Aggregate Productivity with Heterogeneous Agents

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How to Aggregate with Heterogenous Agents

- Individuals disagree about the ranking of economy-wide allocations.
- When the economy changes, there are winners and losers.
- In heterogeneous-agent economies, there is no single ranking of economy-wide allocations.
- Basic question: how should we define the aggregate value of a change?
- We define it by the **factor endowments left-over if winners compensate losers.**
- Measures amount of factor savings after we hit the same welfare levels, so we call it the change in **aggregate productivity**.

Relation to Prior Work

- This idea builds on the approach by Allais (1978) and Debreu (1951).
- Related to traditional cost-benefit, which measures aggregate value of a change by:

  How much money is left over if winners compensate losers?
- Usual implementation, summing up CVs, does not answer this question correctly.
- Sum of CVs can be positive even when winners **cannot** compensate losers.
- Cost benefit also assumes costless transfers are available, which may not be the case.
- We provide a GE counterpart to cost benefit.

Difference to Social Welfare Functions

- The dominant approach in many literatures is to use a SWF.
- Although our definition is not a SWF function, it can accomodate SWF-like analysis.
- If agents' preferences over economy-wide allocations are all the same, then our measure is the change in that commonly-held SWF in TFP-equivalent terms.
- Hence, our measure automatically accounts for equity concerns of agents in the model without requiring that they all agree.
- No need for analyst to impose externally specified Pareto weights or cardinal utility functions.

Research Agenda

- This paper provides framework and shows how to translate rep agent results like Hulten, misallocation formulas, gains from trade formulas, so that they apply to aggregate productivity in heterogeneous agent economies.

- We show how to characterize aggregate productivity in terms of observables (expenditure shares, price elasticities, etc.)

- Our broader research agenda applies this analysis to different settings:
  - *“Misallocation due to Incomplete Markets”* (in closed & open economies)
  - *“Aggregate Welfare with Discrete Choice Across Places and Jobs”*
  - *“An Approximate General Theory of Second Best”* (optimal policy to max A with distortions)
  - *“Monetary Policy without Redistributive Concerns”* (in HANK environments)

Economic Environment: Arrow-Debreu with Distortive Wedges

- Households $h \in H$ with preferences $\succeq_h$ over consumption vectors $\boldsymbol{c}_h$.
- There are N goods and F factor endowments.
- Producer of good i minimizes costs given general neoclassical production function:

$$y_i = z_i G_i \left(\{y_{ij}\}, Z \{l_{if}\} \right).$$

  where z_i is i's productivity and Z is total factor productivity.
- Distorting implicit or explicit bilateral tax, denoted by $\boldsymbol{\mu}$, so $p_i = \mu_i \times mc_i$. Revenues rebated.
- Standard resource constraints for goods and primary factor endowments.
- Environment is very general and flexible
  (e.g. production networks, incomplete markets, nominal rigidities, international trade, etc.)

Definition of Aggregate Productivity given Lump Sum Transfers

- Index techn. and wedges by t; let $t = 0$ be status quo and $t > 0$ counterfactual of interest.
- So $\boldsymbol{z}(t)$ and $\boldsymbol{\mu}(t)$ are some counterfactual technologies and distortions.
- Let $\mathcal{C}(t, Z)$ be set of allocations supported as equil. given $(\boldsymbol{z}(t), \boldsymbol{\mu}(t), Z)$ and some transfers.
- $A(t)$ is maximum reduction in Z at t needed to make everyone indifferent to initial allocation.

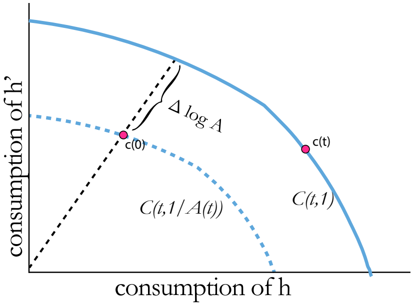
$$A(t) \equiv \max\left\{Z \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \mathcal{C}(t, 1/Z) \text{ and } \boldsymbol{c}_h \succeq_h \boldsymbol{c}_h(0) \text{ for every } h\right\}.$$

- If $A(t) > A(0) = 1$, there exists a potential Pareto improvement.

  e.g. if $A(t) = 1.1$, it is possible to discard 10% of every factor and keep everyone indifferent.

Graphical illustration

- Shock pushes frontier but in equilibrium, one agent is worse off. After transfers, goods left over, so $A > 1$.



- No stance on who gets the left-over resources.

Nesting Other Aggregate Measures

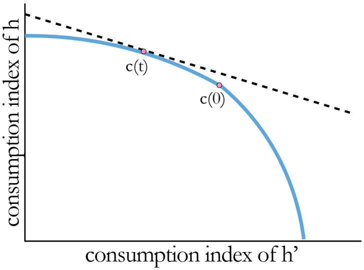
- Start by considering benchmark where $A(t)$ coincides with existing popular measures.
- If everyone has identical, homothetic preferences & faces same prices, then

$$A(t) = \underset{\text{for positive rep agent}}{\text{consumption equivalent}}(t) = \text{real GDP}(t) = \underset{\text{to compensating income}}{\text{income relative}}(t).$$

- But lots of examples violate one of these assumptions (e.g. non-homothetic, incomplete markets, different tastes).
- Outside of these restrictive assumptions, other measures go haywire (violate Pareto criterion)!
- Rest of the talk focuses on characterizing $A(t)$ when these conditions do not hold.

Sum of Compensating Variations with Different Preferences

- Consider transfer from h' to h with different preferences.



$$\frac{\text{income at (t)}}{\text{compensating income at (t)}} > 1.$$

- After winners compensate losers, they have money left over (Boadway 1974). But, unfeasible.
- But $A(t) = 1$ — this economy is efficient.

Sum of Compensating Variations with Different Prices

- Suppose there is unit endowment of consumption good.
- Agent 1 pays higher markup μ_1 than agent 2 who pays μ_2.
- A lump-sum transfer from agent 2 to agent 1 raises one's spending share $\Delta\omega_1$, and

$$\Delta \log \frac{\text{income at (t)}}{\text{compensating income at (t)}} \approx \bar{\mu} \left[\frac{\mu_1 - \mu_2}{\mu_1 \mu_2} \right] \Delta\omega_1,$$

- One unit of 1's consumption is worth $\mu_1/\mu_2 \geq 1$ units of agent 2's consumption.
- But $A(t) = 1$ — this economy is efficient.

Compensated Representative Agent

- In general, aggregate productivity can be computed directly using its definition.
- But in many cases, easier solve for a dual problem.
- *Equilibrium with compensated rep agent* is equilibrium with the same technologies & wedges as the original economy, but with a rep agent that has preferences

$$U(\boldsymbol{c}) = \min\left\{\tilde{u}_1(\boldsymbol{c}_1), \ldots, \tilde{u}_H(\boldsymbol{c}_H)\right\},$$

  where $\tilde{u}_h(\boldsymbol{c}_h)$ is h's individual consumption-equivalents (invariant to monotone transforms).
- Agent h's consumption-equivalent $\tilde{u}_h(\boldsymbol{c}_h)$ solves

$$u_h\left(\frac{\mathbf{c}_h}{\tilde{u}_h}\right) = u_h(\mathbf{c}_h(0))$$

Computing $A(t)$ via Equilibrium with Representative Agent

- **Theorem**

  Given some mild conditions

$$A(t) = U(\boldsymbol{c}^{comp}(t)) \qquad \text{for every } t,$$

- At $t = 0$, prices and quantities in equil. with comp. rep agent the same as decentralized equil:

$$\boldsymbol{p}^{\text{comp}}(0) = \boldsymbol{p}(0), \qquad \boldsymbol{c}^{\text{comp}}(0) = \boldsymbol{c}(0).$$

- So we can use representative agent results to study $A(t)$ in heterogenous agent economies.

  (e.g. Hulten; Baqaee-Farhi; Arkolakis-Costinot-Rodriguez-Clare; Harberger; Hsieh-Klenow, etc.)

Effect of Productivity Shocks in Competitive Economy

Hulten's Theorem (1978)

With perfect competition $\boldsymbol{\mu} = 1$, in response to $\Delta \log z_i$, to a first-order

$$\Delta \log U(\boldsymbol{c}^{comp}) \approx \frac{sales_i}{GDP} \times \Delta \log z_i \equiv \lambda_i \Delta \log z_i$$

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- Under perfect competition, $A(t)$ obeys Hulten with arbitrary heterogenous agents.

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Beyond Hulten's Theorem, Baqaee & Farhi (2019)

With perfect competition, in response to $\Delta \log z_i$, to a second-order

$$\Delta \log A \approx \lambda_i \Delta \log z_i + \frac{1}{2} \Delta \lambda_i^{\text{comp}} \Delta \log z_i.$$

where $\Delta \lambda_i^{\text{comp}}$ is change in sales share with as-if rep agent (paper characterizes $\Delta \lambda^{\text{comp}}$).

Porting Arkolakis et al. (2012) for Gains from Trade

- Example application: Losses from autarky with heterogeneous agents.

- With one agent, then Arkolakis et al. (2012) show losses from autarky are:

$$\Delta \log A = \frac{\log \overbrace{s_h}^{\text{share of spending on domestic}}}{\underbrace{\theta_h - 1}_{\text{trade elasticity}}}.$$

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- With heterogeneous agents, we instead get

$$\Delta \log A \approx \underbrace{\mathbb{E}_\omega \left[\frac{\log s_h}{\theta_h - 1} \right]}_{\text{first order}}$$

where ω is cross-sectional distribution of spending.

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$$\Delta \log A \approx \underbrace{\mathbb{E}_\omega \left[\frac{\log s_h}{\theta_h - 1} \right]}_{\text{first order}} \underbrace{- \frac{1}{2} Var_\omega \left[\frac{\log s_h}{\theta_h - 1} \right]}_{\text{second order}}.$$

  where ω is cross-sectional distribution of spending.

- Loss greater with heterog. import shares. More exposed agents need more compensation.

Porting Misallocation Results

- Use theorem to characterize losses from distortions.

Harberger's Triangles (Harberger, 1971; Baqaee & Farhi, 2020)
Suppose we eliminate all wedges, to a second-order:

$$\Delta \log A = \Delta \log U(\boldsymbol{c}^{comp}) \approx -\sum_i \frac{sales_i}{GDP} \times \underbrace{\left(\frac{1}{2} \log \mu_i \Delta \log q_i^{comp} \right)}_{\substack{\text{area of} \\ \text{deadweight loss triangle}}},$$

where Δq_i^{comp} is first-order change in quantity of i in equilibrium with compensated rep agent.

- In paper we characterize Δq_i^{comp} in terms of expenditure shares and price elasticities.

Example: Misallocation with Heterogeneous Agents

- Suppose

$$u_h(\boldsymbol{c}_h) = \left[\sum_i c_{hi}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

- Household h pays markup μ_{hi} on good i. Goods made from labor endowment.

- If there is a single agent, as in Hsieh-Klenow (2009), then:

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- If there is a single agent, as in Hsieh-Klenow (2009), then:
$$\Delta \log A \approx \frac{\theta}{2} Var\left[\log \boldsymbol{\mu}\right].$$
- With heterogeneous agents, misallocation depends on average **conditional** variance:
$$\Delta \log A \approx \frac{\theta}{2} \mathbb{E}_\omega[Var\left[\log \boldsymbol{\mu}_h | h\right]].$$
- No comparison of average markups across individuals — those are purely redistributive.
- Companion paper uses this idea to measure *misallocation due to incomplete markets*.

Aggregate Productivity with Costly Redistribution

$A(t)$ is how much we can reduce TFP to make everyone indifferent given lump-sum transfers.

- But we can handle cases without lump-sum too.

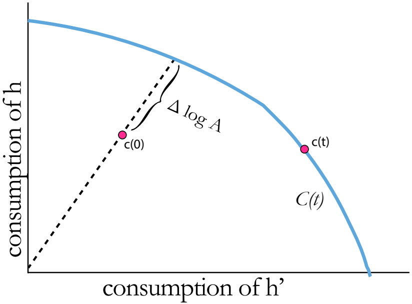
$A^{\text{costly}}(t)$ is how much we can reduce TFP to make everyone indifferent given costly redistribution.

- Consider distorting taxes be $\boldsymbol{\tau}$ and restricted lump-sum transfers $\boldsymbol{T}$ in some feasible set $\mathcal{T}$.
- Let $\mathcal{C}^{\text{costly}}(t, Z)$ be set of allocations given $(\boldsymbol{z}(t), \boldsymbol{\mu}(t), Z)$, and some transfers & taxes in $\mathcal{T}$.
- $A^{\text{costly}}(t)$ uses $\mathcal{C}^{\text{costly}}(t, Z)$ in place of $\mathcal{C}(t, Z)$.

Aggregate Productivity with Costly Redistribution

- Graphical illustration:

  shock pushes out frontier but in equilibrium, one agent worse off.
  after transfers, goods left over, so $\Delta \log A > 0$.



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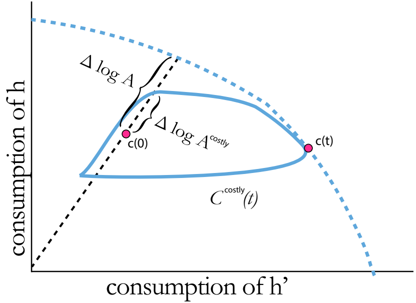
- Graphical illustration:

  if only distortionary labor tax available, compensating loser causes misallocation.



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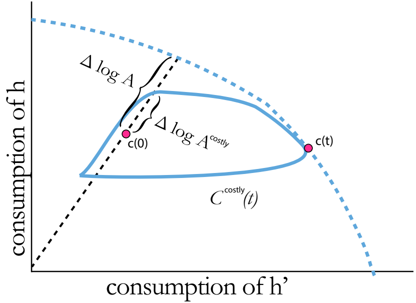

- In the paper, we show

$$\Delta \log A^{\text{costly}} \approx \Delta \log A - \text{deadweight loss triangles from redistribution.}$$

ACR Example Revisited with Distortionary Taxes

- Consider again the gains from trade relative to autarky with heterogeneous import shares.
- Now agents have preferences over consumption bundle (of domestic & foreign) and leisure.
- Substitution elast. between cons-leisure is ρ, all work same number of hours in status-quo.
- Suppose compensations must be financed by linear labor income tax.
- The losses from autarky are:

$$\Delta \log A^{\text{linear tax}} \approx \Delta \log A^{\text{lump-sum}} - \underbrace{\frac{1}{2}\rho \Omega_c (1 - \Omega_c)(d \log \tau^*)^2}_{\text{2nd order losses from distorting taxes}},$$

with two agents, this is $d \log \tau^* = \frac{\omega_h}{\theta - 1} [\log s_{h'd} - \log s_{hd}] > 0$.

- Losses from limited redist. growing in ρ and heterogeneous exposure and falling in θ.

Quantitative Example: Impact of Rise of China on the US

- Off-the-shelf quant. trade model, (Baqaee-Farhi, 2024) — 10 regions & 30 industries, IO.
- Low-, medium-, high-skill labor & capital in each industry. Each factor-industry is an agent.
- China grows from 5% of world GDP to 20% (2008 to 2023). What is effect on the US?
- Compensations among US agents from lump-sum transfers or from import tariffs.

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	National labor market	Industry-level labor market
Available Instruments	$\Delta \log A$	$\Delta \log A$
Lump-sum transfers	0.010	0.008
Tariffs with targeted rebates		
Tariffs with untargeted rebates		

- Change in aggregate productivity depends on:
  1. How heterogeneous is exposure of workers to the shock (in this case, flexibility of labor market).
  2. How distortionary is redistributive tool.

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Summary

- Household heterogeneity central to much of modern macro.
- Yet, we still want meaningful aggregate measure that summarizes welfare-relevant outcomes and guides policy.
- Cost-benefit in general equilibrium with policy limits on transfers.
- Counterfactual question about observables, no Pareto weights or cardinal utility.
- Characterized in terms of supply and demand curves.
- Lots of open questions to explore with this type of measure.

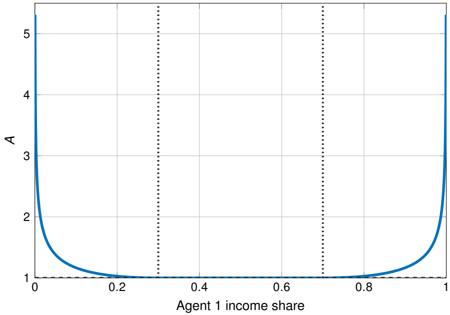
Appendix Slides

Altruism and Misallocation

- Consider two agents with altruistic preferences:

$$u_i(c_i, c_{-i}) = (1-\beta)\log c_i + \beta \log c_{-i}.$$

- Resource constraint: $c_1 + c_2 = 1$. Status quo: $c_1 = z_1$, $c_2 = 1 - z_1$.
- Misallocation, $A > 1$, only if inequality is large enough relative to altruism: $z_1 \notin [\beta, 1-\beta]$.



- If $\beta = 1/2$, then A is misallocation according to a utilitarian SWF.

Formal Economic Environment

- Households $h \in \{1, ..., H\}$ with preferences $\succeq_h$ over consumption vectors $\boldsymbol{c}_h \in \mathbb{R}^N$:

$$\max u_h(\boldsymbol{c}_h) \text{ such that } \sum_i p_i c_{hi} \leq \sum_f \omega_{hf} w_f L_f + T_h.$$

- Producer of good i minimizes costs given general neoclassical production function:

$$\min \sum_j p_j y_{ij} + \sum_f w_f l_{if}, \text{ such that } y_i = z_i G_i \left(\{y_{ij}\}, Z \{l_{if}\} \right).$$

  Z is total factor-augmenting productivity.

- Wedges μ_i on i, explicit (e.g. tax/markup) or implicit (e.g. borrowing constraint).

$$p_i = \mu_i mc_i.$$

Formal Economic Environment

- Resource constraint for goods and factors:

$$\sum_j y_{ji} + \sum_h c_{hi} \leq y_i, \qquad \text{and} \qquad \sum_i l_{if} \leq z_f L_f.$$

- Aggregate budget balance.

$$\sum_h T_h = \sum_i p_i y_i \left(1 - \mu_i^{-1}\right).$$

- Decentralized equilibrium with wedges:

Utility maximization, cost minimization, $p_i = \mu_i mc_i$, budget balance, & resource constraints.

Formal Definitions of Other Aggregate Measures

- Chain-weighted real GDP:

$$\log \text{Real GDP}(t) = \int_0^t \sum_i \frac{p_i(s)c_i(s)}{\sum_{i'} p_{i'}(s)c_{i'}(s)} \frac{d \log c_i(s)}{ds} ds.$$

- Sum of compensating variations (Kaldor-Hicks or cost-benefit analysis)

$$\frac{\text{income at (t)}}{\text{compensating income at (t)}} = \frac{\sum_h e_h(\boldsymbol{p}(t), u_h(t))}{\sum_h e_h(\boldsymbol{p}(t), u_h(0))}.$$

- Consumption-equivalent variation for positive representative agent (if it exists)

$$u^{RA}\left(\boldsymbol{c}^{RA}(t)/A^{RA}(t)\right) = u^{RA}\left(\boldsymbol{c}^{RA}(0)\right).$$