Efficiency Costs of Incomplete Markets

David Baqaee   Ariel Burstein

UCLA

Quantifying Misallocation due to Incomplete Markets

- Financial market incompleteness is inescapable feature of data and modern theory:
  - In domestic economies, failure of permanent-income hypothesis.
  - In international, violations of perfect risk-sharing (Backus-Smith).
- Inability to share resources across states and time is a form of misallocation.
- How much of society's resources are wasted due to this type of misallocation?

  If financial markets were completed and everyone was compensated, how much resources would be left over?

- Provide answers for closed & open economies with incomplete financial markets.
  - Exact formulas & approximations using sufficient statistics w/o fully-specified model.
- Quantify misallocation from incomplete domestic (US) & international markets.

Selection of Related Papers

- Measuring Aggregate Efficiency with Heterogeneous Agents:

  Coeff. of resource utilization, (Debreu, 1951); Baqaee and Burstein, (2025).

- Efficiency Losses from Incomplete Markets without SWF:

  Benabou (2002); Floden (2001); Farhi & Werning (2012); Aguiar, Amador, Arellano (2024); Fitzgerald (2025).

- Aggregate Welfare in Incomplete Market Models:

  Imrohoroglu (1989); Heathcote, Storesletten, Violante (2008); Conesa, Kitao, & Krueger (2009); Davila, Hong, Krusell, & R. Rull (2012); Karahan and Ozcan (2013); Aguiar, Itskhoki & Mukhin (2024); Constantinides (2025).

- Decompositions of Social Welfare Functions:

  Bhandari, Evans, Golosov & Sargent (2021); Davila & Schaab (2023, 2024).

Agenda

Definition & Characterization of Misallocation

Closed Economy with Idiosyncratic Risk
- Theory
- Empirical Application

Open Economy with Country-Level Risk
- Theory
- Empirical Application

Conclusion

Agenda

Aggregate Efficiency

- Consider collection of H agents with preferences $\succeq_h$ over consumption vectors $\boldsymbol{c}_h \in \mathbb{R}^N$.
- Let $\boldsymbol{c} = [\boldsymbol{c}_1, \cdots, \boldsymbol{c}_H] \in \mathbb{R}^{H \times N}$ denote a consumption allocation matrix.
- Let $\boldsymbol{c}^0$ be status-quo, and $\mathcal{C}$ be set of feasible consumption allocations.

  We typically set $\mathcal{C}$ to be dynamic Pareto-efficient allocations under complete markets.
- By how much can we contract $\mathcal{C}$ while keeping everyone indifferent to status-quo?

Aggregate Efficiency

- Consider collection of H agents with preferences $\succeq_h$ over consumption vectors $\boldsymbol{c}_h \in \mathbb{R}^N$.
- Let $\boldsymbol{c} = [\boldsymbol{c}_1, \cdots, \boldsymbol{c}_H] \in \mathbb{R}^{H \times N}$ denote a consumption allocation matrix.
- Let $\boldsymbol{c}^0$ be status-quo, and $\mathcal{C}$ be set of feasible consumption allocations.

  We typically set $\mathcal{C}$ to be dynamic Pareto-efficient allocations under complete markets.
- By how much can we contract $\mathcal{C}$ while keeping everyone indifferent to status-quo?

Misallocation

$$A(\boldsymbol{c}^0, \mathcal{C}) \equiv \max\left\{\phi \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \phi^{-1}\mathcal{C} \text{ and } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h\right\}.$$

- e.g. if $A = 1.1$, it is possible to compensate everyone and have $\approx 10\%$ of every good left over.
- no stance on how surplus should be distributed.

Aggregate Efficiency

- Consider collection of H agents with preferences $\succeq_h$ over consumption vectors $\boldsymbol{c}_h \in \mathbb{R}^N$.
- Let $\boldsymbol{c} = [\boldsymbol{c}_1, \cdots, \boldsymbol{c}_H] \in \mathbb{R}^{H \times N}$ denote a consumption allocation matrix.
- Let $\boldsymbol{c}^0$ be status-quo, and $\mathcal{C}$ be set of feasible consumption allocations.

  We typically set $\mathcal{C}$ to be dynamic Pareto-efficient allocations under complete markets.
- By how much can we contract $\mathcal{C}$ while keeping everyone indifferent to status-quo?

Misallocation

$$A(\boldsymbol{c}^0, \mathcal{C}) \equiv \max\left\{\phi \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \phi^{-1}\mathcal{C} \text{ and } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h\right\}.$$

- e.g. if $A = 1.1$, it is possible to compensate everyone and have $\approx 10\%$ of every good left over.
- answer depends on ordinal preference relation; monotone transformations of utility irrelevant.

How to Solve for A

- Use main theorem in Baqaee and Burstein (2025) to attack problem.

- Let $\tilde{u}_h(\mathbf{c}_h)$ be h's individual consumption equivalent:

$$u_h\left(\mathbf{c}_h/\tilde{u}_h\right) = u_h(\mathbf{c}_h^0).$$

  $\tilde{u}_h(\mathbf{c}_h)$ is H.O.D. 1 function of $\mathbf{c}_h$.

Proposition

Misallocation is

$$A(\boldsymbol{c}^0, \mathcal{C}) = \max_{\boldsymbol{c}\in\mathcal{C}} \left[\min\left\{\tilde{u}_1(\boldsymbol{c}_1), \cdots, \tilde{u}_H(\boldsymbol{c}_H)\right\}\right].$$

Problem of calculating misallocation converted into one of maximizing utility for a fictional agent.

Call maximizer of problem $\boldsymbol{c}^{\text{comp}}$. Not interesting per se, useful device to calculate A.

Agenda

Baseline Closed Economy

- Specialize preferences to CRRA over state-contingent consumption streams:

$$u(\boldsymbol{c}_h) = \frac{1}{1-1/\eta} \sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}(s)^{1-\frac{1}{\eta}}.$$

- Production in each t and s is statically efficient. No capital or leisure for now.

- Production efficient: removing distortions won't alter aggregate output, so take $y_t(s)$ as given:

  General neoclassical production technologies with any pattern of technological shocks.

- Consumption possibility set is dynamic feasible set:

$$\mathcal{C} = \left\{ \boldsymbol{c} : \sum_h c_{ht}(s) \leq y_t(s), \text{ for every } t \text{ and } s \right\}.$$

- Let status-quo $\boldsymbol{c}^0$ be equilibrium under incomplete markets.

- $\log A$ is output leftover, in every date & state, if markets completed & everyone compensated.

Exact Characterization

- Given these preferences, we have

$$\tilde{u}_h(\boldsymbol{c}_h) = \frac{\mathrm{CE}(\boldsymbol{c}_h)}{\mathrm{CE}(\boldsymbol{c}_h^0)},$$

  where $\mathrm{CE}(\boldsymbol{c}_h)$ is certainty-equivalent that solves $u(\mathbf{c}_h) = u(\mathbf{1}\, CE)$.

- Using theorem, misallocation is

$$A = \frac{\mathrm{CE}(\sum_h \boldsymbol{c}_h^0)}{\sum_h \mathrm{CE}(\boldsymbol{c}_h^0)},$$

  Similar to Benabou (2002), but as a result rather than a definition.

- "Misallocation" according to utilitarian social welfare function is

$$A^U = \frac{\mathrm{CE}(\sum_h \boldsymbol{c}_h^0)}{\left(\sum_h \left(\mathrm{CE}(\boldsymbol{c}_h^0)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}.$$

- Permanent deterministic inequality implies $A^U < 1$ but is Pareto efficient ($A = 1$).

Exact Characterization: Sketch of derivation

- Allocations on Pareto frontier satisfy $c_{ht}(s) = \alpha_h y_t(s)$ with $\sum_h \alpha_h = 1$, so

$$A = \max_{\boldsymbol{c} \in \mathbf{C}} \min_h \{\tilde{u}_h(\boldsymbol{c}_h)\} = \max_{\substack{\boldsymbol{\alpha} \in \mathbb{R}^H \\ \alpha_h \geq 0, \sum_h \alpha_h = 1}} \min_h \{\alpha_h \tilde{u}_h(\mathbf{y})\} .$$

- Solve this utility-maximizing problem and use

$$\tilde{u}_h(\boldsymbol{c}_h) = \frac{\mathrm{CE}(\boldsymbol{c}_h)}{\mathrm{CE}(\boldsymbol{c}_h^0)}.$$

  yields

$$\alpha_h = \frac{\mathrm{CE}(\boldsymbol{c}_h^0)}{\sum_{h'} \mathrm{CE}(\boldsymbol{c}_{h'}^0)}.$$

- Substituting this into $A = \alpha_h \tilde{u}_h(\boldsymbol{y})$ and using $\sum_h \boldsymbol{c}_h^0 = \boldsymbol{y}$, yields the expression.

Approximate Characterization

- Use second-order approximation to build sufficient statistics.

- Replicate status-quo allocation using Arrow-Debreu with consumption taxes, $\mu_{ht}(s)$.

- Let $\omega_{ht}(s)$ be h's share of consumption in date t and state s, taxes must be

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \omega_{ht}(s) - \log \omega_{h0}\right].$$

Approximate Characterization

- Use second-order approximation to build sufficient statistics.
- Replicate status-quo allocation using Arrow-Debreu with consumption taxes, $\mu_{ht}(s)$.
- Let $\omega_{ht}(s)$ be h's share of consumption in date t and state s, taxes must be

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \omega_{ht}(s) - \log \omega_{h0}\right].$$

- **Harberger's Triangles (Harberger, 1971; Baqaee & Farhi, 2020)**
  Suppose we eliminate all wedges, to a second-order:

$$\log A \approx -\mathbb{E}_0 \sum_{t,h} \underbrace{\frac{r}{(1+r)^{t+1}}\omega_h}_{\text{NPV of expenditures}} \times \underbrace{\left(\frac{1}{2}\log \mu_{ht}(s)\left(\log c_{ht}^{comp}(s) - \log c_{ht}^{0}(s)\right)\right)}_{\substack{\text{area of}\\ \text{deadweight loss triangle}}},$$

where $\log c_h^{comp}$ is h's compensated consumption.

Approximate Characterization

- Special case with one period, $c_h(s) = \bar{c}_h + \varepsilon_h(s)$. Plug terms into Harberger triangle formula:

$$\Delta \log A \approx \frac{1}{2}\frac{1}{\eta}\mathbb{E}_{\omega}\left[Var\left[\log c_h(s)|h\right]\right].$$

  Misallocation depends on average volatility of household consumption (not inequality).

Approximate Characterization

- Special case with one period, $c_h(s) = \bar{c}_h + \varepsilon_h(s)$. Plug terms into Harberger triangle formula:

$$\Delta \log A \approx \frac{1}{2}\frac{1}{\eta}\mathbb{E}_\omega\left[Var\left[\log c_h(s)|h\right]\right].$$

  Misallocation depends on average volatility of household consumption (not inequality).

- Now consider multi-period problem, like Bewley (1972).

- Plug in terms into Harberger triangle formula, misallocation is

$$\Delta \log A \approx \mathbb{E}_0 \sum_{t,h} \underbrace{\frac{r}{(1+r)^{t+1}}\omega_h}_{\text{NPV of expenditures}} \frac{1}{2}\underbrace{\frac{1}{\eta}\left(\log \omega_{ht}(s) - \log \omega_{h0}\right)}_{\log \mu} \underbrace{\left(\log \omega_{ht}(s) - \log \bar{\omega}_h\right)}_{\Delta \log c^{comp}},$$

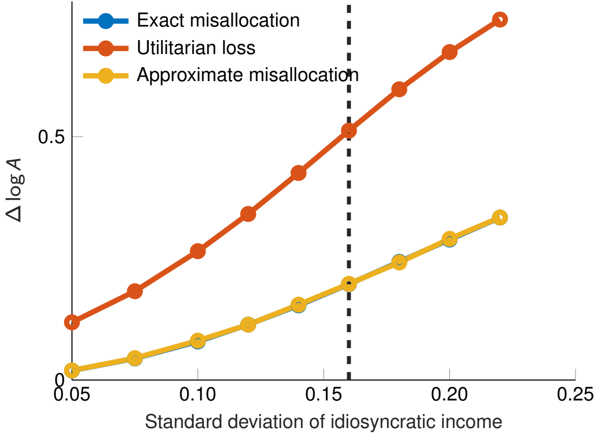
  where $\bar{\omega}_h$ is the NPV of h's consumption share.

- Advantage of formula: no fully-specified model, just a moment of expenditures in status-quo.

- Can apply directly to consumption panel w/out info on portfolio problems or income process.

Example: Off-the-Shelf Calibration of Bewley

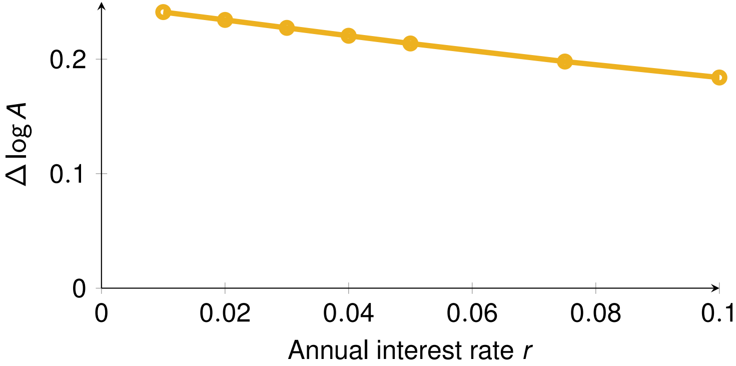
- EIS $= 0.5$, borrowing limit $5\times$ quarterly income, bonds are 140% of annual GDP.



- Baseline: roughly 20% of output left after everyone is kept indifferent.
- Gains more than double under utilitarian SWF, which combines efficiency and "equity."

Sufficient Statistics Applied to PSID, 1999-2020

- Apply triangles formula directly to consumption panel data in the US.
- Estimate $\log \bar{\omega}_h$ by regressing future consumption on household characteristics in 1999.
- Only remaining parameters are EIS, $\eta = 0.5$, and risk-free rate, r.



- Gains from eliminating idiosyncratic volatility: $\approx$ **20**% of output in every period and state.

Dropping covariates when constructing household NPV of expenditures

Eliminated variable	**Estimated misallocation**
None (Baseline)	0.214
Spouse labor income	0.223
Household head labor income	0.223
Business assets (household & spouse)	0.231
Household head college degree	0.239
Household head race and ethnicity	0.244
Household head age	0.249
Renter status	0.252
Household size	0.258
State of residence	0.269
Wealth	0.266

- As we drop covariates, we incorrectly attribute cross-sectional differences to inefficiency.

Extension with Labor-Leisure Choice

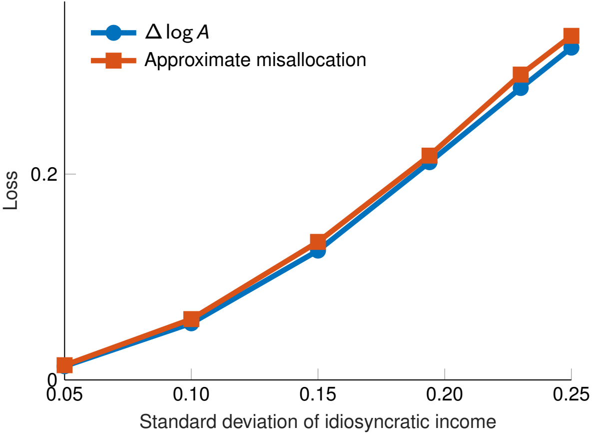
- Household preferences: $u(\boldsymbol{c}_h, \boldsymbol{l}_h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v(c_{ht}(s), l_{ht}(s))$.
- Resource constraint: $\sum_h c_{ht}(s) = \sum_h z_{ht}(s)\,(1 - l_{ht}(s))$.
- Distortionary labor tax finances debt, so $\mathcal{C}$ is no longer Pareto-efficient frontier.
- A measures how much of every good, including leisure, is wasted due to incomplete markets.

Extension with Labor-Leisure Choice

- Household preferences: $u(\boldsymbol{c}_h, \boldsymbol{l}_h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v(c_{ht}(s), l_{ht}(s))$.
- Resource constraint: $\sum_h c_{ht}(s) = \sum_h z_{ht}(s)(1 - l_{ht}(s))$.
- Distortionary labor tax finances debt, so $\mathcal{C}$ is no longer Pareto-efficient frontier.
- A measures how much of every good, including leisure, is wasted due to incomplete markets.
- Benabou (2002) ranks allocations by sum of CE for some fixed leisure.
- Our measure is different because agents optimally choose leisure.
- Our measure assigns $A = 1$ to every point on the Pareto efficient frontier.
- Benabou measure assigns different values to points on Pareto frontier, including strictly preferring more inequality.

Example: Bewley with Labor-Leisure Choice

- Frisch $= 0.5$, calibrate shocks to target baseline cross-sectional variance of consumption.



- Misallocation very similar to the baseline model.
- Original approximation formula works well (even though ignores leisure).

Extension with Capital Accumulation

- Precautionary motive distorts aggregate capital stock.

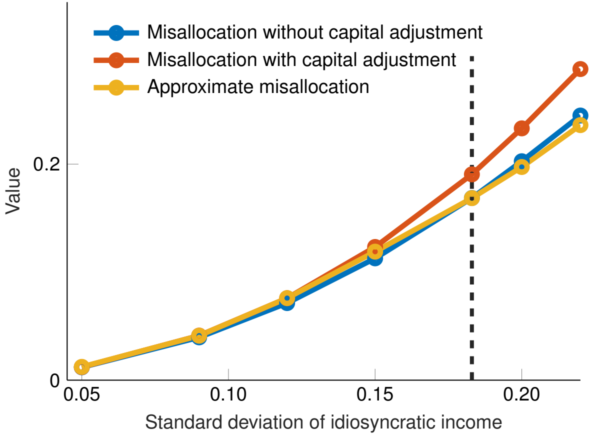
$$A = \frac{\mathrm{CE}(\{c_t^*(s)\}_{t,s})}{\sum_h \mathrm{CE}(\boldsymbol{c}_h^0)},$$

  where $c_t^*(s)$ is agg. consumption in a neoclassical growth model with initial capital stock k_0.

- That is, $c_t^*(s)$ satisfies Euler equation & resource constraints (with usual transversality).

Example: Aiyagari with Capital Accumulation

- Precautionary motive distorts aggregate capital stock.



- Misallocation slightly larger allowing the capital stock to adjust.
- Original approximation formula works well holding capital stock fixed.

Agenda

Definition & Characterization of Misallocation

Closed Economy with Idiosyncratic Risk
- Theory
- Empirical Application

Open Economy with Country-Level Risk
- Theory
- Empirical Application

Conclusion

Gains from Completing International Financial Markets

- Similar question can be asked in open economy, where consumption baskets differ.
- Failure of perfect international risk sharing is well-documented and stark.
- What are welfare implications? How much losses because countries don't insure each other?
- Two challenges for literature:
  - How to quantify "how much are we losing?"
  - How to identify productivity shocks/financial frictions.

Gains from Completing International Financial Markets

- Similar question can be asked in open economy, where consumption baskets differ.
- Failure of perfect international risk sharing is well-documented and stark.
- What are welfare implications? How much losses because countries don't insure each other?
- Two challenges for literature:
  - How to quantify "how much are we losing?" We use A.
  - How to identify productivity shocks/financial frictions. We use sufficient statistics.

Gains from Completing International Financial Markets

- Now allow preferences to differ across h due to e.g. non-tradeables:

$$u_h(\boldsymbol{c}_h) = \frac{1}{1 - 1/\eta} \sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}(s)^{1-\frac{1}{\eta}}.$$

- Goods produced from intermediates and factors (nests Armington model):

$$y_{it}(s) = z_{it}(s) \left(\sum_{j \in N} \alpha_{ij} \left(y_{ijt}(s) \right)^{\frac{\theta_i - 1}{\theta_i}} + \sum_{f \in F} \alpha_{if} \left(l_{ift}(s) \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}}.$$

- Resource constraints for final goods, intermediates, and factors:

$$c_{ht}(s) = y_{ht}(s), \quad \sum_{j \in N} y_{jit}(s) = y_{it}(s), \quad \sum_{j \in N} l_{jft}(s) = z_{ft}(s).$$

Misallocation

- $\mathcal{C}$ dynamic feasible consumption set given process $\boldsymbol{z}$ for productivities & factor endowments.
- $\boldsymbol{c}^0$ status-quo consumption allocation with incomplete markets (i.e. data generating process).
- Measure misallocation as before:

  A is the maximum contraction in $\mathcal{C}$ such that it is possible to keep everyone indifferent.
- Equivalently, A is fraction of factor endowments that is wasted.
- We hold $\boldsymbol{z}$ fixed when we measure A — abstract from changes in innovation, capital accumulation & labor supply when markets completed.
- That is, we focus only on waste from improperly sharing fixed resources.

Approximate Characterization

- Decentralize status-quo by consumption-wedges in Arrow-Debreu:

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \frac{\omega_{ht}(s)/\omega_{h0}}{\omega_{\bar{h}t}(s)/\omega_{\bar{h}0}}\right] + \frac{1-\eta}{\eta}\left[\log \frac{p_{ht}(s)/p_{h0}}{p_{\bar{h}t}(s)/p_{\bar{h}0}}\right]$$

- Wedges capture deviations from Backus-Smith (1993). If status-quo dynamically efficient,

$$\log \frac{c_{ht}(s)/c_{h0}}{c_{\bar{h}t}(s)/c_{\bar{h}0}} = -\eta \log \frac{p_{ht}(s)/p_{h0}}{p_{\bar{h}t}(s)/p_{\bar{h}0}},$$

  consumption growth perfectly negatively correlated with real exchange rate.

Approximate Characterization

- Decentralize status-quo by consumption-wedges in Arrow-Debreu:

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \frac{\omega_{ht}(s)/\omega_{h0}}{\omega_{\bar{h}t}(s)/\omega_{\bar{h}0}}\right] + \frac{1-\eta}{\eta}\left[\log \frac{p_{ht}(s)/p_{h0}}{p_{\bar{h}t}(s)/p_{\bar{h}0}}\right]$$

- Wedges capture deviations from Backus-Smith (1993). If status-quo dynamically efficient,

$$\log \frac{c_{ht}(s)/c_{h0}}{c_{\bar{h}t}(s)/c_{\bar{h}0}} = -\eta \log \frac{p_{ht}(s)/p_{h0}}{p_{\bar{h}t}(s)/p_{\bar{h}0}},$$

consumption growth perfectly negatively correlated with real exchange rate.

- Misallocation is approximately

$$\log A \approx \frac{1}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty} \frac{r}{(1+r)^{t+1}} \sum_{h\in H} \omega_h \log \mu_{ht}(s) \sum_{h'\in H} \mathcal{M}_{hh'}[\log \mu_{h't}(s) - \log \bar{\mu}_h]\right],$$

where $\mathcal{M}_{hh'}$ depends only on the static input-output matrix and elasticities of substitution.

Approximate Characterization: Simple Example

- Symmetric two countries, each country produces one good using labor.
- Misallocation is approximately

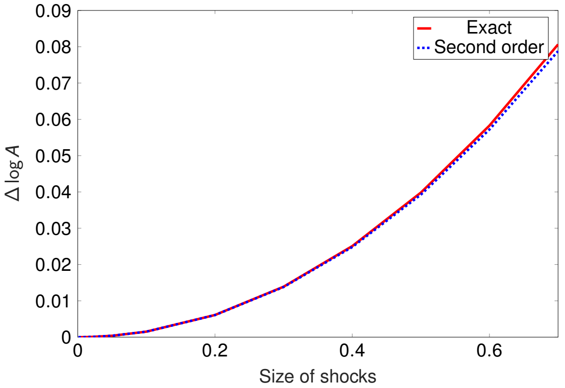
$$\log A \approx \frac{1}{2}\left[\frac{\alpha(1-\alpha)}{\left(\frac{1}{\eta}-\frac{1}{\theta}\right)4\alpha\left(1-\alpha\right)+\frac{1}{\theta}}\right]\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\mathbb{E}_0\left[\log\mu_{1t}(s)\left(\log\mu_{1t}(s)-\log\bar{\mu}_1\right)\right],$$

where α is import share and θ Armington elasticity.

- Misalloc. high if $\alpha \approx 0.5$ or θ high — more missed opportunities to share risk.
- Misalloc. high if EIS η is low — consumption fluctuations more costly.
- $\mathcal{M}$ in previous slide generalizes this example.

Approximate Characterization: Numerical Example

- Armington trade with 15 countries, rep agent within each country, $\theta = 3$, and $\eta = 0.5$.
- Randomize country sizes, I-O matrices, productivity shocks, and Backus-Smith wedges.



- Second-order does not need information on productivity process.

Gains from Completing International Financial Markets

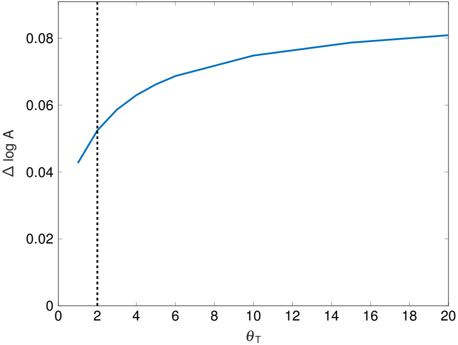
- To apply triangle formula, abstract from within-country heterogeneity.
- Need static IO table, elastic. of subs., and Backus-Smith wedges.
- Use world input-output database with 32 countries and 54 industries from $1970-2019$.
- Use Global Macro Database (Muller et al., 2023) for Backus-Smith wedges.
- Cobb-Douglass aggregator across industries, Armington elasticity $=3$, $EIS=0.5$.
- No information needed on portfolio problems, productivity shocks, ownership of assets.
- No stance on whether consumption fluctuations are due productivity changes or wedges.

Gains From Completing International Financial Markets

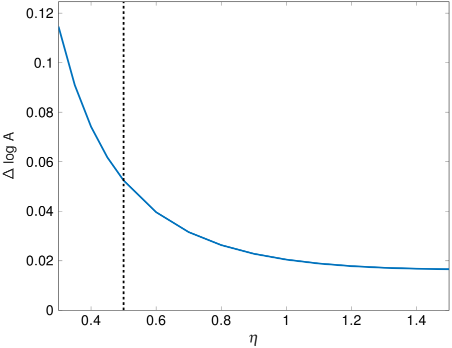
- Benchmark gains: roughly **5**%.
- Driven by differential growth rates between countries (e.g. China vs. Germany).
- Dropping China and India, gains much smaller $\sim$ **1**%.
- Despite stark violations of risk-sharing, small gains from international business-cycle insurance.

Gains From Completing International Financial Markets

- Vary trade elasticity and EIS, holding data fixed.



(a) Armington elasticity



(b) EIS

- Estimated losses increasing in Armington elasticity: more missed opportunities.
- Estimated losses decreasing in EIS: fluctuations more costly.

Agenda

Definition & Characterization of Misallocation

Closed Economy with Idiosyncratic Risk

- Theory
- Empirical Application

Open Economy with Country-Level Risk

- Theory
- Empirical Application

Conclusion

Conclusion

- Quantify misallocation due to inability to share resources across states and time.
- Within countries, these losses are substantial, on the order of 20%.
- Across countries, more limited, especially among similar countries.
- Future research: misallocation rel. to constrained efficient with imperfect instruments.
- Lots of open questions to explore with this type of measures.