + \epsilon_{hr}\right\} - u_h^0\right].\end{aligned}$$

Hence,

$$\frac{\partial e_h}{\partial u_h^0} = -\mu_h.$$

Define the indirect utility function of the agent to be

$$v_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, T_h) = \max\left\{u_h(c_h, l_h) : p_{l_h} c_h = \frac{w_{l_h} a_{hl_h}}{A} + T_h\right\}.$$

Then, by the envelope theorem, we have

$$\frac{\partial v_h}{\partial T_h} = g'\left(\frac{\frac{w_r a_{hl_h}}{A} + T_h}{p_{l_h}}\right)\frac{1}{p_{l_h}}.$$

For households that switch location, the derivative above needs to be evaluated from the appropriate direction, see Milgrom and Segal (2002). However, this issue will not play an important role in the proof, because the set of households that are exactly indifferent between two locations is measure zero. We also have the identity that

$$e_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, v_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, T_h)) = T_h.$$

Differentiating this identity gives

$$\frac{\partial e_h}{\partial u_h}\frac{\partial v_h}{\partial T_h} = 1.$$

Combining equations, we have that

$$-\mu_h = \frac{\partial e_h}{\partial u_h} = \left[\frac{\partial v_h}{\partial T_h}\right]^{-1} = \frac{p_{l_h}}{g'\left(\frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}}\right)}.$$

We know that A satisfies the feasibility condition:

$$\sum_h e_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0) = 0. \tag{32}$$

For households that are not on the margin of switching locations, the expenditure function is totally differentiable in t. We totally differentiate in t and write $\boldsymbol{w}$ and $\boldsymbol{p}$ for the wages and prices in the compensated equilibrium without explicitly writing the superscript comp on every variable. The reason is that in this proof, we never refer to the decentralized equilibrium — only to the compensated one. We have

$$\begin{aligned}
\frac{\partial e_h(\boldsymbol{w}/A,\boldsymbol{p},u_h)}{\partial\left(w_i/A\right)} &= a_{hl_h}\mu_h g'(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})\mathbf{1}\left[l_h(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})=i\right]\frac{1}{p_{l_h}} \\
&= -a_{hl_h}\frac{p_{l_h}}{g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)}\frac{1}{p_{l_h}}g'(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})\mathbf{1}\left[l_h(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})=i\right] \\
&= -a_{hl_h}\mathbf{1}\left[l_h(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})=i\right],
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial e_h(\boldsymbol{w}/A,\boldsymbol{p},u_h)}{\partial p_i} &= -\mu_h g'(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})\mathbf{1}\left[l_h(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})=i\right]\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\frac{1}{p_{l_h}} \\
&= \frac{p_{l_h}}{g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)}g'(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})\mathbf{1}\left[l_h(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})=i\right]\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\frac{1}{p_{l_h}} \\
&= \frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\mathbf{1}\left[l_h(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}})=i\right]
\end{aligned}$$

Total differentiating (32),

$$\sum_r\sum_h\frac{\partial e_h(\frac{\boldsymbol{w}}{A},\boldsymbol{p},u_h^0)}{\partial\left[w_r/A\right]}d\left[\frac{w_r}{A}\right]+\sum_r\sum_h\frac{\partial e_h(\frac{\boldsymbol{w}}{A},\boldsymbol{p},u_h^0)}{\partial p_r}dp_r=0.$$

Substituting the expressions above,

$$\begin{aligned}
\sum_r \sum_h a_{hl_h} \mathbf{1}\left[l_h(\frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}}) = r\right] d\left[\frac{w_r}{A}\right] &= \sum_r \sum_h \frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}} \mathbf{1}\left[l_h(\frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}}) = r\right] dp_r \\
\sum_r L_r^{\text{comp}} d\left[\frac{w_r}{A}\right] &= \sum_r \sum_h \frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}} \mathbf{1}\left[l_h(\frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}}) = r\right] dp_r \\
\sum_r L_r^{\text{comp}} d\left[\frac{w_r}{A}\right] &= \sum_r d\log p_r \sum_h \left[\frac{a_{hl_h} w_{l_h}}{A} + T_h\right] \mathbf{1}\left[l_h(\frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}}) = r\right] \\
\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log\left[\frac{w_r}{A}\right] &= \sum_r E_r^{\text{comp}} d\log p_r \qquad (33) \\
\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log\left[A\right] &= \sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log\left[w_r\right] - \sum_r E_r^{\text{comp}} d\log p_r \\
d\log A &= \sum_r \frac{L_r^{\text{comp}}\left[\frac{w_r}{A}\right]}{\sum_r L_r^{\text{comp}}\left[\frac{w_r}{A}\right]} d\log\left[w_r\right] - \sum_r \frac{E_r^{\text{comp}}}{\sum_r L_r^{\text{comp}}\left[\frac{w_r}{A}\right]} d\log p_r \\
d\log A &= \sum_r \lambda_r d\log w_r - \sum_r \chi_r d\log p_r.
\end{aligned}$$

Shephard's lemma for producer r implies that

$$d\log p_i = -d\log z_i + \sum_j \Omega_{ij} d\log p_j + \sum_r \Omega_{ir} d\log w_r,$$

where w_r is the wage per efficiency unit of labor of type r in the compensated equilibrium. Inverting this system of equations yields

$$d\log p = -\Psi d\log z + \Psi^F d\log w,$$

where $\Psi = (I - \Omega)^{-1}$ is the Leontief inverse and Ψ^F is the matrix of factor contents. Substitute this into the expression for $d\log A$ to get

$$\begin{aligned}
d\log A &= \sum_r \lambda_r d\log w_r - \sum_r \chi_r \left[-\Psi d\log z + \Psi^F d\log w\right] \\
&= \sum_r \lambda_r d\log w_r + \sum_r \chi_r \Psi d\log z - \sum_r \chi_r \Psi^F d\log w \\
&= \sum_r \lambda_r d\log w_r + \sum_{i\in N} \lambda_i d\log z_i - \sum_r \lambda_r d\log w_r \\
&= \sum_{i\in N} \lambda_i d\log z_i.
\end{aligned}$$

By the fundamental theorem of calculus,

$$\log A = \int \sum_{i \in N} \lambda_i d \log z_i,$$

as needed.

□

Proof of Proposition 2. To get (16), log differentiate input demand for CES. To get (17) use Shephard's lemma, constant-returns-to-scale, and the fact that price equals marginal cost. To get (18) log differentiate the definition of λ_r for $r \in R$. To get (19), log differentiate (6) (the equation is the same regardless of whether i is a factor or a good). All these equations are equilibrium conditions that must hold in both the decentralized economy and the compensated equilibrium. □

Proof of Proposition 3. The first part follows from:

$$\begin{aligned} L_r^{\text{comp}}(\boldsymbol{w}/A, p^c, \boldsymbol{u}) &= \int a_{hr} \left[r \in \arg\max_i \left\{ a_{hi} \frac{w_i}{Ap^c} + \frac{e_h(\boldsymbol{w}/A, p^c, u_h)}{p^c} + \epsilon_{hi} \right\} \right] \\ &= \int a_{hr} \left[r \in \arg\max_i \left\{ a_{hi} \frac{w_i}{Ap^c} + \epsilon_{hi} \right\} \right] \\ &= L(\boldsymbol{w}/A, p^c) = L(\boldsymbol{w}/(Ap^c), 1), \end{aligned}$$

showing that the compensated and uncompensated supply systems have the same functional form.

Equation (20) is just the total derivative of L_r^{comp} with respect to t. This pins down the equilibrium in conjunction with equations (15) to (19). The only unknown terms are the ones involving χ^{comp} in (19). However, since $\Omega_{ri} = \Omega_{r'i}$ for every r and r' in R (common consumption good), the terms involving χ^{comp} drop out of (19). In particular,

$$\sum_r d\chi_r \Omega_{ri} + \sum_r \chi_r d\Omega_{ri} = d\Omega_{ri},$$

since we know that $\sum_r d\chi_r = 0$ and $\sum_r \chi_r = 1$. □

Proof of Proposition 4. For the uncompensated equilibrium, the result follows easily from log-differentiating the definition:

$$\chi_r = \frac{w_r L_r}{\sum_{r'} w_{r'} L_{r'}}.$$

Now consider the change in the spending shares by location, $d\chi^{\text{comp}}$,in the compensated equilibrium. To keep the notation more manageable, we do not include the superscript comp on every variable and simply note that everything is in the compensated equilibrium.

Let $T(\epsilon, t)$ be the compensating transfer at t received by an agent with tastes ϵ. Let $B_r(t)$ denote the set of ϵ that choose location r in the compensated equilibrium. Note that it is defined by

$$B_r(t) = \left\{ \epsilon : g(\frac{\frac{w_r(t)}{A(t)} + T(\epsilon, t)}{p_r(t)}) + \epsilon_r - \max \left\{ g(\frac{\frac{w_{r'}(t)}{A(t)} + T(\epsilon, t)}{p_{r'}(t)}) + \epsilon_{r'} \right\} \leq 0 \right\} = \left\{ \epsilon : \phi_r(\epsilon, t) \leq 0 \right\}.$$

Households on the margin of choosing r are defined by $\phi(\epsilon, t) = 0$. Then the change in share of expenditures in each location satisfies:

$$\begin{aligned} d \log \chi_r &= d \log \int [p_r \bar{c}_{hr}] \mathbf{1} [l(h) = r] \, dh - \underbrace{d \log \sum_{r'} \int [p_r \bar{c}_{hr}] \mathbf{1} \left[l(h) = r' \right] dh,}_{\equiv X} \\ &= \frac{1}{\lambda_r} d \int [p_r \bar{c}_{hr}] \mathbf{1} [l(h) = r] \, dh - X \\ &= \frac{1}{\lambda_r} d \left[\int_{B_r(t)} [p_r \bar{c}_r(\epsilon)] f(\epsilon) d\epsilon \right] - X, \end{aligned}$$

where $\bar{c}_r(\epsilon)$ is the consumption-equivalent and $f(\epsilon)$ is the density of households with tastes ϵ. Using the fact that $p_r \bar{c}_r(\epsilon) = \frac{w_r}{A} + T(\epsilon, t)$, we can write

$$\begin{aligned} d \log \chi_r &= \frac{1}{\lambda_r} d \left[\int_{B_r} \left[\frac{w_r(t)}{A(t)} + T(\epsilon, t) \right] f(\epsilon) d\epsilon \right] - X \\ &= \frac{1}{\lambda_r} \int_{B_r} \left[d \frac{w_r}{A} + \frac{\partial}{\partial t} T(\epsilon, t) \right] f(\epsilon) d\epsilon + \frac{1}{\lambda_r} \int \left[\frac{w_r(t)}{A(t)} + T(\epsilon, t) \right] \delta(\phi(z, \epsilon)) f(\epsilon) \| \nabla_\epsilon \phi(z, \epsilon) \| d\epsilon - X \end{aligned}$$

where the second line uses Leibniz' rule. At the status quo, where we differentiate, we have $\frac{w_r(0)}{A(0)} + T(\epsilon, 0) = w_r(0)$. Dropping time subscripts, we can write

$$d \log \chi_r = \frac{1}{\lambda_r} \int_{B_r} \left[d \frac{w_r}{A} + \frac{\partial}{\partial t} T(\epsilon, t) \right] f(\epsilon) d\epsilon + \frac{1}{\lambda_r} \int w_r \delta(\phi(z, \epsilon)) f(\epsilon) \| \nabla_\epsilon \phi(z, \epsilon) \| d\epsilon - X.$$

The first summand is the change in the compensating income of each household in location r and the second summand is the wage in location r times the mass of households that move to location r in the compensated equilibrium. Hence, we can write

$$
\begin{aligned}
d\log\chi_r &= \frac{1}{\lambda_r}\left[\int_{B_r}\left[d\frac{w_r}{A}+dT(\epsilon)\right]f(\epsilon)d\epsilon\right]+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X\\
&= \frac{1}{\lambda_r}\left[\frac{w_r}{p_r}dp_r\int_{B_r}f(\epsilon)d\epsilon\right]+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X\\
&= \frac{1}{\lambda_r}\left[\frac{w_r}{p_r}dp_r\right]L_r+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X\\
&= \frac{1}{\lambda_r}\left[\frac{L_r w_r}{p_r}dp_r\right]+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X\\
&= d\log p_r+\frac{1}{\lambda_r}dL_r w_r-X\\
&= d\log p_r+d\log L_r-X.
\end{aligned}
$$

By definition, we must have

$$\mathbb{E}_\chi\left[d\log\chi\right]=0,$$

hence,

$$X=\mathbb{E}_\chi\left[d\log p_r+d\log L_r\right].$$

This implies that, in the compensated equilibrium, we have

$$d\log\chi_r=\left[d\log p_r+d\log L_r\right]-\mathbb{E}_\chi\left[d\log p_r+d\log L_r\right].$$

□

Proof of Proposition 5. We start with the uncompensated equilibrium and then consider the compensated equilibrium.

Uncompensated Equilibrium. Let $B_i(t)$ denote the set of ϵ that choose i:

$$
\begin{aligned}
B_i(t) &= \left\{\epsilon : g\left(\frac{w_i(t)}{p_i(t)}\right)+\epsilon_i\geq g\left(\frac{w_k(t)}{p_k(t)}\right)+\epsilon_k,\forall\, k\neq i\right\}\\
&= \left\{\epsilon : \phi_i(t,\epsilon)\leq 0\right\},
\end{aligned}
$$

where

$$\phi_i(t,\epsilon)=\max_k\left\{g\left(\frac{w_k(t)}{p_k(t)}\right)+\epsilon_k-g\left(\frac{w_i(t)}{p_i(t)}\right)-\epsilon_i\right\}.$$

The set of movers from i to j is given by

$$\begin{aligned}
L_{i\to j} &= \int \mathbf{1}(\epsilon \in B_i(0))\mathbf{1}(\epsilon \in B_j(t))f(\epsilon)d\epsilon \\
&= \int_{B_j(t)} \mathbf{1}(\epsilon \in B_i(0))f(\epsilon)d\epsilon. \\
dL_{i\to j} &= d\left[\int_{B_j(t)} \mathbf{1}(\epsilon \in B_i(0))f(\epsilon)d\epsilon\right] \\
&= \left[\int_{dB_j(t)} \mathbf{1}(\epsilon \in B_i(0))f(\epsilon)d\epsilon\right] \\
&= \left[\int \mathbf{1}(\epsilon \in B_i(0))\delta(\phi_j(0,\epsilon))\|\nabla_\epsilon \phi_j(0,\epsilon)\|f(\epsilon)d\epsilon\right],
\end{aligned}$$

where δ is the Dirac delta function which means the integral is evaluated at the boundary of $B_j(0)$, that is where $\phi_j(0,\epsilon) = 0$ and $\mathbf{1}(\epsilon \in B_i(0))$. For any vector $\boldsymbol{x}$, let $h(\boldsymbol{x})$ be the density of households with $\epsilon_j - \epsilon_i = x_i - x_j$ and $\epsilon_j - \epsilon_k \geq x_k - x_j$ and $\epsilon_i - \epsilon_k \geq x_k - x_i$. This the density of households that are indifferent between i and j (and prefer i and j to all other options) given payoffs are $x_i = g(c_i)$. Define $\epsilon_{ij} = x_i - x_j$ to be the cut-off between i and j. Then we can write

$$\begin{aligned}
dL_{i\to j} &= -h(\boldsymbol{w}(0)/\boldsymbol{p}(0))d\epsilon_{ij}\mathbf{1}\left[d\epsilon_{ij} \leq 0\right] \\
&= -h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))\left[g'(\frac{w_i(0)}{p_i(0)})d\left[\frac{w_i}{p_i}\right] - g'(\frac{w_j(0)}{p_j(0)})d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[d\epsilon_{ij} \leq 0\right]. \qquad (34)
\end{aligned}$$

We relate this equation to cross-derivatives of the supply function $\boldsymbol{L}(\boldsymbol{p},\boldsymbol{w},\boldsymbol{T})$. In particular, note that in the decentralized equilibrium, $\boldsymbol{T} = 0$, the supply function depends only on the vector of real wages $\boldsymbol{w}/\boldsymbol{p}$. Furthermore, consider a perturbation where only the real wage in location j changes (all other real wages are held constant), then from the equation above, we can write

$$\frac{\partial L_{i\to j}}{\partial(w_j/p_j)} = h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))g'(w_j(0)/p_j(0)).$$

Since

$$\begin{aligned}\frac{\partial L_i}{\partial(w_j/p_j)} &= \sum_{k\neq i}\frac{\partial L_{k\to i}}{\partial(w_j/p_j)} - \sum_{k\neq i}\frac{\partial L_{i\to k}}{\partial(w_j/p_j)}\\ &= -\frac{\partial L_{i\to j}}{\partial(w_j/p_j)}\\ &= -h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))g'(w_j(0)/p_j(0)),\end{aligned} \tag{35}$$

where the second line follows from equation (34). Using these equations, we can write

$$\begin{aligned}dL_{i\to j} &= -h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))\left[g'(w_i(0)/p_i(0))d\left[\frac{w_i}{p_i}\right] - g'(w_j(0)/p_j(0))d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[d\epsilon_{ij}\leq 0\right]\\ &= \left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[d\epsilon_{ij}\leq 0\right]\\ &= \left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))d\epsilon_{ij}\leq 0\right]\\ &= \left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[\left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\geq 0\right],\end{aligned}$$

which is the expression in the proposition (swapping i and j for r and r').

Compensated Equilibrium. We now consider the compensated equilibrium. To show how location choices change, to a first order, let $T(\epsilon, t)$ be the compensating transfer in the compensated equilibrium for households with preferences ϵ. The set of households that are marginal movers between region i and j in the compensated equilibrium is

$$F_{ij}(t) = \left\{\epsilon : g(\frac{w_i(t)+T(\epsilon,t)}{p_i(t)}) + \epsilon_i = g(\frac{w_j(t)+T(\epsilon,t)}{p_j(t)}) + \epsilon_j \geq g(\frac{w_k(t)+T(\epsilon,t)}{p_k(t)}) + \epsilon_k, \forall\, k\notin\{j,i\}\right\}.$$

For every $\epsilon\in F_{ij}(t)$, the following equation holds:

$$\epsilon_{ij}(t) = \epsilon_{hj} - \epsilon_{hi} = g(\frac{w_i(t)/A(t)+T(\epsilon,t)}{p_i(t)}) - g(\frac{w_j(t)/A+T(\epsilon,t)}{p_j(t)}).$$

We call $\epsilon_{ij}(t)$ the cut-off values for i and j (this is different to the cut-off values in the uncompensated decentralized equilibrium because of the compensating transfers). In the status quo, transfers are zero, so the cutoff between regions i and j is

$$\epsilon_{ij}^0 = g(\frac{w_i^0}{p_i^0}) - g(\frac{w_j^0}{p_j^0}).$$

Differentiating the expression for $\epsilon_{ij}(t)$ with respect to t and suppressing dependence on t gives:

$$d\epsilon_{ij} = g'(\frac{w_i/A + T(\epsilon)}{p_i})d\left[\frac{w_i/A + T(\epsilon)}{p_i}\right] - g'(\frac{w_j/A + T(\epsilon)}{p_j})d\left[\frac{w_j/A + T(\epsilon)}{p_j}\right].$$

At the status quo, since $A = 1$ and $T(\epsilon, 0) = 0$, we have

$$d\epsilon_{ij} = g'(\frac{w_i}{p_i})\left[d\left[w_i/Ap_i\right] + \frac{1}{p_i}dT(\epsilon)\right] - g'(\frac{w_j}{p_j})d\left[d\left[w_j/Ap_j\right] + \frac{1}{p_j}dT(\epsilon)\right].$$

Note that

$$T(\epsilon, t) = e(\frac{\boldsymbol{w}(t)}{A(t)}, \boldsymbol{p}(t), u_h^0(\epsilon), \epsilon),$$

where the fourth summand captures how the expenditure function varies as a function of tastes, holding wages, prices, and utility constant. Hence, at the status quo, totally differentiating with respect to t yields:

$$dT(\epsilon) = e_1(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon) \cdot d\left[\frac{\boldsymbol{w}}{A}\right] + e_2(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon) \cdot d\boldsymbol{p} + \left[e_3(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\frac{du_h^0(\epsilon)}{d\epsilon} + e_4(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\right]d\epsilon,$$

where $\epsilon \in F_{ij}(t)$ and $d\epsilon$ is evaluated for some path that remains in $F_{ij}(z, t)$ as t changes. However, in the status quo, $T(\epsilon, 0) = 0$. Hence, moving in the cross-section of households in the status quo, we have

$$e_3(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\frac{du_h^0}{d\epsilon} + e_4(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0(\epsilon), \epsilon) = 0,$$

which means we can ignore change in the transfers due to changes in the identity of the marginal household, and the expression above simplifes to just

$$dT(\epsilon) = e_1(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0(\epsilon), \epsilon) \cdot d\left[\frac{\boldsymbol{w}}{A}\right] + e_2(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0(\epsilon), \epsilon) \cdot d\boldsymbol{p},$$

where ϵ is evaluated for some household that's marginal between i and j. Using the envelope theorem,

$$dT(\epsilon) = \lim_{t \to 0+} \left[\sum_{i \in R} \mathbf{1} \left[l_h(\frac{\frac{w_r}{A} + T(\epsilon_{ij})}{p_r}) = i \right] \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right]$$
$$= \lim_{t \to 0+} \left[\sum_{i \in R} \mathbf{1} \left[l_h(\frac{w_r}{p_r}) = i \right] \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right],$$

where $\epsilon \in F_{ij}(t)$. In particular, at the status quo, the change in compensating transfers dT is the same for every household that is marginal between choosing i and j regardless of the other values ϵ_{hk}. If $d\epsilon_{ij} < 0$ then households move from i to j in the compensated equilibrium, else the reverse. Denote the change in compensating transfers for these marginal households by the following symbol:

$$dT_{ij} = \lim_{t \to 0+} \left[\sum_{i \in R} \mathbf{1} \left[l_h(\frac{w_r}{p_r}) = i \right] \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right]$$

Hence, we can write

$$dT_{ij} = \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \mathbf{1} \left[d\epsilon_{ij} < 0 \right] + \left(-d \left[\frac{w_j}{A} \right] + \frac{w_j}{p_j} dp_j \right) \mathbf{1} \left[d\epsilon_{ij} \geq 0 \right]$$

Substitute this into the expression for $d\epsilon_{ij}$, in the case where $d\epsilon_{ij} < 0$, so households move from i to j, we get

$$d\epsilon_{ij} = g'(\frac{w_i}{p_i}) \left[d \left[w_i / A p_i \right] + \frac{1}{p_i} \left[-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right] -$$
$$g'(\frac{w_j}{p_j}) \left[d \left[w_j / A p_j \right] + \frac{1}{p_j} \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right].$$

Use the fact that

$$d \left[w_i / \left(A p_i \right) \right] + \left[-d \left[w_i / A \right] + \frac{w_i}{p_i} dp_i \right] / p_i = 0$$

to get

$$d\epsilon_{ij} = -g'(\frac{w_j}{p_j}) \left[d \left[w_j / A p_j \right] - \frac{p_i}{p_j} d \left[w_i / \left(A p_i \right) \right] \right]$$

which requires that $p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]$. Similarly, if $d\epsilon_{ij} > 0$, we instead get

$$d\epsilon_{ij} = g'(\frac{w_i}{p_i})\left[d\left[w_j/Ap_j\right] - \frac{p_i}{p_j}d\left[w_i/\left(Ap_i\right)\right]\right]$$

The mass of movers from i to j is then given by

$$\begin{aligned} L_{i\to j} &= \int \mathbf{1}\left[l_h(\boldsymbol{p}(0),\boldsymbol{w}(0),\mathbf{0}) = i\right]\mathbf{1}\left[l_h(\boldsymbol{p}(t),\boldsymbol{w}(t)/A(t),T_h(t)) = j\right]dh. \\ dL_{i\to j} &= -h(\epsilon_{ij}^0)d\epsilon_{ij}\mathbf{1}\left[d\epsilon_{ij} < 0\right] \\ &= h(\epsilon_{ij}^0)g'(\frac{w_j}{p_j})\left[d\left[w_j/Ap_j\right] - \frac{p_i}{p_j}d\left[w_i/\left(Ap_i\right)\right]\right]\mathbf{1}\left[p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]\right], \end{aligned}$$

where $h(\epsilon_{ij}^0)$ is the density of households for whom $\epsilon \in F_{ij}(0)$.

Note that in the uncompensated economy, in response to the perturbation $d\left[w_j/p_j\right] > 0$ — holding all other real wages fixed — (35) implies that

$$-\frac{\partial L_i}{\partial\left[w_j/p_j\right]}d\left[w_j/p_j\right] = h(\epsilon_{ij}^0)g'(w_j/p_j)d\left[w_j/p_j\right].$$

Hence, we can replace

$$-\frac{\partial L_i}{\partial\left[w_j/p_j\right]} = h(\epsilon_{ij}^0)g'(w_j/p_j). \tag{36}$$

Using this we can rewrite

$$\begin{aligned} dL_{i\to j} &= h(\epsilon_{ij}^0)g'(\frac{w_j}{p_j})\left[d\left[w_j/Ap_j\right] - \frac{p_i}{p_j}d\left[w_i/\left(Ap_i\right)\right]\right]\mathbf{1}\left[p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]\right] \\ &= -\frac{\partial L_i}{\partial\left[w_j/p_j\right]}\frac{1}{p_j}\left[p_j d\left[w_j/Ap_j\right] - p_i d\left[w_i/\left(Ap_i\right)\right]\right]\mathbf{1}\left[p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]\right]. \end{aligned}$$

The change in the compensated labor supply of location i then follows from the equation that:

$$dL_i = \sum_{j\neq i} dL_{j\to i} - \sum_{j\neq i} dL_{i\to j}.$$

□

Proof of Proposition 6. By Hulten (1978), to first-order, we have:

$$\Delta\log Y \approx \sum_i \lambda_i(0)\Delta\log z_i + \sum_{r\in R}\lambda_r(0)\Delta\log L_r.$$

We then use the fact that to a first-order,

$$\lambda_r(0)\Delta \log L_r = \frac{w_r(0)}{\sum_{r'} w_{r'}(0)L_{r'}(0)}\Delta L_r.$$

□

Proof of Proposition 7. From Hulten's theorem, we know that

$$\log A^{\mathrm{MFP}}(t) = \int_0^t \sum_i \lambda_i(s)\frac{d\log z_i}{ds}ds$$

whereas from Theorem 1, we know that

$$\log A(t) = \int_0^t \sum_i \lambda_i^{\mathrm{comp}}(s)\frac{d\log z_i}{ds}ds.$$

Note that, since $\epsilon_{hr} = \bar{\epsilon}$ for every h and r, and there is a common consumption good with the law of one price, we have that

$$l_h^{\mathrm{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = l_h(\boldsymbol{w}, \boldsymbol{p}).$$

Furthermore, since there is only a common consumption good, Domar weights do not depend on the distribution of spending (i.e. $\chi(t)$ and $\chi^{\mathrm{comp}}(t)$ result in the same Domar weights). Hence

$$\lambda_i(s) = \lambda_i^{\mathrm{comp}}(s),$$

which implies that

$$A(t) = A^{MFP}(t).$$

Next, since there is only one common consumption good, real output in this economy is just the total production/consumption of that good: $Y(t) = \sum_h c_h(t) / \sum_h c_h(0)$. Since households only value the consumption good, the first welfare theorem implies that the decentralized equilibrium at t maximizes the quantity of the consumption good produced at t. By Hulten's theorem, we know that

$$\frac{d\log Y}{dt} = \sum_{i\in N} \lambda_i(t)\frac{d\log z_i}{dt} + \sum_{r\in R} \lambda_r(t)\frac{d\log L_r}{dt},$$

where the second term is a reallocation effect. However, this term must equal zero, otherwise, the equilibrium at t would not be maximizing Y. Hence,

$$\frac{d \log Y(t)}{dt} = \frac{d \log A^{MFP}(t)}{dt}$$

at every t. The result follows by integrating both sides. □

Appendix B Implications of $g(\cdot)$ function for behavior

Since utility is only pinned down up to monotone transformations, the function f_h has no observable implications. However, the shape of $g(c_h)$ has testable implications, since it controls the way income and substitution effects interact with each other. For example, if $g(c)$ is linear, then household choices are invariant to an additive increase in consumption in every location (regardless of the distribution of tastes). On the other hand, if $g(c)$ is log, then household choices are invariant to scaling consumption in every location (regardless of the distribution of tastes).

The following proposition shows that the conditional labor supply function $\boldsymbol{L}(\boldsymbol{w}|\boldsymbol{a})$, mapping vectors of real wages into labor supply in each location conditional on skills $\boldsymbol{a}$, uniquely pins down $g(c_h)$ up to an affine transformation.

Proposition 8 (Relation between L and g)**.** *Let the efficiency units of labor supplied by workers given real wages $\boldsymbol{w}$ conditional on skill type $\boldsymbol{a}$ to be:*

$$L_i(\boldsymbol{w}|\boldsymbol{a}) = \int a_i \mathbf{1}\left[g(w_i a_i) + \epsilon_{hi} \geq g(a_j w_j) + \epsilon_{hj} \quad \forall j\right] dh.$$

Given knowledge of $\boldsymbol{L}(\boldsymbol{w}|\boldsymbol{a})$, the function $g(c)$ is pinned down up to an affine transformation. A notable implication is the following. The function $\boldsymbol{L}(\boldsymbol{w}|\boldsymbol{a})$ has symmetric cross-derivatives in real wages if, and only if, $g(c)$ is affine.

That is, the functional form of $g(c_h)$ has testable implications. The linear case is noteworthy because, under this assumption, some of our calculations dramatically simplify. Intuitively, if $g(c_h)$ is linear, then a lump-sum transfer (in consumption units) to household h will not change household h's choice of location.[43]

[43]As another example, the population share function without transfers $\boldsymbol{L}(\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{0})$ has symmetric cross semi-elasticities in real wages $\partial L_r / \partial \log w_{r'} / p_{r'} = \partial L_{r'} / \partial \log w_r / p_r$ if, and only if, $g(c_h)$ is a log function of c_h. The commonly used constant-elasticity Fréchet supply system is a special case and requires that $g(c_h)$ be log.

Proof. Suppose that preferences take the form

$$u_h(c,r) = g(c) + \epsilon_{hr}.$$

Consider the set of $\epsilon's$ marginal between i and j that also do not strictly prefer any other region:

$$F_{ij}(\boldsymbol{w}|\boldsymbol{a}) = \{\epsilon : g(a_i w_i) - g(a_j w_j) = \epsilon_{hj} - \epsilon_{hi},\, g(a_i w_i) - g(a_k w_k) \geq \epsilon_{hk} - \epsilon_{hi},\, \forall k \neq i\}.$$

Define the set that prefer region i by $B_i(\boldsymbol{w}|\boldsymbol{a})$. We can write

$$L_i(\boldsymbol{w}|\boldsymbol{a}) = \int a_i \mathbf{1}\left[\epsilon \in B_i(\boldsymbol{w}|\boldsymbol{a})\right] f(\epsilon) d\epsilon,$$

where $f(\epsilon)$ is the density of ϵ. A marginal change in w_j lowers the share in region i by:

$$\frac{\partial L_i(\boldsymbol{w}|\boldsymbol{a})}{\partial w_j} = -a_i f(g(a_i w_i) - g(a_j w_j)) a_j g'(a_j w_j).$$

Similarly

$$\frac{\partial L_j(\boldsymbol{w}|\boldsymbol{a})}{\partial w_i} = -a_j f(g(a_i w_i) - g(a_j w_j)) a_i g'(a_i w_i).$$

Hence,

$$\frac{\frac{\partial L_i(\boldsymbol{w}|\boldsymbol{a})}{\partial w_j}}{\frac{\partial L_j(\boldsymbol{w}|\boldsymbol{a})}{\partial w_i}} = \frac{g'(a_j w_j)}{g'(a_i w_i)}.$$

Define the function

$$g_{ij}(w_i, w_j) = \frac{\partial L_i(\boldsymbol{w}|\boldsymbol{a})/\partial w_j}{\partial L_j(\boldsymbol{w}|\boldsymbol{a})/\partial w_i} = \frac{g'(a_j w_j)}{g'(a_i w_i)}.$$

Hence,

$$g'(a_j w_j) = g_{ij}(w_i, w_j) g'(a_i w_i).$$

By the fundamental theorem of calculus we have

$$g(w_j) - g(1) = g'(a_i w_i) \int_{1/a_j}^{w_j/a_j} g_{ij}(w_i, x) dx.$$

Specifically, at $w_i = 1/a_i$, we get

$$
\begin{aligned}
g(w_j) - g(1) &= g'(1) \int_{1/a_j}^{w_j/a_j} g_{ij}(1,x)dx \\
\frac{g(w_j) - g(1)}{g'(1)} &= \int_{1/a_j}^{w_j/a_j} g_{ij}(1,x)dx = G_{ij}(w_j/a_j) - G_{ij}(1/a_j),
\end{aligned}
$$

where $G_{ij}(x)$ is the antiderivative of the function $g_{ij}(1,x)$. Hence, the function g is pinned down up to an affine transformation. □

Appendix C Special case with homogeneous agents

In this appendix, we consider an instructive special case where agents are homogeneous in preferences and skill levels ($\succeq_h$ does not vary by h and $a_{hr} = 1$ for every h and r). However, agents do have preferences about locations. For concreteness, if preferences are ordinally additively separable, then we can represent every $\succeq_h$ using:

$$
u_h(c_h, l_h) = f_h\left(g(c_h) + \epsilon_{l_h}\right),
$$

where ϵ_r parameterizes the amenity value of location r (which does not vary by h). Characterizing $A(t)$ in this special case is as easy as solving the decentralized equilibrium.

Proposition 9 (Aggregate Welfare with Homogeneous Agents)**.** *Suppose $\succeq_h$ does not vary by h and $a_{hr} = 1$ for every h and r. Define $c_r(\boldsymbol{z}, Z)$ and $L_r(\boldsymbol{z}, Z)$ to be consumption per capita and labor supply in location r in a decentralized equilibrium given productivity vector $\boldsymbol{z}$ and aggregate factor-augmenting productivity Z. Then $A(t)$ solves*

$$
c_r(\boldsymbol{z}(0), 1) = c_r(\boldsymbol{z}(t), \frac{1}{A(t)}), \tag{37}
$$

for any r with both $L_r(\boldsymbol{z}(t), 1/A(t)) > 0$ and $L_r(\boldsymbol{z}(0), 1) > 0$.

If, in addition, $g(c_h) = \log(c_h)$, then $A(t)$ is equal to the change in per capita consumption (i.e. the change in the real wage):

$$
A(t) = \frac{c_r(\boldsymbol{z}(t), 1)}{c_r(\boldsymbol{z}(0), 1)}, \tag{38}
$$

in any region r that is non-empty at both the status quo and at t.

Before presenting the proof, we provide some intuition. Since agents are homogeneous, in a decentralized equilibrium, agents are indifferent between different location choices (unless a location is empty). If we scale aggregate factor-augmenting productivity by $1/A(t)$, every agent is affected in the same way — there are no relative winners or losers — and lump-sum transfers are not needed for compensations. If a location r is non-empty in the status quo and in the allocation that solves $A(t)$, indifference to the status quo requires that $A(t)$ satisfies

$$f_h\big(g(c_r(\boldsymbol{z}(t), 1/A(t))) + \epsilon_r\big) = f_h\big(g(c_r(\boldsymbol{z}(0), 1)) + \epsilon_r\big).$$

The functions f_h and g and scalar ϵ_r disappear from both sides, leading to (37). In words, $A(t)$ must be such that real consumption per capita in each location r is equated to its status quo value. This shows that computing $A(t)$ is as simple as computing the decentralized equilibrium with different productivity parameters (i.e. we do not have to worry about compensating transfers when agents are homogeneous).

In addition, if $g(c) = \log(c)$, then scaling consumption in every location by the same amount leaves the location choice problem of every agent unchanged. Since agents' location choice problems do not respond to a uniform scaling of consumption, they also do not respond to a uniform scaling of factor-augmenting productivity (for more details, see the proof in the appendix). Hence, $c_r(\boldsymbol{z}(t), 1/A(t)) = c_r(\boldsymbol{z}(t), 1)/A(t)$ and (38) follows. In words, if $g(c) = \log(c)$, then $A(t)$ can be computed purely from the growth in real consumption in any location in the decentralized equilibrium: $A(t) = c_r(\boldsymbol{z}(t), 1)/c_r(\boldsymbol{z}(0), 1)$ — one does not even need to solve a counterfactual equilibrium with different factor-augmenting productivity.

One may wonder: why not always use the growth in real consumption to measure the growth in aggregate welfare? The reason is that outside of the $g(c) = \log(c)$, the growth in real consumption in each location, $c_r(\boldsymbol{z}(t), 1)/c_r(\boldsymbol{z}(0), 1)$, can be a different number, even though all agents are homogeneous in terms of welfare.[44] Our measure of aggregate welfare, $A(t)$, in contrast provides a single number.[45]

Proof of Proposition 9. Define $u_h(\boldsymbol{z}, Z)$ to be the utility of household h in a decentralized equilibrium given productivity vector $\boldsymbol{z}$ and aggregate factor augmenting productivity

[44] To see this, note that indifference between locations implies that $g(c_r) - g(c_{r'}) = \epsilon_r - \epsilon_{r'}$ must always hold. Hence, $g(c_r(\boldsymbol{z}(t), 1)) - g(c_r(\boldsymbol{z}(0), 1)) = g(c_{r'}(\boldsymbol{z}(t), 1)) - g(c_{r'}(\boldsymbol{z}(0), 1))$. This does not, in general, imply equal growth rates for real consumption per capita across locations unless $g(c) = \log(c)$.

[45] If $g(c) = \log(c)$, then one can also calculate $A(t)$ using a specific cardinalization of utility. In particular, if we use $f_h(x) = \exp(x)$ so that $u_h(c, r) = c\exp(\epsilon_r)$, then $A(t)$ is equal to the ratio of utils in the decentralized equilibrium at t relative to status-quo. This trick does not work if $g(c)$ is not logarithmic. However, one cannot simply assume that $g(c)$ is logarithmic without loss of generality: if ϵ_r varies by location, then the shape of $g(c)$ is determined by ordinal properties of the underlying agent-level utility functions (see Appendix B).

Z. Since in equilibrium all households have the same utility, irrespective of their chosen location, there are no lump-sum transfers in the compensated equilibrium. Hence, to calculate $A(t)$, we simply need to solve

$$u_h(\boldsymbol{z}(t), 1/A(t)) = u_h(\boldsymbol{z}(0), 1).$$

For any regions r such that $L_r(\boldsymbol{z}(t), 1/A(t)) \times L_r(\boldsymbol{z}(0), 1) > 0$ we have that

$$g(c_r(\boldsymbol{z}(t), 1/A(t))) + \epsilon_r = g(c_r(\boldsymbol{z}(0), 1)) + \epsilon_r.$$

It follows that

$$c_r(\boldsymbol{z}(t), 1/A(t)) = c_r(\boldsymbol{z}(0), 1).$$

This completes the first part of the proposition.

Consider the case where $g(c) = \log c$. We need to show that

$$c_r\big(\boldsymbol{z}(t), 1/A(t)\big) = A(t)^{-1}\, c_r\big(\boldsymbol{z}(t), 1\big).$$

Let factor-augmenting productivity Z change from 1 to $1/A(t)$ while holding $\boldsymbol{z}(t)$ fixed. Conjecture an equilibrium allocation that scales all factor inputs (in efficiency units), outputs, and intermediate inputs by $1/A(t)$, keeping prices and wages (per efficiency unit) unchanged. In this conjectured equilibrium, consumption in every location scales with $1/A(t)$. With logarithmic g, if consumption in every location is scaled by a common factor, then agents do not have any incentive to switch locations. If households do not switch locations, then labor supply (in efficiency units) in each location scales by $1/A(t)$:

$$L_r\big(\boldsymbol{z}, 1/A(t)\big) = A(t)^{-1} L_r\big(\boldsymbol{z}, 1\big).$$

Scaling labor supply (in efficiency units) in every location by $1/A(t)$ makes it feasible to scale up all quantities by $1/A(t)$ (due to constant returns to scale) while keeping all relative prices constant. This confirms the existence of this conjectured equilibrium, which completes the proof.

□

Appendix D Additional Examples

In this appendix we present Example 9, Example 10, and derive optimal place-based policy in Example 7.

Example 9. Cobb-Douglas and Logit Example Every good is produced using a Cobb-Douglas production technology, so (5) implies that the price of each good $i \in N$ can be written as

$$p_i = z_i^{-1} \prod_{j \in N} p_j^{\Omega_{ij}} \prod_{r \in R} w_r^{\Omega_{ir}},$$

where Ω_{ij} and Ω_{ir} are expenditure shares of i on j and r respectively (Cobb-Douglas implies that these expenditure shares are constant). We can solve out for λ_r in the market clearing condition, (6), to get that for every factor r:

$$\lambda_r = \sum_{r'} \chi_r \Psi_{c(r')r},$$

where $\Psi = (I - \Omega)^{-1}$ is the Leontief inverse (which, again, is constant due to the Cobb-Douglas assumption). This equation states that income of r must equal the dollar-weighted average factor content of final consumption $\sum_{r' \in R} \chi_{r'} \Psi_{c(r')r}$, where $\Psi_{c(r')r}$ is the total factor r content of consumption by agents in location r'.

Suppose that all consumers consume the same consumption good, so that the factor content Ψ_r is the same for every r'. The previous equation simplifies to

$$\lambda_r = \frac{w_r L_r}{\sum_{r''} w_{r''} L_{r''}} = \Psi_{0r}, \tag{39}$$

where $c(r') = 0$ is the index for the common consumption good and the right-hand side is a constant, depending only on the Cobb-Douglas share parameters. That is, we use the fact that $\sum_{r'} \chi_r \Psi_{c(r')r} = \sum_{r'} \chi_r \Psi_{0r} = \Psi_{0r}$.

Suppose skills are homogeneous, $a_{hr} = 1$ and normalize $Z = 1$. Utility functions are given by $u_h(c_h, l_h) = f_h(c_h + \epsilon_{hr} \mathbf{1}[l_h = r])$ where ϵ_{hr} are drawn from type I extreme value distribution and f_h is any strictly increasing function. The labor supply function is

$$L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \frac{\exp(\theta w_r / p^c + B_r)}{\sum_{r'} \exp(\theta w_r' / p^c + B_r')}, \tag{40}$$

where p^c is the price of the final consumption good, and θ and B_r are parameters of the distribution of ϵ_{hr}. Given this functional form, equation (39) can be rewritten as

$$\frac{w_r \exp(\theta w_r / p^c + B_r)}{\sum_{r'} w_{r'} \exp(\theta w_r' / p^c + B_r')} = \Psi_{0r}$$

where the consumer price index satisfies

$$p^c = \prod_j z_j^{-\Psi_{0j}} \prod_r w_r^{\Psi_{0r}}.$$

These equations pin down all equilibrium prices, up to the choice of numeraire, which can then be used to pin down quantities.

Example 10. Occupational Choice Example Suppose that the common consumption good is a CES bundle of outputs from different industries:

$$y = \left(\sum_r \Omega_{0r}^{\frac{1}{\theta_0}} x_{0r}^{\frac{\theta_0 - 1}{\theta_0}} \right)^{\frac{\theta_0}{\theta_0 - 1}},$$

where the units of quantities are chosen so that Ω_{0r} are expenditures in the status quo. Industry r's output is

$$x_{0r} = z_r L_r.$$

Suppose that workers utility functions can be written as

$$u_h(c_h, r) = c_h + \epsilon_{hr},$$

where ϵ_{hr} is type I extreme value. In this case, $L(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ has the standard logit functional form. This implies that

$$\frac{\partial L_r}{\partial \left[w_{r'} / p^c \right]} = \theta L_r L_{r'}.$$

Substituting this into (20) yields

$$d \log L_i^{\text{comp}} = \theta \left(d \left[w_i / \left(A p^c \right) \right] - \mathbb{E}_{L^{\text{comp}}} \left[d \left[w_i / \left(A p^c \right) \right] \right] \right), \tag{41}$$

where the wages and prices are evaluated in the compensated equilibrium. We now apply Proposition 2. The sales share of industry r, in the compensated equilibrium, is given by its share of the wage bill:

$$d \log \lambda_l^{\text{comp}} = d \log w_l + d \log L_l^{\text{comp}} - \mathbb{E}_{\lambda^{\text{comp}}} \left[d \log w + d \log L^{\text{comp}} \right]. \tag{42}$$

At the same time, the sales of share industry r, in the compensated equilibrium, is also given by the share of household spending on industry r:

$$d\log\lambda_l^{\text{comp}} = (\theta_0 - 1)\left[d\log z_l - d\log w_l - \mathbb{E}_{\lambda^{\text{comp}}}\left[d\log z - d\log w\right]\right], \tag{43}$$

where we use the fact that $d\log p_r = d\log w_r - d\log z_r$. Shephard's lemma implies that the consumer price index in the compensated equilibrium is

$$d\log p^c = \sum_l \lambda_l^{\text{comp}}\left[d\log w_l - d\log z_l\right]. \tag{44}$$

Finally, from Theorem 2, the change in TFP-equivalent aggregate welfare is:

$$d\log A = \sum_i \lambda_i^{\text{comp}} d\log z_i. \tag{45}$$

Equations (41)-(45) form a system of ordinary differential equations that can be solved to obtain $A(t)$ without simulation methods. The boundary conditions are that at $t = 0$, expenditure and population shares coincide with the status quo, and $A(0) = 1$. Solving the system is simple: discretize the productivity shocks, and iterate on the linear system, updating variables each time.

Deriving optimal place-based policy in Example 7 In this example, there is a measure–1 continuum of agents, R locations, no differences in idiosyncratic skills, and a single good produced with labor in each location. Agent h has utility

$$u_h(c) = \max_{r\in R}\{c_r \bar{e}_h \varepsilon_{hr}\},$$

where $\varepsilon_{hr} > 0$ are independent Fréchet random variables with common shape parameter θ and location-specific scale parameters $B_r > 0$. Given a vector of consumption by location, $\{c_r\}$, the supply of labor in location r is

$$L_r = \frac{B_r c_r^\theta}{\sum_{r'} B_{r'} c_{r'}^\theta}.$$

Under the cardinalization $\bar{e}_h = 1$ for all h, the utilitarian social welfare function is

$$W = \int u_h(c)\, dh = \kappa\left[\sum_r B_r c_r^\theta\right]^{1/\theta},$$

where $\kappa = \Gamma(1 - 1/\theta)$.

Derivation of (31)**.** Consider the problem of choosing $\{c_r^0\}$ to maximize W, subject to individual rationality, which implies the labor-supply system and the resource constraint

$$\sum_r L_r^0 c_r^0 = \sum_r L_r^0 z_r.$$

Using the labor-supply function, the resource constraint can be written as

$$\sum_r B_r (c_r^0)^{\theta+1} = \sum_r B_r (c_r^0)^{\theta} z_r.$$

Since W is increasing in $\sum_r B_r c_r^\theta$, the Lagrangian is

$$\mathcal{L}(c, \lambda) = \sum_r B_r c_r^\theta + \lambda \left(\sum_r B_r c_r^{\theta+1} - \sum_r B_r c_r^\theta z_r \right).$$

The FOC for c_r is

$$\theta B_r c_r^{\theta-1} + \lambda B_r \left[(\theta+1) c_r^\theta - \theta c_r^{\theta-1} z_r \right] = 0.$$

Dividing by $\theta B_r c_r^{\theta-1} > 0$ gives

$$1 + \frac{\lambda}{\theta} \left[\frac{\theta+1}{\theta} c_r - z_r \right] = 0.$$

Hence $(\theta+1)c_r - \theta z_r$ is constant across locations. Denote this constant by C:

$$c_r^0 = \frac{\theta}{\theta+1} z_r + \frac{C}{\theta+1}.$$

Multiplying by L_r^0 and summing over r, using $\sum_r L_r^0 = 1$ and the resource constraint, gives

$$C = \sum_r L_r^0 c_r^0.$$

Therefore, we obtain (31).

The following proposition shows that, starting at the "optimal" allocation (31), there exists a schedule of lump-sum transfers such that every agent can be made better off. For simplicity, we consider the version of the model with two regions, but the same argument applies in an economy with R regions.

Proposition 10. *Suppose $R = 2$ and $z_1 \neq z_2$. If the status quo is the "optimal" allocation implied* *by* (31)*, then this allocation is Pareto inefficient and* $A > 1$*.*

Proof. Let $l_h^0 \in \arg\max_{r=1,2} c_r^0 \varepsilon_{hr}$ denote the initial location choice. If household h is assigned to location r, the consumption required to keep her indifferent to the status quo is

$$\bar{c}_{hr} = c_{l_h^0}^0 \frac{\varepsilon_{hl_h^0}}{\varepsilon_{hr}}.$$

Applying Theorem 1 and using $w_r = z_r$ and $p_r = 1$, aggregate welfare can be written as

$$A = \max_l \frac{\sum_r \int z_r \mathbf{1}[l_h = r]\, dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh}.$$

By Dinkelbach's theorem, the solution A satisfies $F(A) = 0$, where

$$F(x) = \int \max_{r=1,2} \{z_r - x\bar{c}_{hr}\}\, dh.$$

We show that $F(1) > 0$, which implies $A > 1$. Define

$$\phi_h(r) = z_r - \bar{c}_{hr}.$$

Under the initial location choice,

$$\phi_h(l_h^0) = z_{l_h^0} - c_{l_h^0}^0.$$

Therefore, using the resource constraint,

$$\int \phi_h(l_h^0)\, dh = \sum_r L_r^0 (z_r - c_r^0) = 0.$$

Since $\max_r \phi_h(r) \geq \phi_h(l_h^0)$ for every h, we have $F(1) \geq 0$. It remains to show that the inequality is strict. Without loss of generality, suppose $z_1 > z_2$. From (31),

$$c_1^0 - c_2^0 = \frac{\theta}{\theta + 1}(z_1 - z_2) > 0.$$

Consider agents initially in location 2. Initial choice of location 2 over location 1 requires

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}} \geq \frac{c_1^0}{c_2^0}.$$

For such agents,

$$\phi_h(2) = z_2 - c_2^0, \qquad \phi_h(1) = z_1 - c_2^0 \frac{\varepsilon_{h2}}{\varepsilon_{h1}}.$$

Thus $\phi_h(1) > \phi_h(2)$ if and only if

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}} < 1 + \frac{z_1 - z_2}{c_2^0}.$$

The interval

$$\left[\frac{c_1^0}{c_2^0},\, 1 + \frac{z_1 - z_2}{c_2^0}\right)$$

is nonempty because

$$1 + \frac{z_1 - z_2}{c_2^0} - \frac{c_1^0}{c_2^0} = \frac{z_1 - z_2 - (c_1^0 - c_2^0)}{c_2^0} = \frac{z_1 - z_2}{(\theta + 1)c_2^0} > 0.$$

Since $(\varepsilon_{h1}, \varepsilon_{h2})$ has a continuous positive density, there is a positive-measure set of agents with

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}} \in \left[\frac{c_1^0}{c_2^0},\, 1 + \frac{z_1 - z_2}{c_2^0}\right).$$

For these agents, $l_h^0 = 2$ and $\phi_h(1) > \phi_h(2) = \phi_h(l_h^0)$. For all other agents, $\max_r \phi_h(r) \geq \phi_h(l_h^0)$. Therefore,

$$F(1) = \int \max_r \phi_h(r)\, dh > \int \phi_h(l_h^0)\, dh = 0.$$

Since $F(x)$ is decreasing in x and satisfies $F(A) = 0$, it follows that $A > 1$. Hence, starting from the allocation in (31), there exists another feasible allocation, implemented with lump-sum transfers, that makes every household strictly better off. The case $z_2 > z_1$ is symmetric. □

Appendix E Relation to the social surplus function

Consider the cardinalization of utility functions $f_h(x) = x$, which results in the following utilitarian social welfare function

$$U(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} g(c_{l_h}) + \epsilon_{h,l_h}].$$

This social welfare function is called the "*Social Surplus Function*" in the discrete choice literature (see McFadden, 1981).[46]

The proposition below shows that under Assumption 1, the social surplus function can be used to calculate $A(t)$.

Proposition 11 (Using Social Surplus to Calculate A)**.** *Suppose Assumption 1 holds. Let* $U(\boldsymbol{w}/p^c) = \mathbb{E}[\max_r a_{hr} w_r / p^c + \epsilon_{hr}]$ *be the expected social surplus function. Then, aggregate welfare in terms of TFP-equivalents,* $A(t)$*, satisfies the following equation:*

$$U(\boldsymbol{w}(t)/(A(t)p^c(t))) = U(\boldsymbol{w}(0)/p^c(0)),$$

where $\boldsymbol{w}(t)/(A(t)p^c(t))$ *are real wages in a decentralized equilibrium with productivities* $\boldsymbol{z}(t)$ *and aggregate factor-augmenting productivity* $1/A(t)$ *and* $\boldsymbol{w}(0)/p^c(0)$ *are real wages in the status quo. If, in addition, real wages do not depend on labor supplied in each region (e.g. as in the one-good economy), then we also have that* $A(t) = A^U(t)$*, where* $A^U(t)$ *is the consumption-equivalent variation defined via* $U(\boldsymbol{c}(t)/A^U(t)) = U(\boldsymbol{c}(0))$.

The proof is at the end of this subsection. In words, $A(t)$ can be computed as the reduction in labor productivity in every location such that the social surplus function, with productivity shocks $\boldsymbol{z}(t)$, is equal to social surplus under the status quo (without the need to explicitly compute any compensating transfers). Assumption 1 is critical and if it does not hold, then the social surplus function $U(\boldsymbol{c})$ cannot be used to compute $A(t)$ (see Example 12 below).

Notably, even if Assumption 1 holds, $A(t)$ may still differ from the consumption-equivalent under the social surplus function, $A^U(t)$. This is because real wages with productivities $(\boldsymbol{z}(t), 1/A(t))$ are not the same as real wages with productivities $\boldsymbol{z}(t)$ divided by $A(t)$. Intuitively, a reduction in aggregate factor productivity can cause agents to switch locations, and if real wages are a function of labor supply, then this can cause real wages to change and $A(t)$ to be different than $A^U(t)$. The last part of the proposition points out that if, in addition to Assumption 1, real wages do not respond to changes in labor supplies, as in the one-good economy where they are pinned down by productivity parameters, then $A(t)$ coincides with $A^U(t)$.

Proposition 11 relates to a well-known property of discrete choice models where consumers choose among a discrete set of options given fixed prices and incomes. In that

[46]The social surplus function plays an important role in the literature in part because of the Williams-Daly-Zachary theorem (Williams, 1977; Daly and Zachary, 1978), which gives conditions under which the social surplus function acts like a potential function (i.e. its derivatives are equal to aggregate location choices).

literature, it is known that if the indirect utility function is linear in the price of the good, then the social surplus function can be used to calculate the sum of compensating variations (Small and Rosen, 1981).[47]

The example below illustrates Proposition 11 using a simple example.

Example 11 (Using Social Surplus to Calculate Aggregate Welfare). Consider the one-good economy example from Example 3 again. There is a single freely-traded consumption good produced linearly from labor and productivity in each location r is z_r. We impose Assumptions 1 and 2. The real wage per efficiency unit in each location is simply $w_r = z_r$. Hence, by Proposition 11, we have

$$U(\boldsymbol{z}(t)/A(t)) = \mathbb{E}[\max_r \frac{z_r(t)}{A(t)} + \epsilon_{hr}] = \mathbb{E}[\max_r z_r(0) + \epsilon_{hr}] = U(\boldsymbol{z}(0)).$$

If we assume that ϵ are drawn from a Type I extreme value distribution, then we get

$$U(\boldsymbol{z}(t)/A(t)) = \sum_r \exp(\theta z_r(t)/A(t) + B_r) = \sum_r \exp(\theta z_r(0) + B_r) = U(\boldsymbol{z}(0)),$$

which coincides with equation (22). Furthermore, in this one-good economy, real wages in each location are equal to productivity and independent of labor supply. Hence, scaling all productivities by $1/A(t)$ is observationally equivalent to dividing all real wages by $A(t)$. This makes $A(t) = A^U(t)$.

Proposition 11 breaks down if Assumption 1 does not hold: i.e. if either $g(c)$ is nonlinear or if consumption prices vary by location. Intuitively, when Assumption 1 holds, we have a transferable utility model where the sum of $g(c_h)$ across households is interpretable as an aggregate resource constraint and can be split across households freely using lump-sum transfers. However, if either $g(c)$ is nonlinear or agents in different locations consume different consumption goods, then utility is not transferable and the sum of $g(c_h)$ across households cannot be interpreted as an aggregate resource constraint.

The following example demonstrates that when Assumption 1 does not hold, as in the commonly studied Fréchet case, then $A^U(t)$ and $A(t)$ need not agree even to a first order approximation (and can even have different signs).

[47] In the absence of linearity, calculating the sum of compensating variations typically requires resorting to simulation methods. See Hortaçsu and Joo (2023), and the references therein, for a recent textbook discussion. As discussed by Hortaçsu and Joo (2023), many recent studies in the industrial organization literature ignore individual compensating variations and directly use the social surplus function as the starting point of their analysis, even if the quasi-linear justification for this approach does not hold. In this case, the social surplus function is interpreted like a social welfare function. Dubé et al. (2025) analyze the welfare properties of this social surplus function for a broad class of demand systems.

Example 12 (When Proposition 11 Fails). Consider Example 6 again. There is a common freely-traded consumption good produced linearly from labor, with productivity z_r in location r. Assumption 1 does not hold because $g(c)$ is logarithmic. Assume that $\exp(\epsilon)$ is distributed according to a Fréchet distribution. Utilitarian welfare, as measured by $A^U(t)$, is the solution to

$$U(\boldsymbol{z}(t)/A^U(t)) = \mathbb{E}[\max_r \log(z_r(t)/A^U(t)) + \epsilon_{hr}] = \mathbb{E}[\max_r \log(z_r(0)) + \epsilon_{hr}] = U(\boldsymbol{z}(0)).$$

One can show that $A^U(t) = \left(\sum_r z_r^\theta(t)/\sum_r z_r^\theta(0)\right)^{\frac{1}{\theta}}$.[48] To a first-order approximation, changes in $\log A^U$ are a population-weighted sum of changes in productivities:

$$\Delta \log A^U(t) \approx \sum_r L_r(0)\Delta \log z_r,$$

where $L_r(0)$ is the share of the population choosing location r at the status quo. This does not coincide, even to a first-order, with changes in aggregate welfare as measured by A, because $\Delta \log A \approx \sum_r \lambda_r(0)\Delta \log z_r$, where the Domar weights in this example are $\lambda_r(0) = L_r(0)z_r(0)/(\sum_{r'} L_{r'}(0)z_{r'}(0))$. It can easily be the case that $\Delta \log A > 0$, so that winners can compensate losers and have resources left over, and yet $\Delta \log A^U < 0$.

Proof of Proposition 11. Without loss of generality, let the common consumption good price p^c be the numeraire. The social surplus function is

$$U(\boldsymbol{w}) = \mathbb{E}\left[\max_r \{a_{hi}w_r + \epsilon_{hr}\}\right],$$

where w_r is now the real wage in location r. Then,

$$\begin{aligned}
\frac{\partial U}{\partial w_r} &= \frac{\partial}{\partial w_r}\int \max_i \{a_{hi}w_i + \epsilon_{hi}\} f(\epsilon)d\epsilon \\
&= \int \frac{\partial}{\partial w_r}\left[\max_i \{a_{hi}w_i + \epsilon_{hi}\}\right] f(\epsilon)d\epsilon \\
&= \int a_{hr}\mathbf{1}\left\{a_{hr}w_r + \epsilon_{hr} \geq a_{hj}w_j + \epsilon_{hj}\forall j\right\} f(\epsilon)d\epsilon \\
&= L_r(\boldsymbol{w}).
\end{aligned}$$

[48] Compare this with the right hand side of (30), which calculates $A^W(t)$ under the $f_h(x) = \exp(x)$ cardinalization. The fact that the consumption-equivalent variation of the social welfare function $U(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} \log(c_{l_h}(t)) + \epsilon_{hl_h}]$ is equal to that of the social welfare function $W(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} c_{l_h}(t)\exp(\epsilon_{hl_h})]$ is a special implication from the assumption that $\exp(\epsilon)$ is distributed according to a Fréchet distribution.

Equation (33) in the proof of Theorem 2 states that with a common consumption good

$$\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log\left[\frac{w_r}{A}\right] = \sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log p^c,$$

where the wages and prices are the ones in the compensated equilibrium. Since the consumer price index is the numeraire, this is the same as:

$$\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log\left[\frac{w_r}{A}\right] = 0.$$

Equivalently,

$$\sum_r L_r^{\text{comp}} d\left[\frac{w_r}{A}\right] = 0.$$

Integrating between 0 and t gives:

$$\int_{w(0)/p^c(0)}^{w(t)/(A(t))} \sum_r L_r^{\text{comp}}(\boldsymbol{x})d\boldsymbol{x} = 0.$$

By Proposition 3, the compensated and uncompensated labor supply functions are the same. Furthermore, given productivities $\boldsymbol{z}(t)$ and aggregate factor-augmenting productivity $1/A(t)$, the compensated and uncompensated equilibrium real wages are the same. Hence, we can write

$$\int_{w(0)}^{w(t)/(A(t))} \sum_r L_r(\boldsymbol{x})d\boldsymbol{x} = 0,$$

substituting in $\frac{\partial U}{\partial w_r}$ from above gives

$$\int_{w(0)}^{w(t)/(A(t))} \sum_r \frac{\partial U}{\partial w_r}(\boldsymbol{x})d\boldsymbol{x} = 0,$$

which is equal to

$$U(w(t)/(A(t))) - U(w(0)) = 0,$$

by the fundamental theorem of calculus.

□

Appendix F Calculating $A(t)$ via system of linear differential equations

In this appendix we describe how to calculate $A(t)$ using a system of linear differential equations implied by Theorem 2 and Proposition 2, as discussed in Section 5. We present the log-linearized system and then give a step-by-step algorithm.

System of equations

TFP-equivalent welfare

$$\frac{d \log A(t)}{dt} = \sum_i \lambda_i^{\text{comp}}(t) \frac{d \log z_i(t)}{dt}, \qquad A(1) = 1. \tag{46}$$

Input-output and prices

$$d \log \Omega_{ij} = (1-\theta_i) \left[d \log p_j - \sum_{k \in N} \Omega_{ik} \, d \log p_k - \sum_{r \in R} \Omega_{ir} \, d \log w_r \right]. \tag{47}$$

$$d \log \boldsymbol{p} = -\Psi \, d \log \boldsymbol{z} + \Psi^F \, d \log \boldsymbol{w}, \tag{48}$$

where $\Psi = (I - \Omega)^{-1}$ is the Leontief inverse and Ψ^F the matrix of factor contents.

Compensated supply and demand derivatives Let

$$L_r^{\text{comp}} = L_r^{\text{comp}}\left(\tfrac{\boldsymbol{w}}{A}, \boldsymbol{p}\right), \qquad \chi_r^{\text{comp}} = \chi_r^{\text{comp}}\left(\tfrac{\boldsymbol{w}}{A}, \boldsymbol{p}\right),$$

and define

$$\eta_{r,s}^{L} = \frac{\partial \log L_r^{\text{comp}}}{\partial \log(w_s/A)}, \quad \eta_{r,i}^{L,p} = \frac{\partial \log L_r^{\text{comp}}}{\partial \log p_i},$$
$$\eta_{r,s}^{\chi} = \frac{\partial \chi_r^{\text{comp}}}{\partial \log(w_s/A)}, \quad \eta_{r,i}^{\chi,p} = \frac{\partial \chi_r^{\text{comp}}}{\partial \log p_i}.$$

Then

$$d \log L_r^{\text{comp}} = \sum_s \eta_{r,s}^{L} \left(d \log w_s - d \log A \right) + \sum_i \eta_{r,i}^{L,p} \, d \log p_i, \tag{49}$$

$$d \chi_r^{\text{comp}} = \sum_s \eta_{r,s}^{\chi} \left(d \log w_s - d \log A \right) + \sum_i \eta_{r,i}^{\chi,p} \, d \log p_i. \tag{50}$$

Factor Domar weights

$$d\log\lambda_r^{\text{comp}} = d\log w_r + d\log L_r^{\text{comp}} - \sum_{r'} \lambda_{r'}^{\text{comp}} \Big[d\log w_{r'} + d\log L_{r'}^{\text{comp}} \Big]. \tag{51}$$

Using (49), this can be written in terms of $d\log w_s$, $d\log p_i$, and $d\log A$.

Goods/intermediate Domar weights

$$d\log\lambda_i^{\text{comp}} = \left(\lambda_i^{\text{comp}}\right)^{-1} \left(\sum_r d\chi_r^{\text{comp}}\, \Omega_{c(r)i} + \sum_r \chi_r^{\text{comp}}\, d\Omega_{c(r)i} + \sum_j d\big[\lambda_j^{\text{comp}}\, \Omega_{ji}\big] \right), \tag{52}$$

with

$$d\big[\lambda_j^{\text{comp}}\, \Omega_{ji}\big] = \lambda_j^{\text{comp}} \Omega_{ji}\, d\log\lambda_j^{\text{comp}} + \lambda_j^{\text{comp}} \Omega_{ji}\, d\log\Omega_{ji}.$$

Here $d\chi_r^{\text{comp}}$ comes from (50), $d\Omega$ from (47), and $d\log\lambda_j^{\text{comp}}$ from (51)–(52). Given $d\log A$ from (46) and $d\log z_i$, the system (47)–(48)–(49)–(50)–(51)–(52) is linear in

$$d\log p_i, \quad d\log w_r, \quad d\log\Omega_{ij}, \quad d\log\lambda_i^{\text{comp}}.$$

Algorithm

Step 0 (initialization, $t = 1$)

i. Solve the decentralized equilibrium at $t = 1$ to obtain

$$\boldsymbol{p}(1),\ \boldsymbol{w}(1),\ \boldsymbol{\Omega}(1),\ \boldsymbol{\lambda}(1).$$

ii. Set compensated objects at $t = 1$:

$$\lambda_i^{\text{comp}}(1) = \lambda_i(1),$$

and use the same $\boldsymbol{p}(1)$, $\boldsymbol{w}(1)$, $\boldsymbol{\Omega}(1)$.

iii. Set $A(1) = 1$.

Step 1 (shocks and grid)

i. Specify a path $\boldsymbol{z}(t)$ and compute $d\log z_i(t)/dt$.

ii. Choose a time grid $1 = t_0 < \cdots < t_K$ with $\Delta t_k = t_{k+1} - t_k$.

Step 2 (for each $k = 0, \ldots, K-1$) Given

$$\boldsymbol{p}(t_k),\ \boldsymbol{w}(t_k),\ \boldsymbol{\Omega}(t_k),\ \boldsymbol{\lambda}^{\text{comp}}(t_k),\ A(t_k),$$

proceed as follows.

2.1 Hulten step

Compute

$$\left.\frac{d\log A}{dt}\right|_{t_k} = \sum_i \lambda_i^{\text{comp}}(t_k)\ \left.\frac{d\log z_i}{dt}\right|_{t_k}.$$

2.2 Compensated demand and supply

i. Evaluate $L_r^{\text{comp}}(t_k)$ and $\chi_r^{\text{comp}}(t_k)$ at

$$\left(\frac{\boldsymbol{w}(t_k)}{A(t_k)}, \boldsymbol{p}(t_k)\right)$$

using the compensated choice problem.

ii. Compute

$$\eta^L_{r,s},\ \eta^{L,p}_{r,i},\ \eta^\chi_{r,s},\ \eta^{\chi,p}_{r,i}$$

at that point (analytically or by finite differences).

iii. Use (49) and (50) to express $d\log L_r^{\text{comp}}$ and $d\chi_r^{\text{comp}}$ as linear functions of $d\log w_s$, $d\log p_i$ and the known $d\log A\big|_{t_k}$.

2.3 General equilibrium

i. Plug (49), (50), (47), and (48) into (51) and (52).

ii. This yields a linear system for

$$\left.\frac{d\log p_i}{dt},\quad \frac{d\log w_r}{dt},\quad \frac{d\log \Omega_{ij}}{dt},\quad \frac{d\log \lambda_i^{\text{comp}}}{dt}\right|_{t_k},$$

where $\left(d\log A/dt\right)\big|_{t_k}$ and $\left(d\log z_i/dt\right)\big|_{t_k}$ are known. Solve this system.

2.4 Time update

For any variable $x \in \{p_i, w_r, \Omega_{ij}, \lambda_i^{\text{comp}}, A\}$,

$$\log x(t_{k+1}) = \log x(t_k) + \left.\frac{d\log x}{dt}\right|_{t_k} \Delta t_k.$$

Repeat until t_K.

Appendix G Aggregate welfare with distortions and externalities

In the body of the paper, we consider perfectly competitive discrete choice models. In this appendix we discuss how Theorem 1 extends beyond the perfectly competitive baseline. We first introduce wedge distortions and then agglomeration externalities.

Wedge distortions

The household block of the model is unchanged. Each agent chooses consumption and location to maximize utility given prices, wages, and transfers, where transfers now include revenues from wedges (e.g. tax revenues). Aggregating individual choices gives rise to labor supply and final demand functions, $L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ and $\chi_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$.

Producers choose quantities to minimize costs, taking prices as given. The price of good i is now given by

$$p_i = \mu_i z_i^{-1} \text{mc}_i(\boldsymbol{p}, \boldsymbol{w}), \tag{53}$$

where $\mu_i > 0$ is a tax, markup, or any other wedge between price and marginal cost for producer of good i.[49] Lump sum transfers across households must equal total revenues generated by the wedges,

$$\int T_h dh = \sum_i p_i y_i (1 - \frac{1}{\mu_i}). \tag{54}$$

In the decentralized equilibrium, transfers are distributed across households according to some rule.

To solve for a decentralized equilibrium with wedges, the market clearing conditions (6) and (7) are unchanged, but when defining the ratio of sales, factor payments, and

[49] Even though we assume that wedges are on gross output only, this is without loss of generality. This is because we can recreate buyer-seller wedges by treating firm i's purchases of an input from j as a distinct good (made linearly using j's output). A wedge on this good is then isomorphic to a buyer-seller wedge.

spending to GDP ,

$$\lambda_i = \frac{p_i y_i}{GDP}\mathbf{1}[i \in C+N] + \frac{Z w_i L_i}{GDP}\mathbf{1}[i \in R], \qquad \text{and} \qquad \chi_r = \frac{Z w_r L_r + \int T_h \mathbf{1}[l_h = r] dh}{GDP},$$

we use the fact that GDP (or aggregate income) is given by

$$GDP = \sum_r Z w_r L_r + \sum_i p_i y_i (1 - \frac{1}{\mu_i}).$$

Given wedges, some transfer rule, and the aggregate supply and demand functions, equilibrium prices, wages, and quantities satisfy (6), (7), and (53).

We parameterize technologies and wedges at t by $\boldsymbol{z}(t)$ and $\boldsymbol{\mu}(t)$. Define the set of feasible allocations given productivity shifters $\boldsymbol{z}(t)$, wedges $\boldsymbol{\mu}(t)$, and aggregate factor-augmenting productivity Z to be

$$\mathcal{C}(t, Z) \equiv \{\{\boldsymbol{c}_h, l_h\}_h \text{ supported via equilibrium with wedges given } \boldsymbol{z}(t), \boldsymbol{\mu}(t), Z, \text{and some transfers.}\}$$

The change in productivity $A(t)$ is defined exactly as in Equation (11). In response to a change in technologies and wedges, we consider a hypothetical rescaling of Z by a factor $1/A(t)$, and define $A(t)$ as the largest contraction such that every agent can be made at least as well off as in the initial equilibrium. Misallocation due to distortions in the status quo can be quantified by setting $\boldsymbol{\mu}(t) = 1$ and $\boldsymbol{z}(t) = \boldsymbol{z}(0)$.

We can calculate $A(t)$ using a compensated equilibrium with wedges, extending the results in Section 3.2. In the compensated equilibrium with wedges, every agent receives a transfer that keeps them indifferent, the sum of transfers equals the sum of wedge revenues, and TFP is contracted by $A(t)$. Compensated choices, $l_h^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ and the expenditure function, $e_h(Z\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ are obtained using Proposition 1, and these individual choices are aggregated to get location-level variables, $L_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)$, $E_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)$, and $\chi_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)$, exactly as in the baseline model without wedges. The generalization of Theorem 1 is:

Theorem 3 (Aggregate Welfare). *TFP-equivalent aggregate welfare satisfies*

$$\sum_r E_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0) = \frac{1}{1 - \sum_i \lambda_i (1 - \frac{1}{\mu_i})} \sum_r (w_r / A(t)) L_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0). \tag{55}$$

Given compensated labor supply $\boldsymbol{L}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0)$ and demand $\boldsymbol{\chi}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0)$, the vector of prices $\boldsymbol{p}$ in the compensated equilibrium satisfies (53), *and the vector of wages $\boldsymbol{w}$ and sales shares $\boldsymbol{\lambda}$ satisfy market clearing conditions* (6) *and* (7).

To prove this result, we follow the same steps in the proof of Theorem 1, except for condition (54) that the sum of transfers is not equal to zero but equal to total revenues from wedges.

Introducing agglomeration externalities

We now briefly discuss how to account for agglomeration externalities. For simplicity, we only consider local externalities determined by the workers located in the location where the good is produced, but it is straightforward to also allow for spillovers between regions. Suppose that the production function of good i produced in region r is given by

$$y_i = \tilde{z}_i F_i \left(\{x_{ij}\}_{j \in N}, \{L_{ir}\}_{r \in R} \right), \tag{56}$$

where the Hicks-neutral productivity shifter is

$$\tilde{z}_i = z_i L_r^{\gamma_i}.$$

Producers take $\tilde{z}_i$ as given when minimizing costs. If $\gamma_i > 0$, then there is a positive externality from a larger mass of agents, weighted by their skills exclusive of the aggregate TFP shifter Z, in location r on the production of good i. If $\gamma_i < 0$, then there is a congestion externality from a larger mass of agents in location r on the production of good i.

For example, in the one-good economy, each region produces output according to

$$y_r = z_r L_r^{1+\gamma_r} = \tilde{z}_r L_r,$$

and the equilibrium wage is equal to the private marginal product of labor,

$$w_r = z_r L_r^{\gamma_r} = \tilde{z}_r,$$

since producers take $\tilde{z}_r$ as given. When solving for equilibrium and for $A(t)$, we must therefore take into account how changes in labor supply affect productivities $\tilde{\boldsymbol{z}}$.

Theorem 1 remains unchanged, with $\tilde{\boldsymbol{z}}$ in place of $\boldsymbol{z}$, and with the additional constraint

$$\tilde{z}_i = z_i \left(L_r^{\text{comp}} \right)^{\gamma_i}. \tag{57}$$

If there are multiple compensated equilibria, choose the one that implies the highest value of $A(t)$. The solution algorithm described after Theorem 1 remains unchanged, with $\tilde{\boldsymbol{z}}$ in place of $\boldsymbol{z}$ and where each iteration must also impose condition (57).

We illustrate for the one-good economy of Example 2. TFP-equivalent aggregate welfare $A(t)$ is given by a modified version of (13):

$$A(t) = \frac{\sum_r \int \tilde{z}_r a_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh},$$

where compensated location choices satisfy:

$$l_h^{\text{comp}} \in \arg\max_{r \in R} \left\{ \tilde{z}_r a_{hr} / A(t) - \bar{c}_{hr} \right\},$$

and

$$\tilde{z}_r = z_r(t) \left(L_r^{\text{comp}} \right)^{\gamma_r}, \qquad \text{where} \qquad L_r^{\text{comp}} = \int a_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh.$$

Appendix H Aggregate welfare with place-based compensation

In the body of the paper, we measure aggregate welfare by the amount of factor endowments that can be saved after winners compensate losers using lump-sum transfers. In this appendix, we briefly discuss how to extend this definition of aggregate welfare to cases where individual-level lump-sum transfers are not available, but place-based redistributive policies are.

Suppose that instead of individual-specific transfers, location-level consumption taxes, denoted by τ_r, can be levied on households that choose location r. In this case, the budget constraint of agent h is now:

$$\sum_r (1 + \tau_r) p_r c_h \mathbf{1} \left[l_h = r \right] = Z \sum_r w_r a_{hr} \mathbf{1} \left[l_h = r \right] + T,$$

where T is a uniform lump-sum rebate (the same for all households). Budget balance requires that

$$T = \int \sum_r \tau_r p_r c_h \mathbf{1} \left[l_h = r \right] dh. \tag{58}$$

All other conditions defining equilibrium are the same as in Section 2. We can now define the feasible set of allocations that can be supported using these place-based tax instruments:

$$\mathcal{C}^{pb}(t, Z) \equiv \left\{ \{ \boldsymbol{c}_h, l_h \}_{h \in H} \text{ supported via equilibrium given } \boldsymbol{z}(t), Z, \text{ and some consumption taxes} \right\}.$$

Given this feasible set, the definition of TFP-equivalent aggregate welfare, given place-based compensations, is the same as before replacing $\mathcal{C}(t, Z)$ with $\mathcal{C}^{pb}(t, Z)$.

Definition 7. TFP-equivalent aggregate welfare with place-based compensation at t is

$$A^{pb}(t) = \max\left\{Z^{-1} : \{c_h, l_h\}_{h\in H} \in \mathcal{C}^{pb}(t, Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h\right\}. \tag{59}$$

where $c_h(0)$ and $l_h(0)$ are the consumption and location of h in the status quo.

To see how this works in practice, consider the following one-good economy example.

Example 13 (One-Good Economy with Place-Based Policies). Consider the one-good economy again: there is a single freely-traded consumption good produced linearly from labor and productivity in each location r is z_r. Assume all agents have homogeneous skills, as in Assumption 2. Hence, production in each location is

$$y_r = z_r(t)L_r = z_r(t)\int Z\mathbf{1}[l_h = r]dh.$$

The following shows that, in the one-good economy, $A^{pb}(t)$ is bounded above by $A(t)$ but that, to a first-order approximation, the two are the same.

Proposition 12 (Hulten's Theorem for One-Good Economy). *In the one-good economy with $a_{hr} = 1$ for every h and r, we have*

$$A^{pb}(t) \leq A(t) \quad \text{for all } t.$$

Moreover, around the status quo $t = 0$, the two measures coincide to a first-order approximation and both obey Hulten's theorem:

$$\Delta \log A^{pb} \approx \Delta \log A \approx \sum_r \lambda_r(0)\, \Delta \log z_r.$$

Before providing a formal proof (below), we provide a heuristic discussion. To calculate A^{pb}, rearrange the budget-balance condition, (58), as

$$Z^{-1} = \frac{\sum_r w_r L_r}{\sum_r p_r c_r L_r} = \frac{\sum_r z_r L_r}{\sum_r c_r L_r}, \tag{60}$$

where c_r denotes per-capita consumption in location r, and we use the fact that the real wage in location r is equal to $z_r(t)$ in this one-good economy. Using equation (59) and

(60), we can write

$$A^{pb}(t) = \max_{\boldsymbol{c}} \left\{ \frac{\sum_r z_r(t) L_r(\boldsymbol{c})}{\sum_r c_r L_r(\boldsymbol{c})} : u_h(c_r, l_h(\boldsymbol{c})) \geq u_h^0 \text{ for every } h \right\},$$

where $L_r(\boldsymbol{c})$ is labor supply in location r as a function of the vector of location-level per capita consumption $\boldsymbol{c}$.

If, in the solution to the problem above, for every location $r \in R$, some agent $h \in H$ stays in the same location as in the status-quo and all locations are non-empty, then compensating the stayers requires ensuring that consumption per capita in each location is at least as high as in the status quo. Hence, we can replace the H inequalities above with just R inequalities:

$$A^{pb}(t) = \max_{\boldsymbol{c}} \left\{ \frac{\sum_r z_r(t) L_r(\boldsymbol{c})}{\sum_r c_r L_r(\boldsymbol{c})} : c_r \geq c_r^0 \text{ for every } r \right\}.$$

This is relatively simple constrained optimization maximization problem given the supply function.

To make it more concrete, suppose that $\boldsymbol{L}(\boldsymbol{c})$ is the iso-elastic labor supply function:

$$\frac{L_r(\boldsymbol{c})}{\sum_{r'} L_{r'}(\boldsymbol{c})} = \frac{c_r^\theta}{\sum_{r'} c_{r'}^\theta}.$$

Then, it is straightforward to show that the solution to $A^{pb}(t)$ satisfies:

$$c_r^{pb}(t) = z_r(0) \mathbf{1} \left[\frac{z_r(t)/z_r(0)}{A^{pb}(t)} \leq \frac{\theta+1}{\theta} \right] + \frac{\theta}{\theta+1} \frac{z_r(t)}{A^{pb}(t)} \mathbf{1} \left[\frac{z_r(t)/z_r(0)}{A^{pb}(t)} > \frac{\theta+1}{\theta} \right].$$

If the shocks are sufficiently small, $\frac{z_r(t)/z_r(0)}{A^{pb}(t)} \leq \frac{\theta+1}{\theta}$ for every r, then the solution sets consumption in each location equal to its status quo value $c_r^{pb}(t) = z_r(0)$. This is achieved by scaling aggregate factor productivity by $1/A^{pb}(t)$ and setting consumption taxes appropriately. Since consumption is equal to its status-quo value in every location, no agent switches locations, and hence every agent is exactly indifferent to the status-quo.

However, if region r experiences much faster productivity growth than average — in the sense that $\frac{z_r(t)/z_r(0)}{A^{pb}(t)} > \frac{\theta+1}{\theta}$ — then consumption in region r must exceed its status-quo level to incentivize additional workers to move there. In this case, stayers in region r are strictly better off than in the status quo. Given the optimal choices $\boldsymbol{c}^{pb}(t)$ and the implied labor supplies $L_r(\boldsymbol{c}^{pb}(t))$, the place-based aggregate welfare measure $A^{pb}(t)$ is obtained from (60).

The proof of Proposition 12 makes use of the following lemma:

Lemma 1. *In the one-good economy, for any preferences $u_h(c_h, l) = f_h(g(c_h) + \epsilon_{hl})$, the following is true. The problem where aggregate welfare is calculated by choosing feasible locations agent by agent,*

$$\tilde{A} = \max_{\boldsymbol{l}} \frac{\sum_r \int z_r \mathbf{1}[l_h = r]\, dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh}, \tag{61}$$

yields the same answer as A with the same location choices. That is, $\tilde{A} = A$.

Proof of Lemma 1. Let $\tilde{A}$ be the value of the maximization problem in (61), and denote by $\tilde{\boldsymbol{l}}$ the corresponding optimal location profile. By Dinkelbach (1967), $\tilde{\boldsymbol{l}}$ also solves the alternative problem

$$\max_{\boldsymbol{l}} \left[\sum_r \int \frac{z_r}{\tilde{A}} \mathbf{1}[l_h = r]\, dh - \sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh \right],$$

or equivalently

$$\max_{\boldsymbol{l}} \sum_r \int \left(\frac{z_r}{\tilde{A}} - \bar{c}_{hr} \right) \mathbf{1}[l_h = r]\ dh.$$

Given $\tilde{A}$, this problem is separable across h, so it can be solved household by household:

$$\tilde{l}_h \in \arg\max_{r \in R} \left\{ \frac{z_r}{\tilde{A}} - \bar{c}_{hr} \right\}.$$

By Proposition 1, the compensated location choice of household h when aggregate welfare is $1/\tilde{A}$ is

$$l_h^{\text{comp}} \in \arg\max_{r \in R} \left\{ \frac{z_r}{\tilde{A}} - \bar{c}_{hr} \right\},$$

so $\tilde{l}_h = l_h^{\text{comp}}$ for all h. By definition of A, the value of aggregate welfare that supports this compensated allocation is precisely $A = \tilde{A}$, as claimed. □

Proof of Proposition 12. The first part of the proposition is a consequence of Lemma 1. The lemma shows that, in a one-good economy, the value of aggregate welfare $A(t)$ obtained by maximizing over all feasible location profiles $\{l_h\}_{h \in H}$ coincides with $A(t)$ defined using individual-specific lump-sum transfers. Any allocation implementable with place-based policies corresponds to some location profile, and hence is feasible in the feasible agent-by-agent location problem. Therefore, place-based policies cannot achieve a strictly larger value of aggregate welfare than individual-specific transfers, and

$$A^{pb}(t) \le A(t) \quad \text{for all } t.$$

For the second part, we construct an explicit feasible place-based policy that yields a lower bound for $A^{pb}(t)$ and then apply a squeezing argument. Let L_r^0 denote labor supply in location r in the status quo. Define

$$A^{lb}(t) \equiv \frac{\sum_r z_r(t) L_r^0}{\sum_r z_r(0) L_r^0}. \tag{62}$$

Consider the following policy. Scale productivities by $1/A^{lb}(t)$ and choose a vector of consumption tax rates $\{\tau_r^{lb}\}_{r\in R}$ and a common lump-sum rebate T^{lb} such that, in the resulting equilibrium,

$$\frac{z_r(t)/A^{lb}(t) + T^{lb}}{1+\tau_r^{lb}} = c_r^0 = z_r(0) \quad \text{for every } r. \tag{63}$$

In words, after taxes and transfers, consumption per capita in every location is exactly equal to its status-quo level c_r^0. This implies that every household is exactly indifferent to the status quo and has no incentive to move. Hence, location choices remain at their status-quo values. Substituting $(1+\tau_r^{lb})c_r^0 = z_r(t)/A^{lb}(t) + T^{lb}$ into the budget-balance condition (58), one can verify that such a pair $(T^{lb}, \{\tau_r^{lb}\}_r)$ exists if $A^{lb}(t)$ is given by (62).

Since this policy keeps everyone indifferent to the status quo and is implementable with place-based consumption taxes, it is a feasible solution for the problem that defines $A^{pb}(t)$. Therefore,

$$A^{pb}(t) \geq A^{lb}(t) \quad \text{for all } t.$$

In the one-good economy with exogenous labor supplies $\{L_r^0\}$, the lower-bound $A^{lb}(t)$ coincides with a standard aggregate productivity index: $A^{lb}(t) = \sum_r z_r(t)L_r^0 / \sum_r z_r(0)L_r^0$. By Hulten's theorem, its derivative at the status quo satisfies

$$\frac{d}{dt}\log A^{lb}(t)\Big|_{t=0} = \sum_r \lambda_r(0)\,\frac{d}{dt}\log z_r(t)\Big|_{t=0}, \tag{64}$$

where $\lambda_r(0) = z_r(0)L_r^0/\sum_{r'} z_{r'}(0)L_{r'}^0$. On the other hand, the full-transfer measure $A(t)$ also satisfies Hulten's theorem at the status quo (by Corollary 1). So,

$$\frac{d}{dt}\log A(t)\Big|_{t=0} = \sum_r \lambda_r(0)\,\frac{d}{dt}\log z_r(t)\Big|_{t=0}. \tag{65}$$

Combining $A^{lb}(t) \leq A^{pb}(t) \leq A(t)$ for all t with $A^{lb}(0) = A^{pb}(0) = A(0) = 1$, we have for t close to zero and $t \neq 0$,

$$\frac{\log A^{lb}(t) - \log A^{lb}(0)}{t} \leq \frac{\log A^{pb}(t) - \log A^{pb}(0)}{t} \leq \frac{\log A(t) - \log A(0)}{t}.$$

Using (64) and (65), the left and right terms converge to the same limit. By the squeeze theorem, the middle term must converge to the same limit. Hence $\log A^{pb}(t)$ is differentiable at $t = 0$ and

$$\frac{d}{dt} \log A^{pb}(t)\Big|_{t=0} = \sum_r \lambda_r(0) \frac{d}{dt} \log z_r(t)\Big|_{t=0}.$$

Rewriting this in terms of finite changes yields the first-order approximation in the statement of the proposition. □

Appendix I Alternative definition of Kaldor-Hicks efficiency

In Section 6.3 we defined Kaldor-Hicks efficiency as

$$S(t) \equiv -\int_h e_h(\boldsymbol{w}(t), \boldsymbol{p}(t), u_h^0)\, dh$$

where

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min\left\{T_h : u_h(c_h, l_h) \geq u_h^0, \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\right\}.$$

The following lemma shows that $S(t)$ is equal to the maximum difference between total income and the sum of compensating incomes (minimum income to attain indifference with the status quo), evaluated at t equilibrium wages and prices.

Lemma 2 (Equivalent definition of $S(t)$)**.**

$$S(t) = \max_{\{(c_h, l_h)\}_{h \in H}} \left\{\int_h \left[w_{l_h}(t)\, a_{l_h} - p_{l_h}(t)\, c_h\right] dh : \ u_h(c_h, l_h) = u_h^0\ \forall h\right\}.$$

Proof. By local non-satiation in c_h, the minimizer in the definition of $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ must satisfy $u_h(c_h, l_h) = u_h^0$. Thus

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \left\{T_h : \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\right\}.$$

For any (c_h, l_h), the budget constraint implies $T_h \geq p_{l_h} c_h - w_{l_h} a_{hl_h}$, so at the minimum $T_h = p_{l_h} c_h - w_{l_h} a_{hl_h}$. Hence

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \{p_{l_h} c_h - w_{l_h} a_{hl_h}\},$$

and therefore

$$-e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \max_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \{w_{l_h} a_{hl_h} - p_{l_h} c_h\}.$$

Summing over h,

$$S(t) = \int_h \max_{u_h(c_h, l_h) = u_h^0} \left\{ w_{l_h}(t) a_{l_h} - p_{l_h}(t) c_h \right\} dh.$$

Because choices and payoffs are additively separable across households, maximizing the integral over all feasible profiles $\{(c_h, l_h)\}_{h \in H}$ (i.e. those that satisfy $u_h(c_h, l_h) = u_h^0 \ \forall h$) is equivalent to choosing, for each h, a maximizer of the integrand, giving the stated result. □

Consider now an alternative definition of Kaldor-Hicks efficiency as the maximum *ratio* — rather than the difference in terms of the numeraire — between aggregate income and the sum of compensating incomes, at t wages and prices:

$$A^{KH}(t) \equiv \max_{\{(c_h, l_h)\}_{h \in H}} \left\{ \frac{\int_h w_{l_h}(t)\, a_{l_h}\, dh}{\int_h p_{l_h}(t)\, c_h\, dh} \ : \ u_h(c_h, l_h) = u_h^0 \ \forall h \right\}.$$

Note that, in general, the allocations that maximize $S(t)$ do not coincide with those that maximize $A^{KH}(t)$. However, the following proposition shows that Kaldor-Hicks efficiency rises (i.e. $S(t) > 0$) if and only if $A^{KH}(t) > 1$. Therefore, $A^{KH}(t)$ inherits the same undesirable properties in general equilibrium as $S(t)$. For example, as discussed in Section 6.3, pure transfers cause Kaldor-Hicks efficiency to rise.

Proposition 13 (Equivalence in sign between $S(t)$ and $A^{KH}(t)$.)**.**

$$\operatorname{sign}\big(A^{KH}(t) - 1\big) = \operatorname{sign}\big(S(t)\big).$$

Proof. We can write

$$\int_h \left[w_{l_h}(t)\, a_{l_h} - p_{l_h}(t)\, c_h\right] dh = \int_h w_{l_h}(t)\, a_{l_h}\, dh - \int_h p_{l_h}(t)\, c_h\, dh$$
$$= \left(\int_h p_{l_h}(t)\, c_h\, dh\right) \left(\frac{\int_h w_{l_h}(t)\, a_{l_h}\, dh}{\int_h p_{l_h}(t)\, c_h\, dh} - 1\right).$$

Since in equilibrium $\int_h p_{l_h}(t)\, c_h\, dh > 0$, the sign of the left-hand side is the same as the sign of the term in parentheses. Thus, for every feasible profile, the sign of

$$\int_h \left[w_{l_h}(t)\, a_{l_h} - p_{l_h}(t)\, c_h\right] dh$$

coincides with the sign of

$$\frac{\int_h w_{l_h}(t)\, a_{l_h}\, dh}{\int_h p_{l_h}(t)\, c_h\, dh} - 1.$$

By Lemma 2, $S(t)$ is the maximum of the former over all feasible allocations, while $A^{KH}(t)$ is the maximum of the latter over all feasible allocations. Since the sign equivalence holds allocation by allocation and both $S(t)$ and $A^{KH}(t)$ are maxima over the same feasible set, it follows that

$$S(t) > 0 \iff A^{KH}(t) > 1, \quad S(t) = 0 \iff A^{KH}(t) = 1, \quad S(t) < 0 \iff A^{KH}(t) < 1,$$

which is equivalent to $\text{sign}(A^{KH}(t) - 1) = \text{sign}(S(t))$. □

The virtue of defining Kaldor-Hicks as a ratio, rather than a difference, is that when real wages are pinned by technologies — i.e., they do not depend on labor supply $\boldsymbol{L}$ or on demand $\boldsymbol{\chi}$ — then $A^{KH}(t) = A(t)$. The assumption that real wages are independent of $\boldsymbol{L}$ and $\boldsymbol{\chi}$, that holds in the one-good economy of Example 2, is very strong. It implies that the real marginal product of labor is constant, which is to say that labor in each location can be converted in consumption goods in that location using a linear technology.

Proposition 14 (Equivalence between $A(t)$ and $A^{KH}(t)$)**.** *Suppose that real wages and relative prices in the decentralized equilibrium are independent of labor supply $\{L_r\}$ and demand $\{\chi_r\}$. Then $A^{KH}(t) = A(t)$.*

Proof. We use the representation

$$A^{KH}(t) = \frac{\sum_r w_r(t)\, L_r^{\text{comp}}\Big(\frac{\boldsymbol{w}(t)}{A^{KH}(t)}, \boldsymbol{p}(t), \boldsymbol{u}^0\Big)}{\sum_r E_r^{\text{comp}}\Big(\frac{\boldsymbol{w}(t)}{A^{KH}(t)}, \boldsymbol{p}(t), \boldsymbol{u}^0\Big)}. \tag{66}$$

This is a special case of Equation (12) in Theorem 1: there, $A(t)$ is characterized as the maximum proportional reduction in aggregate factor-neutral productivity such that winners can compensate losers, with wages and prices evaluated in the compensated equilibrium. Here, $A^{KH}(t)$ is the maximum proportional reduction in wages in all locations such that it is possible for winners to compensate losers (based on prices and wages at t) and there is no money left over (sum of net transfers equals zero).

By assumption, equilibrium real wages and relative prices do not depend on labor supply or demand. Hence the wages and prices in the compensated equilibrium coincide with those in the decentralized equilibrium without transfers. Under this condition, Equation (12) reduces exactly to (66). Since (66) characterizes $A^{KH}(t)$ and Equation (12) characterizes $A(t)$, it follows that $A^{KH}(t) = A(t)$. □