Long-Run Comparative Statics

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September 29, 2025

Abstract

What are the long-run effects of permanent changes to productivities, taxes, and other economic parameters, accounting for changes in the capital stock? We show that balanced growth paths of dynamic open economies can be represented as equilibria of equivalent static economies, in which capital services are intermediate inputs subject to wedges that capture deviations from the Golden Rule of savings. Hence, tools developed for distorted static economies can be used to characterize long-run comparative statics. The long-run impact of any shock on consumption has two components. First, a mechanical technological impact captured by a cost-based Domar weight, which we show far exceeds the sales share for investment industries and their suppliers. Second, there is a resource reallocation effect, which increases consumption if it raises capital intensity, with the magnitude determined by how much the reallocation lowers the aggregate labor share. These reallocations can be quantitatively significant: tariffs, for example, generate long-run consumption losses that far exceed their static GDP impact, by reallocating resources away from capital formation. The magnitude of losses from tariffs depends primarily on the economy's internal elasticities – such as the elasticity of substitution between capital and labor and the elasticity of household asset demand to interest rates – rather than on the trade elasticities that are central in static models.

*We thank Yutong Zhong for outstanding research assistance. We thank Anmol Bhandari, Timo Boppart, Ariel Burstein, Mike Elsby, Axel Gottfries, Per Krusell, Oleg Itskhoki, Ellen McGrattan, Ezra Oberfield, Todd Schoellman, Kjetil Storesletten, Ludwig Straub, Gustavo Ventura, and seminar participants for their comments. We are especially grateful to Xiang Ding for sharing his data. This paper subsumes Baqaee and Malmberg (2025).

1 Introduction

This paper studies the long-run macroeconomic effects of permanent shocks. How, for instance, does the level of aggregate consumption respond to a permanent change in productivity, taxes, or tariffs, once capital and other accumulated factors have fully adjusted? Questions of this nature are central to empirical macroeconomics. In cross-sectional work, researchers investigate how permanent differences in policies or technologies explain persistent differences in outcomes between countries or regions. In time-series analysis, they ask how changes in technologies or economic regimes shift the long-run level of consumption or GDP.[1]

While these long-run outcomes may not be direct measures of aggregate intertemporal welfare (accounting for transitions), they remain a primary focus of empirical work for a practical reason: they are far easier to measure. Calculating changes in dynamic welfare requires taking a stand on things like discount factors and social welfare weights, whereas long-run shifts in consumption or GDP can be readily quantified using standard national accounts data.[2]

Despite their empirical importance, a general characterization of long-run outcomes in dynamic general equilibrium models is lacking. This contrasts sharply to the analysis of static networked economies, where a rich literature provides powerful characterization results that map microeconomic parameters to aggregate outcomes in general settings (Harberger, 1964; Hulten, 1978; Baqaee and Farhi, 2019; Liu, 2019; Bigio and La'O, 2020; Baqaee and Farhi, 2020). This paper develops a dynamic analogue to these results.

We provide a general characterization for the long-run effects of permanent shocks, with formulas stated in terms of observable primitives like expenditure shares and microeconomic elasticities, without strong functional form assumptions. Our framework allows for multiple countries, each with potentially overlapping generations of households

[1] For example, the development accounting literature seeks to explain cross-country differences in outcomes – such as hours worked and GDP per capita – by appealing to differences in taxes, endowments, and technologies (e.g., Prescott, 2004; Hsieh and Klenow, 2010). In the time-series dimension, quasi-experimental methods like difference-in-differences and synthetic controls are used to estimate the long-run impact of permanent shocks, such as joining the EU (Grassi, 2017) or Brexit (Born et al., 2019), on consumption, investment, and GDP.

[2] Measures such as aggregate consumption and GDP can be defined in terms of static prices and quantities, and are routinely constructed by statistical agencies. In contrast, intertemporal welfare is notoriously difficult to measure empirically. Its calculation requires researchers to make assumptions about numerous unobservables, including the expected path of future prices and quantities, social welfare weights for different households or generations, and the specific rates used to discount the future. Recent work continues to explore these complexities from both a macro perspective (e.g., Basu et al., 2022, assuming a representative agent) and a micro perspective that allows for household heterogeneity (Del Canto et al., 2023; Fagereng et al., 2022; Baqaee et al., 2024)

facing uninsurable idiosyncratic risk. The model's production side is also general, accommodating flexible neoclassical production functions, multiple types of capital goods, arbitrary input-output networks, and any pattern of tax-like distortions.

The starting point for our analysis is a result establishing an equivalence between dynamic and static economies. We show that the prices and quantities along a balanced growth path (BGP) can be represented as the equilibrium of an as-if static economy with wedges. In the equivalent static economy, the flow of capital services is treated as an intermediate input produced from investment goods and sold at a markup. For each capital good i, this as-if markup wedge μ_i is given by the ratio of capital income to investment expenditures. Intuitively, since capital income is the revenue generated by the capital stock, and investment is the cost of maintaining the stock, the ratio between them is isomorphic to a markup, and the size of the wedge μ_i quantifies the deviation from the Golden Rule of savings.[3]

The equivalence between dynamic and static economies means that the long-run effect of a permanent shock to any parameter can be characterized by a corresponding shift in the parameters and wedges of the as-if static model. This allows us to import the rich analytical toolkit developed for distorted static economies to characterize balanced growth comparative statics.

We begin by analyzing the long-run effect of productivity improvements when rates of return are pinned down by consumer preferences, so that the capital wedges μ are fixed. With Cobb-Douglas production and preferences, we show that the long-run consumption impact of a sectoral productivity shock is fully summarized by that sector's cost-based Domar weight. This weight measures a sector's importance not by its sales, but by tracing its full contribution to the costs of consumption goods through the input-output network. When the economy is not at the Golden Rule, the measure exceeds a sector's simple sales share because the rental costs paid by users of capital exceed the sales of investment sectors. The deviations between cost and sales shares are largest for industries that produce investment goods or that supply inputs to them. For example, we show that while the construction sector's sales are 13% of world consumption, its cost-based Domar weight is 36%; for machinery, the corresponding figures are 5% and 14%.

The Cobb-Douglas case is special because productivity shocks do not alter the economy's resource allocation: the distribution of each factor and output across different uses remains fixed. When we deviate from this case, the direct technological effect – still summarized by the cost share – is complemented by a reallocation effect, reflecting long-run

[3]This is the same criterion as the one introduced by Abel et al. (1989) for determining whether the capital stock is below its Golden Rule value.

consumption gains from moving resources towards more capital-intensive uses. We show that this additional effect can be fully summarized by a measure of the decline in the aggregate labor share. For example, if capital and labor are gross substitutes, productivity improvements in investment goods trigger a positive reallocation effect by reducing the labor share. The effect is reversed if capital and labor are gross complements, with Cobb-Douglas being a neutral special case. This summary statistic for reallocation effects is highly general: for example, it remains true even when considering changes in global aggregate consumption in open-economy models.

For productivity effects, we also derive a dynamic analogue to Hulten's theorem for long-run consumption: when economies operate at the Golden Rule, the long-run impact of a productivity shock simplifies precisely to its sales share relative to consumption. As in the original theorem from Hulten (1978), this result is independent of the underlying production network or substitution elasticities. It is also independent of whether rates of return adjust. The intuition is twofold: First, at the Golden Rule, the "as-if" markups on capital disappear, causing cost shares to collapse to sales shares. Second, because a Golden Rule economy already maximizes long-run consumption, the envelope theorem applies, and first-order effects from resource reallocation are zero. The combination of these two facts leaves the simple sales share as the sole determinant of a sector's long-run importance for consumption.

Having analyzed productivity shocks with fixed rates of return, we generalize our framework in two directions. First, we allow for imperfectly elastic asset demand, which makes rates of return endogenous. Second, we consider the effect of permanent changes to exogenous distortions like tariffs and markups. We show that in both cases — endogenous changes in returns or exogenous changes in taxes — are isomorphic to changing wedges in the equivalent static economy. We use this equivalence to show that the consumption gain from changes in wedges is positive if resources shift toward more capital-intensive uses, but only if capital income exceeds investment costs. Furthermore, these effects are summarized by how much the reallocation of resources reduces the aggregate labor share (this is the change in the aggregate labor share minus the mechanical effect coming from changes in wedges).

We demonstrate these points using two analytical examples. First, we consider productivity changes in an economy with imperfectly elastic asset demand, where returns are endogenous. We show that reallocation effects are dampened by countervailing changes in returns; for example, if capital and labor are gross substitutes, the reallocation effect from a productivity increase is counteracted by a rising rate of return. Second, we analyze a symmetric tariff and show it has first-order negative effects on long-run consumption, even starting from free trade, but only if capital income exceeds investment. Furthermore, the first-order losses depend on the elasticity of substitution between labor and capital, but not the trade elasticity. A tariff makes investment goods relatively more expensive, and if labor and capital are not perfect complements, this reduces demand for capital and capital accumulation. If capital income exceeds investment, then the initial capital stock is below its Golden Rule value, so a reduction in the capital stock reduces long-run consumption.

We conclude the paper by using a quantitative model to study how the forces we analyze play out in a complete model of the world economy. The model features a rich international input-output structure, as in Costinot and Rodriguez-Clare (2014) and Baqaee and Farhi (2024), and overlapping generations of households in each country that accumulate capital subject to undiversifiable idiosyncratic investment risks, as in Angeletos and Panousi (2011). The model delivers endogenous risk premia that vary by country and industry, as well as a common endogenous risk-free interest rate on a global bond in zero net supply. We calibrate our model using expenditure shares from the World-Input Output Database (Timmer et al., 2015), augmented with investment flows data from Ding (2022). We characterize comparative statics in terms of parameters and expenditure shares, and solve for nonlinear effects by integrating these first-order effects.

We use the model to conduct two experiments to validate the intuition gained from the analytical examples. First, we analyze the long-run consumption impact of permanent, industry-specific productivity shocks. In a benchmark case where Cobb-Douglas aggregation of capital and labor limits the scope for reallocation, we find that an industry's importance for long-run consumption is extremely well summarized by its cost-based Domar weight, and that it is poorly approximated by its sales share. The distinction is stark for industries upstream of investment. For example, the elasticity of long-run consumption to a permanent productivity increase in construction is 0.37 even though its sales relative to consumption is 0.13. In contrast, the effect of a shock to a large downstream sector like food is well approximated by its sales relative to consumption.

We then show how these impacts are modified by reallocation effects once we move beyond the Cobb-Douglas benchmark. The elasticity of substitution between capital and labor becomes central. For a productivity shock in an investment-related sector such as machinery, the consumption gain is amplified when capital and labor are substitutes (as the economy shifts to more capital-intensive production, lowering the labor share) and is dampened when they are complements. This reallocation channel, however, is only potent for shocks that alter the relative price of capital. It is therefore negligible for downstream industries like food or health care.

Our second quantitative experiment analyzes the long-run consequences of a global trade war. We consider the effect of the US imposing the tariffs announced by the Trump administration on April 2, 2025, with symmetric retaliation from other countries. The trade war reduces long-run global consumption by approximately 1%. The effect is almost four times bigger than when capital is held fixed. We decompose consumption-losses analytically into a component driven by capital adjustment, and components driven by terms-of-trade and changes in the current account. We find that almost all effects are driven by changes in capital accumulation, rather than by the other two traditional trade mechanisms.

As highlighted by our analytical examples, the quantitative dominance of the capital channel also means that the key parameters governing long-run consumption losses are different from those in a static analysis. The magnitude of the consumption decline depends primarily on the economy's internal elasticities – such as the elasticity of substitution between capital and labor and the elasticity of asset demand from saving – rather than on the traditional Armington trade elasticities that are central in static models.

Related Literature. Our paper is related to a long tradition on the treatment of investment and capital in national income accounting. One recurring theme in this literature is that since consumption is the only true final good, investment goods should be viewed as intertemporal intermediate inputs (Kuznets, 1941; Hulten, 1979), prompting suggestions for different ways of netting out investment costs from GDP, to be consistent with the treatment of other intermediates (Weitzman, 1976; Barro, 2021). Our paper provides a precise sense in which capital goods are equivalent to intermediates for the purpose of long-run comparative statics.

Our paper is also related to Foerster et al. (2022) and Ding (2022) who study balanced-growth paths and steady states of multi-sector models. Foerster et al. (2022) work with a closed-economy Cobb-Douglas model with an infinitely-lived representative agent. Our analysis relaxes these assumptions by having an open economy, arbitrary elasticity structure, imperfectly elastic capital supply, and heterogeneous returns across sectors. Ding (2022) constructs investment flow tables for the world economy, and uses this data to study the gains from trade relative to autarky allowing for adjustments in capital. We relax the assumption of infinitely-lived agents, no growth, financial autarky, and homogeneous returns across capital goods. We also characterize the response of the economy to a different set of counterfactuals.

Our paper is also related to quantitative dynamic disaggregated and international general equilibrium models, pioneered by Long Jr and Plosser (1983) and Backus et al. (1992).

Some recent contributions include Alvarez (2017), Kehoe et al. (2018), Ravikumar et al. (2019), Dix-Carneiro et al. (2023), Lyon and Waugh (2019), Vom Lehn and Winberry (2022), and Kleinman et al. (2023).[4] We complement this literature by providing analytical characterizations for balanced-growth outcomes. Second, in contrast to our paper, this literature tends to work with infinitely-lived representative agents which results in a capital supply curve that is infinitely elastic and a rate of return that is fully pinned down by preferences and the growth rate.

In terms of methodology, we draw on and generalize tools from the literature on shock propagation in static production networks, in particular, Costinot and Rodriguez-Clare (2014), Baqaee and Farhi (2020), and Baqaee and Farhi (2024). They consider static production networks with exogenous wedges and exogenous trade imbalances. We extend these frameworks to account for capital accumulation. In our paper, wedges and trade imbalances are determined endogenously by equilibrium in capital markets. Other papers in this literature include Foerster et al. (2011), Gabaix (2011), Acemoglu et al. (2012), Di Giovanni et al. (2014), Buera and Trachter (2024) and Dávila and Schaab (2023).

Our paper identifies that a central determinant of the long-run response of consumption to shocks is the aggregate elasticity of capital-to-labor income. This is an object that has previously been extensively studied for its role in determining inequality and structural transformation (e.g. Antras, 2004; Rognlie, 2016; Oberfield and Raval, 2021).

Finally, our approach to modeling trade imbalances is based on the intertemporal approach to the current account from international macroeconomics (Obstfeld and Rogoff, 1995). As in Angeletos and Panousi (2011) and Cuñat and Zymek (2024), we use a model where demand for savings is not infinitely elastic in steady state, and we are able to solve for the long-run outcomes without having to solve transition dynamics. By allowing for financial frictions, we can relate global imbalances in the current account to financial development, similar to Caballero et al. (2008) and Mendoza et al. (2009).

2 Equivalence between BGPs and Static Economies

We first establish an equivalence between BGPs of dynamic economies and equilibria of static economies. Our results apply to a wide class of macroeconomic models. The class of models is implicitly defined by a set of necessary conditions that the balanced

[4]In this paper, we abstract from fixed costs and entry/exit decisions of firms, for example, as in Hopenhayn (1992), Melitz (2003). Alessandria et al. (2021) review this literature as it pertains to international trade. Barkai and Panageas (2021) study how the distribution of the types of entering firms affects long-run consumption near the Golden Rule. Although we do not explicitly study this class of models, we provide an example of how our results can be extended to models with firm entry in Section 2.

growth variables must satisfy. Any model satisfying these conditions falls within the class. We first prove a characterization result for competitive closed economies without implicit or explicit taxes, before extending the analysis to incorporate distorting taxes and international trade.

2.1 Environment

Consider a class of closed, competitive, economies. The economies have primary factor endowments $L_1, \ldots, L_F$ whose effective aggregate supply grows at a constant and common rate g:

$$L_f(t) = e^{gt} L_f,$$

where the growth rate g incorporates both increases in physical supply and factor-augmenting technological progress. All other technologies and preferences are time-invariant, and we consider economies having BGPs where all aggregate quantities grow at this common rate g, while prices and rates of return are constant.[5] We further require that the balanced growth paths in the class satisfy the following equations.

Production and profit maximization:

$$Y_i = A_i F_i \left[\{L_{if}\}_{f \in F}, \{Y_{ij}\}_{j \in N}, \{K_{ij}\}_{j \in N} \right], \tag{1}$$

$$\max_{\{Y_i, L_{if}, Y_{ij}, K_{ij}\}} \pi_i = p_i Y_i - \sum_{f \in F} w_f L_{if} - \sum_{j \in N} p_j Y_{ij} - \sum_{j \in N} R_j K_{ij}. \tag{2}$$

Resource constraints:

$$Y_i = C_i + X_i + \sum_{j \in N} Y_{ji}, \quad X_i = (g + \delta_i) K_i, \quad \sum_{i \in N} L_{if} \leq L_f, \quad \sum_{j \in N} K_{ij} \leq K_j. \tag{3}$$

User cost of capital:

$$R_j = (r + \delta_j) p_j. \tag{4}$$

Consumption.

$$\max_{\{C_i^h\}} U^h(C_1^h, \ldots, C_N^h) \quad \text{s.t.} \quad \sum_{i \in N} p_i C_i \leq \sum_{f \in F} w_f L_f + (r - g) B. \tag{5}$$

Asset market clearing and asset demand.

[5]Constant relative prices are consistent with growth in our framework because all growth is factor-augmenting, which ensures constant expenditure shares for a general class of technologies and preferences. The main case this assumption rules out is an economy with Cobb-Douglas technologies and preferences but heterogeneous TFP growth across sectors, where relative prices change systematically even as expenditure shares remain constant. Appendix A.2 discusses how our analysis can be extended to cover this scenario.

$$B = \sum_{j \in N} p_j K_j, \qquad \mathcal{A}^d(r) = \frac{B}{\sum_f w_f L_f}. \tag{6}$$

We describe each block of equations below.

Production and profit maximization. Equation (1) is the production technology for good i. We assume that the production function F_i has constant-returns-to-scale.[6] Each i is subject to a Hicks-neutral technology shifter A_i. Although we impose that productivity shifters, A_i, are Hicks-neutral, we can still capture input-specific technology shocks by relabeling. For example, to capture a shock to i's use of input j, introduce a fictitious producer whose sole role is to buy from j and sell to i, and subject it to a Hicks-neutral shock.

Equation (2) states that production choices are made to maximize profits taking the output price, p_i, wages of primary factors, w_f, intermediate input prices, p_j, and the rental price of capital, R_j, as given. If firms own their capital, rather than renting, this assumption still holds provided a shadow rental rate exists under which the balanced growth allocation is profit-maximizing. For example, in Appendix A.3, we show how to nest models with costly firm entry and entrepreneurial capital as in Hopenhayn (1992).

Resource constraints. The resource constraints for the economy are given in (3). The output of each good (Y_i) is allocated to final consumption (C_i), investment (X_i), and for use as an intermediate input by other producers (Y_{ji}). The second equation states that investment must be sufficient to maintain the capital stock at the balanced growth path, and is generally true in dynamic models where capital is accumulated linearly and subject to a constant depreciation rate. Finally, the last two constraints impose market clearing for all primary factors and capital goods.

User cost of capital. Equation (4) is a standard user cost of capital formula, saying that the rental cost of capital consists of a depreciation cost $\delta_i K_i p_i$ and a time cost $p_i r$, where the time cost depends on a required rate of return r, which we for now assume to be constant across capital goods (relaxed later). When capital prices are constant and all assets earn the same returns, as on our balanced growth paths, this equation holds across many dynamic models, since it follows from an indifference condition between holding capital good i and other assets.

Consumption. Equation (5) states that the vector of consumption goods maximizes a homothetic utility function given aggregate consumption expenditure. This assumption

[6]This setup admits decreasing-returns-to-scale, since decreasing returns are equivalent to having producer-specific fixed factor endowments.

holds both in models where there is a representative household as well as in models with heterogeneous households that have homothetic preferences (e.g. Aiyagari, 1994). Note that the expression for aggregate consumption is implied by zero profits of firms, the resource constraints, and asset market clearing, which means that it holds in any models where those other relationships hold. In particular, for models with heterogeneous households, the expression follows from integrating individual budget constraints and imposing market clearing.

Asset market clearing. The first condition in (6) is the asset market clearing condition, which states that household asset holdings, B, equal the total value of the capital stock, $\sum_j p_j K_j$; it holds in models where there are no outside assets and where the value of firms equal the replacement value of their capital, i.e. where Tobin's Q is 1 on a balanced growth path.[7] Later, we consider models where all assets are not perfect substitutes, yielding multiple asset market clearing conditions.

Asset demand. The last condition in (6) states that household asset holdings on a balanced growth path can be represented using an asset demand correspondence $\mathcal{A}_i^d(r)$. The correspondence does not depend on production parameters directly and summarizes the role of household savings across different models of asset accumulation. For example, in Aiyagari (1994), the asset demand for each r is an integral over the ergodic distribution of assets implied by that r. In overlapping generations models, asset demand is instead given by the integral across age groups of the asset holdings coming out of an optimal savings decision given r. In a neoclassical growth model, asset demand is infinitely elastic at some fixed r determined by preferences and the growth rate. The assumption that asset holdings normalized by total labor income only depend on returns is in line with many standard models, where steady-state asset accumulation is homothetic in labor income (see Auclert and Rognlie, 2018).[8] Below are two examples of asset demand correspondences in different dynamic models.

1. *Neoclassical growth model.* With a representative infinitely-lived household maximizing discounted utility with discount rate ρ, no population growth, labor-augmenting growth rate g, and elasticity of intertemporal substitution γ, the consumption Euler equation implies that the only return consistent with balanced growth is $r =$

[7] In models where there are pure profits and there are tradable claims to those profits, the capitalized value of profits need to be included in asset supply.

[8] Our assumption that $\mathcal{A}$ only depends on r and household parameters simplifies exposition and can be relaxed.

$\rho + g/\gamma$. In this case, asset demand is given by

$$\mathcal{A}(r) = \begin{cases} -\frac{1}{r-g} & \text{if } r < \rho + g/\gamma \\ \left[-\frac{1}{r-g}, \infty\right) & \text{if } r = \rho + g/\gamma \\ \varnothing & \text{if } r > \rho + g/\gamma \end{cases} . \tag{7}$$

2. *Perpetual-youth model.* Consider the overlapping-generations model laid out in Blanchard (1985). Agents have log utility over time, can save in riskless capital, there is no growth, and agents die at a constant hazard rate of ν. In steady state, Blanchard (1985) shows that

$$\mathcal{A}(r) = \frac{\frac{(r-\rho)}{\nu(\nu+\rho)}}{\left[1 - \frac{(r-\rho)}{\nu(\nu+\rho)}\right]},$$

which is an upward sloping function of r.

2.2 Equivalent Static Economy

Since our class of dynamic models is implicitly defined through the equations (2)-(6), these equations can be used to derive conclusions holdings for all models in the class. In particular, it is possible to show that if we have a BGP of a model that belongs to our class, its prices and normalized quantities are also the equilibrium of an equivalent static economy.

Proposition 1 (Equivalence Between BGPs and Distorted Static Economies)**.** *Consider a BGP equilibrium with a rate of return r. The following two statements hold.*

1. *Prices and wages $\{p_i, w_f\}$ and quantities $\{Y_i, C_i, K_{ij}, L_{if}, Y_{ij}\}$ are also the equilibrium of a static economy where (i) the production functions of goods and the preferences of the representative household are the same as in the dynamic economy; (ii) capital goods are intermediate inputs produced with a linear technology from investment goods $K_i = A_{K_i} X_i$, with productivity shifter $A_{K_i} = 1/(g+\delta_i)$; (iii) capital goods are sold at a markup*

$$\mu_i = \frac{r+\delta_i}{g+\delta_i} = \frac{R_i K_i}{p_i X_i},$$

with profits distributed to households.

2. *The rate of return r clears the asset market $\mathcal{A}^d(r) = \frac{\sum_i p_i K_i}{\sum_f w_f L_f}$.*

The equivalence in the first part of the proposition follows from two observations. First, maintaining a unit of capital good i along a BGP requires continuous investment of $(g +$

δ_i) units, which maps to the linear production technology (ii). The BGP marginal cost of producing capital is thus $(g + \delta_i)p_i$, while firms pay a rental price of $p_i(r + \delta_i)$. The ratios between rental prices and production costs are $(r + \delta_i)/(g + \delta_i)$, which map to the markups in (iii). These as-if markups equal 1 if the economy is at the Golden Rule ($r = g$) or if inputs are not durable ($\delta_i = \infty$). Empirically, μ_i is simply the ratio of capital income to investment on a BGP, exceeding unity whenever capital income exceeds investment.[9]

Economically, the production functions in (ii) reflect that capital goods and regular intermediates are similar on a technological level, with both being produced inputs. The markups in (iii), in turn, reflect the difference introduced by discounting and durability. For intermediate inputs, the zero profit condition implies that production costs and prices are the same. However, for capital, the zero profit condition is that investment costs equal the present value of future rental payments. Since rental payments occur in the future and are discounted, their undiscounted sum exceeds investment costs – creating the wedges manifested in our as-if markups.[10]

While the first part of the proposition characterizes the balanced growth path up to a rate of return r, the second part pins down that rate of return from asset market clearing. Conceptually, the right-hand side is aggregate demand for capital, which is determined by the equivalent static model as a function of the as-if markups $\mu_i(r) = (r + \delta_i)/(g + \delta_i)$. The left-hand side comes from the households' asset demand correspondence.

2.3 Balanced Growth Comparative Statics

Using Proposition 1, we can use the equivalent static economy to conduct long-run comparative statics. Formally, consider a change in a vector of parameters. For now, we assume that this is the vector of productivity shifters, A. Later we extend the argument to include other shocks like changes in implicit or explicit taxes.

Assume that the economy is initially at a BGP with prices and quantities X_0 for some A_0. Consider the effect of a perturbation ΔA to the parameter vector. Assuming that balanced growth paths are locally unique and attractive, the economy eventually converges to a new balanced growth path in the neighborhood of X_0. This procedure defines a function, denoted by $X^{BGP}(A)$, mapping parameters to long-run prices and quantities. Define the long-run effect of ΔA as the change between the initial and terminal BGP, as illustrated

[9] This is the same criterion for capital being below its Golden Rule level as in Abel et al. (1989).

[10] Since markups reflect discounting, they do not imply that the dynamic equilibrium is necessarily Pareto inefficient. For example, in a standard undistorted neoclassical growth model with $r > g$, the as-if markup on capital exceeds one even though the economy is efficient.

in Figure 1.[11]

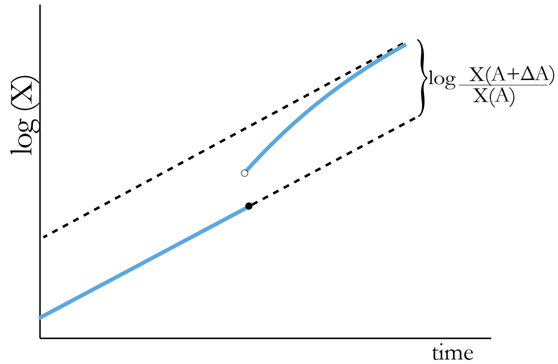

Figure 1: Response of BGP equilibrium to a permanent change in some parameter Θ.

Proposition 1 implies that

$$X^{BGP}(A) = X^{static}[A, \mu(r(A)],$$

where $X^{static}(A, \mu)$ is a function that maps productivities and wedges into prices and quantities of the equivalent static economy.[12] The advantage of this formulation is that the mapping $X^{static}(A, \mu)$ and its derivatives are well understood from the literature on static economies with distortions. The term $\mu[r(A)]$ indicates that wedges depend on the interest rate $r(A)$, which in turn depends on A through the auxiliary asset market clearing condition 2 in Proposition 1. The following proposition summarizes.

Proposition 2 (Comparative Statics using Static Economy)**.** *Suppose that a balanced growth path exists in the neighborhood of A_0 and that the solution of the static economy is locally unique around A_0 and $\mu[r_0]$. Then, the balanced growth path is locally unique, with its prices and quantities $X^{BGP}(A)$ and its rate of return $r(A)$ satisfying the following pair of equations*

$$X^{BGP}(A) = X^{static}(A, \mu[r(A)]) \tag{8}$$

$$\mathcal{A}^d[r(A)] = \frac{\mathcal{K}^{static}(A, \mu[r(A)])}{\mathcal{L}^{static}(A, \mu[r(A)])}. \tag{9}$$

where $\mu[r] = \frac{r+\delta}{g+\delta}$ are the as-if wedges associated with the rates of return r. Further, $\mathcal{L}^{static} =$

[11] If a BGP equilibrium exists with parameter values A_0 and outcomes $X^{BGP}(A_0)$, then we can guarantee the existence of locally isolated BGP equilibria in the neighborhood of A_0 by the inverse function theorem. However, we do not prove that such equilibria necessarily are locally attractive.

[12] To see why this is true, note that $X^{BGP}(A)$ is a balanced growth path with rates of return $r(A)$, and that the implication of the first part of Proposition 1 is precisely that such balanced growth paths are equilibria of static economies with the same A and as-if markups associated with $r(A)$.

$\sum_f w_f L_f$ and $\mathcal{K}^{static} = \sum_i p_i K_i$ are aggregate labor income and the aggregate capital stock in the equivalent static economy given productivities A and wedges $\boldsymbol{\mu}$.

In words, (8) implies that the long-run effect of a permanent shock can be characterized by a corresponding shift in the parameters and wedges of the as-if static model. This means that long-run comparative statics can be analyzed using, for example, hat algebra or exact hat algebra (see, e.g., Jones, 1965; Dekle et al., 2008; Baqaee and Farhi, 2024). The as-if static model also determines total demand for capital, $\mathcal{K}^{static}/\mathcal{L}^{static}$ as a function of $\mu[r]$. The equilibrium rate of return is then pinned down by (9), using the asset demand correspondence $\mathcal{A}[r]$. Hence long-run comparative statics can be computed from the distorted static model plus the return r pinned down by asset-market clearing $\mathcal{A}^d[r(A)] = \mathcal{K}^{static}/\mathcal{L}^{static}$.

2.4 Generalization of Basic Framework

We now extend the results from Proposition 1 and 2 to a more general class of models. Specifically, we introduce tax-like wedges τ_i to nest models with distortions, heterogeneous capital returns r_i to nest economies without return equalization, and multiple household types $h \in H$ which lets us nest open economies (each h representing a different country). Formally, we consider a class of models defined by having the following balanced growth path equations:

Production and profit maximization:

$$Y_i = A_i F_i \left[\{L_{if}\}_{f \in F}, \{Y_{ij}\}_{j \in N}, \{K_{ij}\}_{j \in N} \right], \tag{10}$$

$$\max_{\{Y_i, L_{if}, Y_{ij}, K_{ij}\}} \pi_i = (1 - \tau_i) p_i Y_i - \sum_{f \in F} w_f L_{if} - \sum_{j \in N} p_j Y_{ij} - \sum_{j \in N} R_j K_{ij}. \tag{11}$$

Resource constraints:

$$Y_i = \sum_{h \in H} C_i^h + X_i + \sum_{j \in N} Y_{ji}, \quad X_i = (g + \delta_i) K_i, \quad \sum_{i \in N} L_{if} \leq \sum_h L_f^h, \quad \sum_{j \in N} K_{ij} \leq K_j. \tag{12}$$

User cost of capital:

$$R_j = (r_j + \delta_j) p_j. \tag{13}$$

Consumption.

$$\max_{\{C_i^h\}} U^h(C_1^h, \dots, C_N^h) \quad \text{s.t.} \quad \sum_{i \in N} p_i C_i^h \leq \sum_{f \in F} w_f L_f^h + \sum_i (r_i - g) B_i^h + T^h. \tag{14}$$

Asset market clearing, asset demand, and wedge revenue.

$$\sum_{h \in H} B_i^h = p_i K_i, \qquad \mathcal{A}_i^{d,h}(\boldsymbol{r}) = \frac{B_i^h}{\sum_f w_f L_f^h + T^h}, \qquad T^h = \sum_{i \in N_h} \tau_i p_i Y_i \tag{15}$$

The differences compared to the baseline model equations (1)-(6) in Section 2.1 are the following. First, we introduce distortions using wedges τ_i in the profit maximization problem of producers. While the wedges τ_i are on output, we can accommodate input-specific wedges by re-labeling. To place a wedge on i's purchases of inputs from j, introduce an output wedge on a fictitious middle-man whose sole role is to buy from j and sell to i. Second, there are now multiple final consumers – indexed by h – in the resource constraint. Third, there are also different rates of return r_i in the user cost of capital. Fourth, in the consumer problem, utility functions are now indexed by type h, and households hold a portfolio of different assets B_i^h with heterogeneous returns. Households also receive wedge income T^h, which comes from taxes on some subset of goods denoted by N_h.[13] Fifth, the asset market clearing condition requires that household asset holdings are consistent with capital stocks asset-by-asset, and household asset demand is a correspondence $\mathcal{A}_i^{d,h}$ that maps vectors of capital returns to vectors of desired asset holdings. Last, total wedge revenue across households should be consistent with total wedge income.

The following proposition generalizes Proposition 1 to this broader class of models.

Proposition 3 (Generalized Equivalence Between BGPs and Static Economies)**.** *Consider a BGP of a model satisfying* (10)-(15) *with rates of returns r_i, and with $\pi^h = \frac{\sum_i (r_i - g) B_i^h}{\sum_{h'} \sum_i (r_i - g) B_i^{h'}}$ being the share of net capital income earned by household h. Then, the following holds.*

1. *Prices and wages $\{p_i, w_f\}$, quantities $\{Y_i, C_i^h, K_{ij}, L_{if}, Y_{ij}\}$, and wedge payments T_h also form an equilibrium of a static economy where: (i) production functions of goods, tax wedges τ_i, and the preferences of households in each country are the same as in the dynamic economy; (ii) capital goods are intermediates produced with linear technology from investment $K_i = A_{K_i} X_i$, with productivity shifter $A_{K_i} = 1/(g + \delta_i)$; (iii) all other goods are sold with a tax $(1 - \tau_i)$, with tax revenue rebated to the corresponding household; (iv) capital goods are sold at a markup $\mu_i = (r_i + \delta_i)/(g + \delta_i)$ with profits rebated to households according to $\{\pi^h\}$.*

2. *The rates of returns r_i and the net capital income shares π^h satisfy*

$$\sum_{h \in H} \mathcal{L}_h^{static} \mathcal{A}_i^{d,h}(\boldsymbol{r}) = \mathcal{K}_i^{static}$$

[13]The subsets N_h partition the set of all goods to ensure that aggregate household wedge income equals aggregate wedge payments by firms. The partition assumption is natural when households types correspond to countries and the wedges represent taxes.

$$\pi^h = \frac{\mathcal{L}_h^{static} \sum_i (r_i - g) \mathcal{A}_i^{d,h}(\boldsymbol{r})}{\sum_{h'} \mathcal{L}_{h'}^{static} \sum_i (r_i - g) \mathcal{A}_i^{d,h'}(\boldsymbol{r})},$$

where $\mathcal{K}_i^{static} = p_i K_i$ is the value of the capital stock of i, and $\mathcal{L}_h^{static} = T_h + \sum_f w_f L_f^h$ is the non-capital income of h, both in the equivalent static economy with productivities A, wedges μ, and profit shares π.

Just like Proposition 1, this proposition allows us to conduct long-run comparative statics using the equivalent as-if static economy, using for example, hat algebra. The static as-if economy features endogenous markups, $\mu(r)$, and an endogenous distribution of profit income, π, which are pinned down by the auxiliary equations from asset market clearing in the second part of the proposition. Crucially, these auxiliary equations only feature variables that are determined by the static as-if economy and the asset demand correspondences.

3 Changes in Productivities with Constant Rates of Return

This section characterizes the comparative statics in the special case when only productivities change and, because asset demand is perfectly elastic, rates of return are constant. (We consider imperfectly elastic asset demand later). We show that the effect of a productivity shock is the sum of two effects:

1. A technological effect, which captures the impact of a productivity shock fixing the allocation of resources. The effect equals the sector's *cost-based* Domar weight, which generally exceeds the sales share, especially for producers of investment goods and their suppliers.

2. A reallocation effect that is positive when the economy is not at the Golden Rule and when reallocation favors more capital-intensive sectors. The effect can be summarized by a measure of the fall in the labor share of the economy.

Further, we note that when the economy operates at the Golden Rule, there is neither a reallocation effect, nor a distinction between cost weights and sales shares, meaning that a dynamic analogue to Hulten's theorem applies: the long-run impact of a productivity shock to a sector is precisely the sector's sales share relative to consumption.

We illustrate the logic of cost versus revenue shares using an economy with a chain network structure, and the logic of reallocation using a neoclassical growth model where the production function is not Cobb-Douglas. We also use the World Input-Output tables

(Timmer et al., 2015) and the capital flows table from Ding (2022) to calculate revenue and cost shares of different industries, showing that cost shares can be dramatically larger than revenue shares in industries such as machinery, construction, and basic metals that produce investment goods or are important inputs to the production of investment goods.

3.1 Input-Output Framework and Definitions

The results in Section 2 show that balanced growth outcomes can be expressed in terms of outcomes in an equivalent static economy with markups. Here, we provide an input-output framework and key definitions for such economies.

To begin, we partition the economy into industries according to the use of their output. We define four distinct sets of sectors:

- **Consumption goods (*H*):** Industries whose outputs are exclusively used for final consumption, one for each household type h. Because there is one consumption industry per type, we index these industries by $h \in H$.
- **Intermediate goods (*N*):** Industries whose output is exclusively used as intermediate inputs.
- **Capital goods (*K*):** Industries that use intermediates to produce investment goods.
- **Primary factors (*F*):** The economy's primary factors.

This partitioning can be done without loss of generality. For each household type, we can create a "consumption industry" in set H by treating its homothetic utility function as the production function for a final composite good. Similarly, if an investment good uses capital or labor in addition to intermediates, the bundle of capital and labor can be treated like a separate intermediate good.

Cost-based input output matrix $\tilde{\Omega}$. Given this partition, we can construct a cost-based input output matrix $\tilde{\Omega}$, where $\tilde{\Omega}_{ij}$ is the share of costs of i that comes from j. The structure of $\tilde{\Omega}$ is depicted below:

$$\tilde{\Omega} = \begin{pmatrix} \mathbf{0}_{H\times H} & \tilde{\Omega}_{H\times N} & \mathbf{0}_{H\times K} & \mathbf{0}_{H\times F} \\ \mathbf{0}_{N\times H} & \tilde{\Omega}_{N\times N} & \tilde{\Omega}_{N\times K} & \tilde{\Omega}_{N\times F} \\ \mathbf{0}_{K\times H} & \tilde{\Omega}_{K\times N} & \mathbf{0}_{K\times K} & \mathbf{0}_{K\times F} \\ \mathbf{0}_{F\times H} & \mathbf{0}_{F\times N} & \mathbf{0}_{F\times K} & \mathbf{0}_{F\times F} \end{pmatrix}, \tag{16}$$

In the first line, the only non-zero block is $[\tilde{\Omega}_{H\times N}]_{h,n} = p_n C_n^h / \sum_{n'} p_{n'} C_{n'}^h$, where each row contains household expenditure shares by goods for each household h. The second line contains cost shares of intermediate goods N. The first non-zero matrix $[\tilde{\Omega}_{N\times N}]_{n,n'} = p_{n'} Y_{n,n'} / [\sum_m p_m Y_{n,m} + \sum_j R_j K_{n,j} + \sum_f w_f L_{n,f}]$ records the expenditures on intermediate inputs relative to total costs by producer of intermediate good n. The subsequent matrices $\tilde{\Omega}_{N\times K}$ and $\tilde{\Omega}_{N\times F}$ do the same for rental payments to capital and labor expenditures respectively by intermediate good producer n. On the third line, the only non-zero matrix $[\tilde{\Omega}_{K\times N}]_{k,n} = p_n Y_{k,n} / p_k X_k$ records the expenditure on good n as a share of total investment expenditure on capital good k. The final line is all zeros because primary factors use no inputs. Every row of $\tilde{\Omega}$ apart from the last F ones sum to one.

Wedges and the revenue-based input output matrix Ω. With some abuse of notation, define an $(H+N+K+F)\times 1$ wedge vector $\boldsymbol{\mu}$. The first H elements are all ones, the next N elements are $\mu_n = 1/(1-\tau_n)$, where τ_n is the tax on good n, the next K elements are $\mu_k = R_k K_k / p_k X_k$, which is the ratio of capital income to investment costs for capital type k, and, the last F elements are all ones. Define the revenue-based input-output matrix Ω to be

$$\Omega = \text{diag}(\boldsymbol{\mu})^{-1}\tilde{\Omega}.$$

Note that the rows of Ω record expenditures relative to revenues rather than to costs, since $\boldsymbol{\mu}^{-1}$ is the ratios between costs and revenues. Note that if there are no taxes and we are at the Golden Rule, then $\Omega = \tilde{\Omega}$.

Final expenditure shares. Finally, the vector Φ defines the shares of final expenditure. It has dimensions $(H+N+K+F)\times 1$, but as only goods in set H are consumed by households, the vector is non-zero only for its first H entries. Each such entry gives the expenditure share of a country $h \in H$ in global consumption:

$$\Phi_h = \frac{\sum_i p_i C_i^h}{\sum_{i,h} p_i C^h} \qquad h \in H.$$

For a single-country economy, the expenditure share vector reduces to $\Phi = [1, 0, \ldots, 0]'$.

Cost- and revenue-based Domar weights. Define the Domar weight for each $i \in \{H + N + K + F\}$ to be sales of i divided by total consumption expenditures:

$$\lambda_i = \frac{p_i Y_i}{\sum_{h'} \sum_{i'} p_{i'} C_{i'}^{h'}}.$$

Accounting identities imply that

$$\lambda' = \Phi'(I - \Omega)^{-1}, \tag{17}$$

where $\Psi = (I - \Omega)^{-1}$ is the revenue-based Leontief inverse. Intuitively, the Domar weight of i measures the average direct and indirect expenditures of consumption goods on i. Define the cost-based Domar weights to be

$$\tilde{\lambda}' = \Phi'(I - \tilde{\Omega})^{-1}, \tag{18}$$

where $\tilde{\Psi} = (I - \tilde{\Omega})^{-1}$ is the cost-based Leontief inverse. The cost-based Domar weight of i measures the average direct and indirect exposures of final goods to i, measured using costs rather than revenues. It can be shown that the sum of cost-based Domar weights for primary factors always sums to one: $\sum_{f \in F} \tilde{\lambda}_f = 1$. This is because the primary factor content of every consumption good is equal to one.

Changes in aggregate consumption. Given a change in parameters, define the change in real consumption to be the share-weighted change in the quantity of final consumption bundles:

$$d \log C = \sum_h \Phi_h d \log C^h.$$

If there is a single country, then $d \log C$ is simply the change in the consumption of that country.

3.2 The Effects of Productivity Changes

Proposition 3 implies that the effect of changing productivities A with fixed rates of return r and wedges τ is precisely the effect of changing productivities in a static distorted economy holding wedges μ constant (but potentially reallocating the profits from the Golden Rule wedges across households). Using results on comparative statics in distorted open economies (Baqaee and Farhi, 2024), we obtain the following result.

Proposition 4 (Productivity Shocks with Fixed Returns)**.** *Consider an open economy where the rate of return r does not change in response to changes in productivities. Then, the long-run elasticity of aggregate consumption with respect to a productivity change in sector i, A_i, is given by*

$$\frac{\partial \log C}{\partial \log A_i} = \tilde{\lambda}_i - \sum_{f \in F} \tilde{\lambda}_f \frac{\partial \log \lambda_f}{\partial \log A_i}.$$

If preferences and productions are Cobb-Douglas, the expression reduces to $\partial \log C / \partial \log A_i = \tilde{\lambda}_i$.

The first term $\tilde{\lambda}_i$ is the technological effect: the consumption gain resulting from a productivity improvement propagating through the input-output network, holding the allocation of resources fixed. Note that this effect is measured by the cost-based Domar weight, $\tilde{\lambda}_i$, and not the sales weight λ_i. If production functions and preferences are Cobb-Douglas, so that expenditure shares do not respond to shocks, then this is the entire effect. In particular, when a producer's cost share is high relative to its sales share, the producer's importance for long-run consumption is greater than its sales share would suggest. This difference depends on the cumulative wedges between i and the final consumers. In other words, investment sectors and their suppliers tend to have a high $\tilde{\lambda}_i$ relative to λ_i. (See Example 1 below).

When production functions and preferences are not all Cobb-Douglas, changing expenditure shares induce a reallocation effect, as resources are rerouted within the production network. The proposition shows that the net effect of these reallocations is captured by a weighted average of the changes in labor income shares. In the simple case of a single factor (labor), this boils down to the fall in the labor share. More generally, the effect is slightly more involved, since falls in the labor share are more important for factors facing high cumulative capital wedges, indicated by a high ratio of cost to revenue shares.

The next proposition shows that when there are only capital wedges $\mu_i = (r + \delta_i)/(g + \delta_i)$ the difference between the sales share λ_i of producer i and the consumption responses to a productivity shock to i can be related to the capital accumulation induced by such a shock.

Proposition 5 (Induced Capital Accumulation)**.** *If $\boldsymbol{\tau} = \mathbf{0}$, consumption responses to productivity shocks satisfy the following equation:*

$$\frac{\partial \log C}{\partial \log A_i} = \lambda_i + \sum_{j \in K} \frac{R_j K_j - p_j X_j}{\sum_{c,j'} p_{j'} C_{cj'}} \frac{\partial \log K_j}{\partial \log A_i}.$$

With Cobb-Douglas production and preferences, this implies $\tilde{\lambda}_i - \lambda_i = \sum_{j \in K} \frac{R_j K_j - p_j X_j}{\sum_{c,j'} p_{j'} C_{cj'}} \frac{\partial \log K_j}{\partial \log A_i}$.

The proposition shows that the consumption elasticity to a productivity shock exceeds sales by a capital amplification term, which is positive whenever productivity changes induce capital accumulation ($\partial \log K_j / \partial \log A_i > 0$) and capital stocks are below their Golden Rule levels ($R_j K_j > p_j X_j$). Since the consumption elasticity is simply $\tilde{\lambda}_i$ in Cobb-Douglas economies, the capital amplification term is precisely the difference between the cost share $\tilde{\lambda}_i$ and sales λ_i in a Cobb-Douglas model.

3.3 Hulten's Theorem for Long-Run Consumption

The expression in Proposition 5 simplifies considerably in the special case where the economy operates at the Golden Rule and there are no taxes, that is, when $r_i = g$ for all capital goods and $\tau_i = 0$ for every i. Since there are no wedges, cost-based Domar weights coincide with sales shares. Furthermore, consumption must be financed entirely by labor income, meaning that the labor share of consumption is identically one, and that the reallocation term in Proposition 4 disappears.[14] We obtain the following result.

Corollary 1 (Long-Run Hulten's Theorem)**.** *If the assumptions of Proposition 4 hold, the economy is at the Golden Rule ($r_i = g$ for all capital goods i), and there are no taxes, $\tau_i = 0$, then the long-run elasticity of consumption with respect to productivity is given by its sales share relative to consumption:*

$$\frac{\partial \log C}{\partial \log A_i} = \lambda_i.$$

Thus, at the Golden Rule, we obtain a dynamic analogue to Hulten's theorem: the long-run effect of a sectoral productivity shock is fully summarized by its sales share, with no residual role for the underlying network structure, or for elasticities. The one key difference from Hulten is that sales are measured relative to consumption and not to GDP — a result consistent with viewing capital services as intertemporal intermediate inputs and consumption as the economy's final output.[15] In terms of economic intuition, the corollary reflects that since a Golden Rule economy maximizes long-run consumption, the envelope theorem applies. This implies there are no first-order effects from resource reallocations, so the impact of a productivity shock is fully captured by its direct technological effect, which is given by the sector's sales share.

3.4 Illustrative Examples

To illustrate Proposition 4, we consider two examples: one without a reallocation effect but with stark differences between cost- and sales-based weights $\tilde{\lambda}_i$ and λ_i, and one with a reallocation effect on long-run consumption.

[14]Since $\tilde{\lambda}_f = \lambda_f$ for every $f \in F$ and $\sum_f \lambda_f = 1$, it follows that $\sum_{f \in F} \tilde{\lambda_f} \partial \log \lambda_f / \partial \log A_i = 0$.

[15]Note that the corollary should be distinguished from the standard Hulten's theorem, which applies to GDP and not final consumption. The standard Hulten's theorem holds more generally in our class of models model provided that the dynamic economy is efficient. In particular, even when the economy is not at the Golden Rule, the long-run response of real GDP to shocks is $d \log \text{Real GDP} = \sum_i \frac{p_i Y_i}{GDP} d \log A_i + \sum_i \frac{R_i K_i}{GDP} d \log K_i$, where the change in the capital stock is determined by the equilibrium of the as-if static economy.

Example 1 (Cost versus revenue shares in a chain economy.)**.** Consider a production chain where labor produces capital for stage 1, which in turn produces capital for stage 2, and so on, until capital at stage N produces the final consumption good. We assume a no-growth environment where a representative household's savings decisions fix the interest rate r, and where each capital good i depreciates at a rate δ_i.

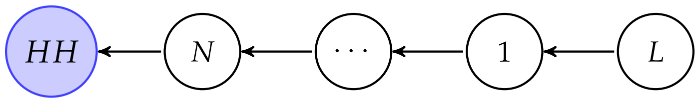

Figure 2: A linear production chain where labor ultimately produces consumption.

At each stage i, the ratio of the rental value of output to investment costs is $\frac{R_i K_i}{p_i X_i} = \frac{r+\delta_i}{\delta_i}$. Applying Proposition 1, the as-if wedge on capital goods is $\mu_i = (r + \delta_i)/\delta_i > 1$. Because of this wedge, the revenue-based Domar weight (sales share) of an upstream firm, λ_i, diminishes with each step in the chain provided there is positive discounting ($r > 0$):

$$\lambda_i = \frac{p_i Y_i}{p_C C} = \frac{1}{\mu_{i+1}\mu_{i+2}\cdots\mu_N} < 1,$$

with $\lambda_N = 1$. In stark contrast, the cost-based weight is simply $\tilde{\lambda}_i = 1$ for all stages, since 100% of the input cost in each stage is paid to upstream suppliers.

Given the linear production structure in the economy, there is no scope for reallocating inputs. Hence, Proposition 4 implies that the long-run consumption elasticity with respect to productivity is one across the entire chain:

$$\frac{\partial \log C}{\partial \log A_i} = \tilde{\lambda}_i = 1,$$

even though revenue-based sales shares λ_i shrink progressively upstream. This is because the sales shares are diminished by the cumulative effect of downstream capital wedges. In economic terms, discounting causes the sales shares of upstream sectors to understate their ultimate importance for long-run consumption, because their final use is far in the future.

The first example illustrates the difference between λ and $\tilde{\lambda}$ for a case without reallocation (i.e., where the change in the factor share $d \log \lambda_f$ is zero). The next example considers a case where the shock can trigger resource reallocation.

Example 2 (Reallocation effects.)**.** To illustrate reallocation effects, we use a standard neoclassical growth model (NGM) with a CES production function for capital (K) and labor

(L), allowing the elasticity of substitution (σ_{KL}) to deviate from the Cobb-Douglas benchmark of one.

Consider a productivity shock to aggregate output, A_Y. Applying our main proposition, the long-run consumption response decomposes into two effects:

$$d\log C = \tilde{\lambda}_Y d\log A_Y - d\log \lambda_L.$$

Here, the first term is the technological effect. Since the cost share of the sector is $\tilde{\lambda}_Y = 1/(1-\alpha)$, where α is the capital share of GDP, this term is the standard capital amplification effect $1/(1-\alpha)$. The second term is the *reallocation effect*, which is active whenever the shock alters the labor share, λ_L.

For a CES production function, solving out for $d\log\lambda_L$ using standard hat algebra methods, gives:

$$d\log C = \frac{1}{1-\alpha} d\log A_Y + \lambda_X(\sigma_{KL}-1)\frac{r-g}{g+\delta} d\log A_Y.$$

where $\lambda_X = \frac{p_x X}{p_C C}$ is the sales of the investment good sector relative to consumption. This equation shows that the reallocation channel is active only when there is a joint deviation from both Cobb-Douglas ($\sigma_{KL} \neq 1$) and the Golden Rule ($r \neq g$). For instance, if capital and labor are gross complements ($\sigma_{KL} < 1$) and $r > g$, then a positive productivity shock increases the labor share ($d\log\lambda_L > 0$). This generates a negative reallocation effect that partially offsets the technological consumption gain.

3.5 Empirical Cost and Revenue Weights

Cost and revenue Domar weights, $\tilde{\lambda}$ and λ, can be constructed based on information from input-output tables and investment flow tables. For example, we use the World Input-Output Database (WIOD) (Timmer et al., 2015) and global investment flow tables from Ding (2022). Using this data, we can populate all the terms in the cost-based IO matrix, $\tilde{\Omega}$, in (16), the consumption expenditure shares of each country, Φ^h, and the vector of as-if markups $\boldsymbol{\mu}$. Given these, we compute $\tilde{\lambda}$ and λ and report a selection of the results in Table 1 (details in Appendix B.3). Shares are constructed by averaging data over the period 1995 to 2009.

Table 1 displays the result for a selected set of industries. We see that for some downstream industries like health, revenue and cost weights are close to each other, reflecting that there are few capital goods on the path from health to the final consumer. In contrast,

Table 1: Cost and revenue based weights for selected industries

Sector	λ_i	$\tilde{\lambda}_i$	$\tilde{\lambda}_i/\lambda_i$
Construction	0.133	0.364	2.74
Machinery	0.054	0.138	2.54
Basic and Fab Metals	0.090	0.194	2.14
Electrical Eqmt	0.081	0.181	2.24
Transport Eqmt	0.083	0.172	2.07
Mining	0.042	0.072	1.71
Prof. Services	0.148	0.235	1.59
Wholesale, Retail, Trade, and Repair	0.371	0.518	1.39
Chemicals, Pharma	0.073	0.096	1.32
Energy, Gas, Water	0.055	0.070	1.28
Finance	0.128	0.161	1.25
Real Estate	0.158	0.180	1.14
Agriculture	0.099	0.111	1.12
Food	0.125	0.130	1.04
Health	0.101	0.102	1.01

investment good producing industries like construction and machinery have cost shares that are dramatically higher than sales shares. For example, sales in construction relative to world consumption is 13.3% of consumption in the economies of the WIOD, but the corresponding cost share is 36.4%. The numbers for machinery are 5.4% and 13.8% respectively. Based on Proposition 4, these differences imply that the effect of raising productivities in those industries is much higher than their sales shares in a Cobb-Douglas model with fixed returns. We return to this example with a more fully-specified quantitative model in Section 5 and check to what extent the results are sensitive to deviations from Cobb-Douglas and fixed returns.

4 General Comparative Statics

Having analyzed productivity shocks with fixed rates of return, this section generalizes the analysis in two directions. First, we allow for imperfectly elastic asset demand, which makes long-run rates of return endogenous. Second, we allow for changes in the tax-like distortions $\boldsymbol{\tau}$. Section 4.1 presents the general result; Section 4.2 discusses how the effects of productivity changes are modified relative to the previous section; Section 4.3 discusses the effect of exogenous changes in exogenous taxes, and Section 4.4 provides illustrative examples.

4.1 General Characterization

Proposition 3 implies that when productivities and wedges change, the effects can be analyzed using the following system of equations

$$X^{BGP} = X^{static}(A, \mu, \pi), \qquad \mu = \frac{r+\delta}{g+\delta}\frac{1}{1-\tau},$$

$$\sum_h \mathcal{L}_h^{static}(A, \mu, \pi)\mathcal{A}^{d,h}(\boldsymbol{r}) = \mathcal{K}^{static}(A, \mu, \pi), \qquad \pi^h = \frac{\mathcal{L}_h^{static}(A, \mu, \pi)\sum_i (r_i - g)\mathcal{A}_i^{d,h}(r)}{\sum_{h'} \mathcal{L}_{h'}^{static}(A, \mu, \pi)\sum_i (r_i - g)\mathcal{A}_i^{d,h'}(r)}.$$

The first line of equations state that prices and quantities on the balanced growth path (normalized by growth) are an equilibrium of the equivalent static economy given productivities A, wedges μ, and profit distribution shares π, with the wedges μ being a function of rates of return r and taxes τ. Here we adopt the convention that $\delta = \infty$ for non-capital goods and $\tau_i = 0$ for capital goods. The second line of equations show how the rates of return and the profit distributions are pinned down by household asset demand.

When all functions are single-valued and smooth, we can totally differentiate these expressions to obtain the following result. For clarity, we use boldface to denote vectors in this proposition.

Proposition 6 (General Comparative Statics)**.** *Suppose $\mathcal{A}^d(\boldsymbol{r})$ and $X^{static}(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi})$ are differentiable, then changes in BGP prices and quantities from changes in productivities $\boldsymbol{A}$ and taxes $\boldsymbol{\tau}$ satisfy*

$$dX^{BGP} = \frac{\partial X^{static}}{\partial \boldsymbol{A}} \cdot d\boldsymbol{A} + \frac{\partial X^{static}}{\partial \boldsymbol{\mu}} \cdot d\boldsymbol{\mu} + \frac{\partial X^{static}}{\partial \boldsymbol{\pi}} \cdot d\boldsymbol{\pi}.$$

The changes in wedges and returns satisfy

$$\frac{d\boldsymbol{\mu}}{\boldsymbol{\mu}} = \frac{d\boldsymbol{\tau}}{(1-\boldsymbol{\tau})} + \frac{d\boldsymbol{r}}{\boldsymbol{r}+\boldsymbol{\delta}}, \tag{19}$$

$$d\boldsymbol{r} = (\epsilon_r^d + \epsilon_r^s)^{-1} \sum_{x \in \{\boldsymbol{\tau}, \boldsymbol{A}, \boldsymbol{\pi}\}} \left[\frac{\partial \mathcal{K}^{static}}{\partial x} - \sum_h \frac{\partial \mathcal{L}_h^{static}}{\partial x} \mathcal{A}^{d,h}(\boldsymbol{r}) \right] dx, \tag{20}$$

where $\epsilon_r^s = -\frac{\partial \mathcal{K}^{static}}{\partial \boldsymbol{r}}$ and $\epsilon_r^d = \frac{\partial}{\partial \boldsymbol{r}}[\sum_h \mathcal{L}_h^{static} \mathcal{A}_h^d]$ are the derivatives of capital demand and asset demand with respect to $\boldsymbol{r}$. Last, the changes in the profit shares of households are given by:

$$\frac{d\pi^h}{\pi^h} = d\log(\mathcal{L}_h^{static}\mathcal{A}^{d,h}(\boldsymbol{r}) \cdot (\boldsymbol{r} - g)) - \sum_{h'} \pi^{h'} d\log(\mathcal{L}_{h'}^{static}\mathcal{A}^{d,h'}(\boldsymbol{r}) \cdot (\boldsymbol{r} - g)). \tag{21}$$

Compared to Section 3, which focused on changes in productivities at fixed rates of return and taxes, the main new feature in Proposition 6 is that it allows for $d\boldsymbol{\mu} \neq 0$. Equation (19) shows that wedges $\boldsymbol{\mu}$ now change for two reasons: changes in exogenous taxes $d\boldsymbol{\tau}$ and changes in endogenous rates of return $d\boldsymbol{r}$.

Equation (20) shows that changes in $\boldsymbol{r}$ depend on the shock to net capital demand $\mathcal{K}^{static} - \sum_h \mathcal{L}_h^{static} \mathcal{A}^{d,h}$ (the gap between demand for capital from firms and appetite for savings from households at fixed returns) and the slope of capital (ϵ_r^s) and asset demand (ϵ_r^d) with respect to $\boldsymbol{r}$. The change in returns is small if the initial shock to net capital demand is limited or if asset and capital demand are highly elastic. The neoclassical growth model is the limit where asset demand is infinitely elastic, $\epsilon_r^d = \infty$, and the rate of return does not respond to capital demand shocks. In this case, we obtain $d\mu = 0$ and recover $dX^{BGP} = \frac{\partial X^{static}}{\partial A} dA$ as in the previous section. In the one-asset case, $\epsilon_r^s + \epsilon_r^d$ are generally positive, which means that positive shocks to net capital demand tend to increase returns and vice versa. The matrix algebra in Proposition 6 generalizes this intuition to the case with multiple assets and returns.

The change in the distribution of income from capital, (21), depends on whether each household's net capital income, which is a product of holdings and net-returns adjusted for growth, changes more quickly or more slowly than the average. This, in turn, depends on initial portfolio holdings, the elasticities of asset demand, and changes in returns.

Note that apart from the derivatives of the asset demand functions $\mathcal{A}^{d,h}(\boldsymbol{r})$, all terms in the proposition are derivatives in the equivalent static economy, which have been characterized previously in terms of expenditure shares and microeconomic elasticities (Baqaee and Farhi, 2024). This means that if we complement information used in static analyses with derivatives of the asset demand functions, the proposition provides a full characterization of long-run balanced growth comparative statics.

Perfectly substitutable assets Proposition 6 assumes that asset demands, $\mathcal{A}^{d,h}(\boldsymbol{r})$, are differentiable mappings from returns to portfolio vectors. This assumption may not hold when households view different assets as perfectly substitutable. For instance, if households treat all assets as identical, asset demand becomes non-differentiable: it jumps when returns diverge, and when returns are equal ($r_i = r$), it is a correspondence rather than a function, encompassing all asset holdings consistent with the level of total wealth.

However, even with perfectly substitutable assets, a version of Proposition 6 remains valid. However, it requires using a reduced set of market clearing conditions – one for each block of perfectly substitutable assets – along with non-arbitrage conditions for returns within each block. For example, when households view assets as identical, we only need one market clearing condition (equating the total capital stock to desired aggregate wealth) combined with non-arbitrage conditions (requiring equal returns across assets). The quantitative model in Section 5 provides an example with multiple blocks and heterogeneous returns.

4.2 The General Effect of Productivity Changes

Proposition 6 allows us to extend the results on the effects of productivity changes from Section 3 to situations where rates of returns are not necessarily constant.

Corollary 2 (General Effect of Productivity Changes)**.** *The first-order effect on long-run consumption from a change in productivity is*

$$\frac{\partial \log C}{\partial \log A_i} = \tilde{\lambda}_i - \sum_f \tilde{\lambda}_f \frac{\partial \log \lambda_f}{\partial \log A_i} - \sum_j \tilde{\lambda}_j \underbrace{\frac{1}{r_j + \delta_j}}_{\frac{d \log \mu_j}{d r_j}} \frac{\partial r_j}{\partial \log A_i},$$

where $\frac{\partial r_j}{\partial \log A_i}$ *is given by Proposition 6.*

The first two terms mirror those in Proposition 4: a direct technology effect and a reallocation effect. The reallocation effect boosts consumption by shifting resources toward more capital-intensive uses, a process captured by a decrease in the labor share.[16] However, since wedges $\boldsymbol{\mu}$ change , there is also a purely mechanical reduction in the labor share through their impact on interest rates. The final term isolates this mechanical effect. The consumption gain from reallocation is then measured by the fall in the labor share net of this mechanical adjustment. This adjustment is non-zero whenever $\boldsymbol{r}$ changes, which happens when productivity shocks affect capital demand ($\partial \mathcal{K}^{static} / \partial A_i \neq 0$) and elasticities of asset demand and capital supply are finite.

4.3 Changes in taxes

In the general model, we can also consider the effect of changing the tax-like distortions τ. In this case, we obtain the following corollary to Proposition 6 characterizing the effect on long-run consumption.

[16]The change in labor shares $\frac{\partial \log \lambda_f}{\partial \log A_i}$ consist both of direct effects from changing A_i in the as-if static economy, as well as the indirect effect through endogenous changes in μ.

Corollary 3 (Changes in Taxes)**.** *The first-order effect on long-run consumption from a change in taxes τ_i is*

$$\frac{\partial \log C}{\partial \tau_i} = -\tilde{\lambda}_i \underbrace{\frac{1}{1-\tau_i}}_{\frac{\partial \log \mu_i}{d\tau_i}} - \sum_f \tilde{\lambda}_f \frac{\partial \log \lambda_f}{d\tau_i} - \sum_{j\in K} \tilde{\lambda}_j \underbrace{\frac{1}{r_j+\delta_j}}_{\frac{d \log \mu_j}{dr_j}} \frac{\partial r_j}{\partial \tau_i},$$

where $\frac{\partial \boldsymbol{r}}{\partial \tau_i}$ is given by Proposition 6.

Unlike a technology shock, a change in taxes τ_i does not alter productivity and thus affects the balanced growth path exclusively through reallocations. Analogous to our previous analysis, the consumption effect of this reallocation is captured by the fall in the labor share, net of a mechanical effect. Here, the mechanical effect has two components: the direct impact of τ_i itself and the indirect impact of τ_i on returns $\boldsymbol{r}$, where the latter are given by (20) in Proposition 6.

When exogenous taxes are all initially zero, $\tau_i = 0$, then we can repurpose the classic formula from Harberger (1964) — derived to study distortions in static economies — to study how long-run consumption responds to changes in wedges in open dynamic economies. We obtain the following result.

Proposition 7 (Real Consumption Response to Wedges)**.** *If $\tau_i = 0$ for every i, then the change in long-run aggregate real consumption to changes in exogenous wedges is*

$$\frac{\partial \log C}{\partial \log \tau_i} = \sum_{j\in K} \frac{R_j K_j - p_j X_j}{\sum_{h,j'} p_{j'} C_{hj'}} \frac{\partial \log K_j}{\partial \log \tau_i}. \tag{22}$$

Furthermore, for each country h, the change in long-run real consumption is

$$\frac{\partial \log C_h}{\partial \log \tau_i} = \underbrace{\sum_{j\in K_h} \frac{R_j K_j - p_j X_j}{p_h C_h} \frac{\partial \log K_j}{\partial \log \tau_i}}_{\text{Golden Rule wedge effect}} + \underbrace{\sum_{j\in N} \frac{NX_{hj}}{p_h C_h} \frac{\partial \log p_j}{\partial \log \tau_i}}_{\text{terms of trade effect}} - \underbrace{\frac{\sum_{j\in N} \partial NX_{hj}/\partial \log \tau_i}{p_h C_h}}_{\text{current account effect}}, \tag{23}$$

where K_h is the set of capital goods and NX_{hj} are the net exports of good j for country h. The endogenous terms $\partial \log K_j/\partial \log \tau_i$, $\partial \log p_j/\partial \log \tau_i$, and $\partial NX_{hj}/\partial \log \tau_i$ are expressible in terms of primitives using Proposition 6.

The first part of Proposition 7 shows that the global long-run consumption response to a wedge depends solely on how capital stocks adjust multiplied by the gap between capital income and investment. Consumption expands from reallocations that increase the size of the capital stock, with the effect scaling in the distance from the Golden Rule. In particular, if the economy operates at the Golden Rule where capital income equals investment for each good $R_j K_j = p_j X_j$, there is no first-order consumption response to a change in an exogenous tax τ_i starting from zero.

The second part, equation (23), generalizes this expression to study long-run consumption of individual countries. The first summand is the same Harberger-"rectangles" formula as in (22), but applied at the country-level. The second summand is a standard terms-of-trade effect capturing whether export prices rise faster or slower than import prices. The final term captures changes in net exports (since trade need not be balanced). The terms-of-trade effect and change in net exports terms are both zero-sum, they add up to zero for the world as a whole. However, the first term, capturing adjustment in the capital stocks weighted by the Golden Rule wedge, is not zero-sum (unless the world is at the Golden Rule, in which case the first term is zero for every country).

We show in Section 5 that the breakdown in Proposition 7 is useful for quantitatively dissecting and understanding how dynamic economies respond to permanent wedge changes in the long run (e.g. tariffs). In particular, we find that the terms associated with capital adjustment are quantitatively very important.

4.4 Examples

Below we provide two examples. The first shows the effect of productivity shocks when returns can adjust. The second shows the effect of changing exogenous taxes like tariffs.

Example 3 (Productivity shock in perpetual youth model.)**.** We first revisit Example 2 that studied the effect of productivity improvements in a neoclassical growth model with CES production. We consider an economy with the same production structure, but instead of having a representative household, the savings block of the model consists of a perpetual youth model as described in Section 2.1. As shown by Blanchard (1985), the asset demand function is

$$\mathcal{A}(r) = \frac{\frac{(r-\rho)}{\nu(\nu+\rho)}}{\left[1 - \frac{(r-\rho)}{\nu(\nu+\rho)}\right]},$$

where ρ is the discount rate and ν is the mortality rate. From proposition Proposition 6, we know that the long-run consumption effect is the productivity effect at fixed returns plus the effect from a change in as-if wedges μ due to changes in the rate of return. Furthermore, from Proposition 7, we know that effects of changes in wedges on consumption are summarized by changes in the capital stock times the Golden Rule wedge, implying

$$d\log C = d\log C^{rep-agent} + \left[\frac{RK - p_X X}{p_C C}\right]\frac{\partial \log K}{\partial \log \mu}\frac{dr}{r+\delta},$$

where $d\log C^{rep-agent}$ is the effect found in Example 2, and $\frac{\partial \log K}{\partial \mu}\frac{dr}{g+\delta}$ is the change in capital from a shock to the wedge in the static economy, multiplied by the change in the wedge $\mu = \frac{r+\delta}{g+\delta}$ implied by a change in the rate of return.

By using Proposition 6, we obtain

$$\frac{\partial \log K^{static}}{\partial \log \mu} = -\frac{\sigma_{KL}}{1-\alpha}, \qquad dr = \frac{\frac{\sigma_{KL}-1}{1-\alpha} d\log A_Y}{\frac{\mathcal{A}'(r)}{\mathcal{A}(r)} + \frac{\sigma_{KL}-1}{1-\alpha}\frac{1}{r+\delta}}.$$

The first expression captures that a higher wedge on capital reduces the use of capital in proportion to the capital-labor substitutability, σ_{KL}, amplified by a roundabout term $1/(1-\alpha)$ that includes both the higher user cost and the effect on investment good prices through the production network. The second term shows that rates of returns change due to the initial shock to net capital demand, $\frac{\sigma_{KL}-1}{1-\alpha} d\log A$, divided by the sum of the elasticities of asset demand (from households) and capital demand (from firms). Note that there is no response if the economy is Cobb-Douglas, $\sigma_{KL} = 1$, since the shock then does not move net capital demand. There is also no response, if, as in the neoclassical growth model, asset demand is infinitely elastic, $\mathcal{A}'(r) = \infty$.

Putting these equations together gives the full comparative static. We note that in general, the adjustment in returns tends to dampen the power of reallocation: if $\sigma_{KL} > 1$, returns rise which dampens the capital response to increases in A, and vice versa when $\sigma_{KL} < 1$.

Example 4 (Effects of a Tariff.)**.** We consider the long-run consumption effect of a two-way, symmetric tariff between two economies. We show that in a simple example, the reduction in long-run consumption from a trade war does not depend on the Armington trade elasticity, but does depend on the elasticity of substitution between labor and capital.

Index the home country by h and the foreign country by f. Home's technology is given by:

$$C_h + X_h = \left((1-\beta)Q_{h,h}^{\frac{\theta-1}{\theta}} + \beta Q_{h,f}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$

$$Q_h = \left(\alpha K_h^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} + (1-\alpha) L_h^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} \right)^{\frac{\sigma_{KL}}{\sigma_{KL}-1}} = Q_{h,h} + Q_{f,h},$$

$$\dot{K}_h = X_h - (\delta + g) K_h.$$

The first equation states that consumption and investment in h use the same bundle of domestic and foreign goods with the Armington trade elasticity $\theta - 1$.

Imports by h from f are $Q_{h,f}$ and exports to f are $Q_{f,h}$. The second equation states that goods produced by h are produced using domestic capital and labor and used either for domestic use or exported. The final equation is the capital accumulation equation. We assume closed capital markets and a representative infinitely-lived household in each country, which ensures balanced trade and fixes the rate of return on capital, r. The foreign country's technology is the mirror image of this technology. We normalize initial prices to ones so that β corresponds to the foreign share of consumption and α the capital share of output.

Initially, trade is free. We then introduce a two-way symmetric tariff, τ, that is rebated to domestic households. The change in the price of an imported good is:

$$d \log p_{imp} = d \log p_Q + d \log(1+\tau)$$

where $d \log(1+\tau)$ is the tariff increase and $d \log p_Q$ is the symmetric change in the local price of goods. Because r is fixed, the rental rate follows the final goods price ($d \log R = d \log p$), and we can derive the following change in the price-to-wage ratio:

$$d \log \frac{p}{w} = \frac{\beta}{1-\alpha} d \log(1+\tau) \tag{24}$$

where p is the price of final expenditure, α is the labor share in the production of the domestic variety, and β is the initial import expenditure share in each country.[17] Since the tariff is a tax, the resulting change in aggregate consumption is given by Proposition 7. Combined with the first-order condition for the capital-labor ratio $d \log K/L = -\sigma_{KL} d \log(p/w)$, the formula (24), and writing λ_X for the ratio of investment to consumption, we obtain:

$$d \log C = \lambda_X \frac{r-g}{g+\delta} d \log K = -\lambda_X \sigma_{KL} \frac{r-g}{g+\delta} d \log \frac{p}{w} = -\lambda_X \sigma_{KL} \frac{r-g}{g+\delta} \frac{\beta}{1-\alpha} d \log(1+\tau).$$

[17] The result is obtained by noting that $d \log p = d \log p_Q + \beta d \log(1+\tau)$ and that $d \log p_Q = (1-\alpha) d \log w + \alpha d \log p$.

When the economy is not at the Golden Rule ($r \neq g$), a tariff has a first-order effect on long-run consumption. The mechanism works by making investment goods (produced from the final good) relatively more expensive than labor. The effect's magnitude depends positively on the import share β, the capital share α, and the elasticity between capital and labor σ_{KL}, but not on the Armington trade elasticity. When the economy is at the Golden Rule, we recover the envelope condition that tariffs have no first-order effect on consumption.

5 A Quantitative Model of the World Economy

To study long-run comparative statics beyond illustrative examples, we set up and calibrate a dynamic general equilibrium model of the world economy. The model features a rich input-output structure with trade as well as overlapping generations of households in each country that accumulate capital subject to undiversifiable idiosyncratic investment risks. The main text provides a high-level summary of the model with details given in the appendix. Sections 5.1 and 5.2 describe the production and accumulation sides of the model. Sections 5.3 and 5.4 present the solution and the calibration. Sections 5.5 and 5.6 use the calibrated model to study the quantitative effects to changes in productivities and tariffs.

5.1 Production Side

There are H countries, with each country producing differentiated varieties of the same set of products. We index producers by two indices: (h, i), where $h \in H$ is the origin country and $i \in N$ is the industry of the producer (e.g. agriculture, mining, and so on).

The production function of industry i in country h is a Cobb-Douglas composite of labor, capital, and intermediate inputs from other industries.[18] Intermediate inputs from industry type j purchased by (h, i) are aggregated using an Armington CES aggregator with elasticity θ across different origins (h', j). Following common terminology in the trade literature, we refer to $\theta - 1$ as the (micro) trade elasticity.

Each country has three factor endowments: low-, medium-, and high-skill labor, with F_h denoting the set of primary factors in country h. Each industry uses a different mix of these labor types using a Cobb-Douglas aggregator. There is uniform labor-augmenting productivity to g_A in every country to ensure the existence of a BGP.

[18] In our experiments, we also consider an extension where labor and capital is aggregated using a CES aggregator with elasticity σ_{KL}.

We assume that each industry in each country, (h, i), has a specialized capital stock produced by a specialized corresponding investment-goods-producing industry. All other goods in the economy are perishable (infinite depreciation). This means that the number of capital stocks, $|K_h|$, in each country is equal to the number of non-investment industries, $|N_h|$, in each country h. The investment good of (h, i) is a Cobb-Douglas composite of inputs from different industries. Just as for perishable goods, investment inputs from industry type j from different origin countries are combined using an Armington CES aggregator with elasticity θ.

5.2 Household Savings and Aggregate Capital

The household side is modeled using a perpetual youth model in the style of Blanchard (1985), where births grow at an exogenous rate $g_L \geq 0$ and households face a constant mortality rate ν_h in country h. The model features idiosyncratic capital risk in the style of Angeletos (2007), where returns on capital in an industry i is subject to idiosyncratic shocks of volatility σ_i that cannot be diversified away. Below, we provide the key results and the relationships that need to hold on a balanced growth path (see appendix for more details).

Households can only accumulate capital in a single domestic industry, and indifference across industries implies that expected returns across industries satisfy a risk price formula

$$r_i = r + S_h \sigma_i,$$

where $S_h \equiv \frac{r_i - r}{\sigma_i}$ is a country-level Sharpe ratio that is equalized within a country. The perpetual youth structure implies that household problems scale in their effective wealth, which is the value of their financial assets and the net present value of future labor income. Households investing in each industry i allocate a fixed share ϕ_i of their total financial and human wealth to risky assets:

$$\phi_i = \frac{\gamma S_h}{\sigma_i},$$

where γ is the inverse of the coefficient of relative risk aversion. The risky share is increasing in the Sharpe ratio, and decreasing in risk-aversion and idiosyncratic volatility.

The remainder of the household's financial wealth is invested in a global risk-free bond, in zero net supply, that yields a return of r. The allocation choices of the households implies that effective wealth for every household in a country grows at a constant rate over their life:

$$g_{\omega,h} = \gamma \left([r - \rho_h] + \frac{\gamma + 1}{2} S_h^2 \right),$$

where ρ_h is the subjective discount rate in country h, which is allowed to vary across countries. Notably, this does not depend on which industry the household chooses to invest in.

The asset market clearing conditions implied by our household block are summarized in the following proposition.

Proposition 8 (Asset Market Clearing)**.** *Along the balanced growth path, there exist country-specific Sharpe ratios S_h, such that the return to capital in each industry i in that country satisfies $r_i = r + \sigma_i S_h$. The Sharpe ratio S_h in each country and the risk-free return r in the world satisfy*

$$\sum_{i \in K_h} \sigma_i p_i K_i = \gamma S_h \frac{\nu_h + g_L}{\nu_h + g - g_{\omega,h}} W_h, \qquad h \in H,$$

$$\sum_{h \in H} \sum_{i \in K_h} p_i K_i = \sum_{h \in H} \frac{g_{\omega,h} - g_A}{\nu_h + g - g_{\omega,h}} W_h,$$

where $g_{\omega,h}$ is the growth rate of effective wealth during an individual's life, and $W_h \equiv \frac{(1+T_h)\sum_{f \in F_h} w_f L_f}{r + \nu_h - g_A}$ is the aggregate human wealth in country h.

The proposition implies that r_i within each country are functions of the global risk free rate, r, and country-level Sharpe ratios, S_h. The first displayed equation in Proposition 8 provides H conditions, one for each country, pinning down the Sharpe ratios by requiring that households desired risky portfolio holdings in each country are equal to the quantity of capital demanded by firms. The second expression requires that risk-free holdings sum to zero, which is equivalent to requiring that total net worth in the world equals the total capital stock. These equations pin down returns, r_i, Sharpe ratios, S_h, and the risk-free rate, r in terms of parameters of the household savings problem and variables from the as-if static economy.

5.3 Solution method

To solve for comparative statics, we combine Proposition 8 with Proposition 6. Appendix D presents the full set of linearized equations in terms of microeconomic primitives. In that appendix, we show that we obtain a system of linear equations of the following form

$$\Xi \begin{pmatrix} d \log \lambda_f \\ dS_h \\ dr \end{pmatrix} + \Gamma \begin{pmatrix} d \log A \\ d \log \tau \end{pmatrix} = \mathbf{0} \Leftrightarrow \begin{pmatrix} d \log \lambda_f \\ dS_h \\ dr \end{pmatrix} = -\Xi^{-1} \Gamma \begin{pmatrix} d \log A \\ d \log \tau \end{pmatrix} \tag{25}$$

where $d \log \lambda_f$ are the vector of changes in labor shares (since the quantity of labor is fixed and nominal consumption is the numeraire, $d \log \lambda_f$ is also the log change in the wage of f), dS_h are vector of changes in the Sharpe ratios, and dr is the change in the risk-free rate. This system determines relative wages and rates of return by combining labor and asset market clearing conditions. The matrices Ξ and Γ only depend on expenditure shares, elasticities of substitution, and elasticities of asset demand. We can express all other endogenous variables in terms of these prices and returns. Since the left-hand side of (25) is sufficient for a full model solution, the equation defines the local comparative statics for an infinitesimal shock in terms of the matrices Ξ and Γ.

These matrices depend on expenditure shares and asset demand elasticities, which are not constant in response to shocks. To solve for the impact of large shocks, we follow the Computational General Equilibrium (CGE) literature by treating (25) as a system of differential equations (Harrison and Pearson, 1996). We discretize a large shock into a sequence of small steps. In each step, we use the current values of the endogenous variables to update how the Ξ and Γ matrices change, and then solve for the incremental changes in prices and returns, and repeat. By iterating this procedure, we trace out the full nonlinear adjustment to obtain an exact nonlinear solution, effectively extending standard CGE methods to the analysis of long-run comparative statics.

5.4 Calibration Strategy

We calibrate the model to a balanced growth path with nine regions (the US, EU, Japan, China, UK, India, Canada, Mexico, and a rest-of-world composite). The targets are expressed in the static distorted economy notation of Section 3, exploiting our equivalence results. Table 2 lists the information needed to solve the model and construct the matrices Ξ and Γ in (25). A more detailed description of the data sources and methodology is provided in Appendix D.6.

As described in Section 3.5, the cost shares $\tilde{\Omega}$ and the capital wedges μ_i are constructed from WIOD and Ding (2022). The net foreign asset positions are from the External Wealth of Nations Database (Lane and Milesi-Ferretti, 2018). The risk-free rate, population growth, and the technology growth rate are based on long-term yields on US treasuries, GDP-weighted working age population growth rates, and per capita growth in US real GDP (UN Population Prospects, FRED, and the Penn World Table). Our results are not very sensitive to the precise values of the interest rate and these growth rates. The Armington trade elasticity, $\theta - 1$ is set equal to 4, following Simonovska and Waugh (2014). We use industry-specific depreciation rates from the Bureau of Economic Analysis, which given

Table 2: Calibration targets

Variable	Description	Value and source
$\tilde{\Omega}$	Cost shares	WIOD, Ding (2022)
μ_i	Capital income / investment	WIOD
B_{0h}	Net foreign asset positions	Lane and Milesi-Ferretti (2018)
r	Risk-free rate	0.025 (FRED)
g_L	Population growth rate	0.004 (UN WPP)
g_A	Labor-augmenting technology	0.02 (Penn World Table)
$\theta - 1$	Armington trade elasticity	4 (Simonovska and Waugh, 2014)
δ_i	Depreciation by industry	BEA
γ	Intertemporal elasticity of sub.	0.5
ρ_h	Impatience parameter	See text.
ν_h	Mortality rate	See text.
S_h	Sharpe ratio	See text.

capital wedges $\mu_{h,i}$ and growth rate g, lets us back out $r_{h,i}$ for each capital good i in each region h.

The remaining parameters in Table 2 are needed to pin down elasticities of the asset demand block. We set the elasticity of intertemporal substitution $\gamma = 0.5$, in line with microeconomic evidence. For the remaining household savings parameters, we calibrate the discount rate, mortality, and Sharpe ratios, (ρ_h, ν_h, S_h), for each country to match three targets: (1) financial wealth relative to labor income, $(B_{0h} + \sum_{i \in K_h} p_i K_i) / \sum_{f \in \mathcal{F}_h} w_f L_f$; (2) a semi-elasticity of total wealth with respect to the risk-free rate (in each country) of 18 (based on Auclert et al., 2021); (3) the total excess returns from risky assets relative to labor income $(\sum_{i \in \mathcal{K}_h} [r_i - r] \, p_i K_i) / \sum_{i \in \mathcal{F}_h} w_f L_f$.[19]

Table 3 summarizes key features of the calibrated steady state for the four largest economies. The results highlight several important features of our benchmark equilibrium. Average returns on capital and, hence, wedges on capital are high (often above 2),

[19]The total wealth of a country relative to its labor income can be expressed as $\frac{1}{r+\nu_h-g_A}\left(\frac{g_{\omega,h}-g_A}{\nu_h+g_L+g_A-g_{\omega,h}}\right)$; we choose parameters to match the level of this expression to $(B_{0h} + \sum_{i \in K_h} p_i K_i) / \sum_{f \in \mathcal{F}_h} w_f L_f$ – which can be calculated from earlier values in Table 2 – and its semi-elasticity with respect to r to 18. The total excess returns from risky assets can be expressed as $\left(\frac{\nu_h+g_L}{\nu_h+g_L+g_A-g_{\omega,h}}\right)\frac{\gamma S_h^2}{r+\nu_h-g_A}$ and we match it to $(\sum_{i \in \mathcal{K}_h} [r_i - r] \, p_i K_i) / \sum_{i \in \mathcal{F}_h} w_f L_f$, which can also be calculated from earlier information in Table 2.

Table 3: Summary of steady-state calibrated values for the four largest economies

Parameter	Description	USA	CHN	EU	JPN
$\bar{r}_h$	Average return on capital	0.123	0.059	0.155	0.120
$\bar{\mu}_h$	Harmonic average wedge on capital	2.356	1.893	2.433	2.198
S_h	Sharpe ratio	0.231	0.139	0.248	0.231
$\rho_h + \nu_h$	Effective discount rate	0.062	0.023	0.082	0.056
b_h/GDP_h	NFA relative to local GDP	-0.309	0.492	-0.254	0.365
NX_h/GDP_h	Trade balance relative to local GDP	-0.000	0.000	-0.000	0.000
χ_h	Ratio of total to human wealth	1.363	1.462	1.299	1.398
$g_{\omega h}$	Wealth growth given survival	0.043	0.043	0.043	0.043

particularly outside of China. This suggests the initial equilibrium is far from the Golden Rule, creating significant scope for reallocation effects and differences between sales share and cost shares. In our calibration, the US and the EU are net borrowers, whereas China and Japan are net savers (this matches the net foreign asset positions of these countries). Since we assume that the economy is on a BGP, this implies that the US, the EU, and the UK must run small trade surpluses, whereas China and Japan run small trade deficits.[20]

In the next two sections, we use the calibrated model to study the effect of technology changes (focusing on industry-level TFPs) and distortions (focusing on tariffs). In both cases, we use our analytical results to interpret the quantitative findings.

5.5 Effects of Productivity Shocks

We analyze the long-run consumption effects of permanent, industry-specific productivity shocks, focusing on the role of revenue versus cost-based Domar weights, and the role of capital-labor substitutability in shaping the degree of reallocation gains.

Cost versus revenue weights. Table 4 shows the elasticity of long-run consumption with respect to industry productivity increases in our baseline calibration. The table reports the elasticity of global consumption to a global productivity shock to a selection of industries. Our baseline calibration has Cobb-Douglas aggregation of everything apart from the Armington nests where varieties of goods from each industry are aggregated across different origins. Accordingly, labor shares do not respond strongly to global productivity

[20] This is counterfactual because, for some countries, net foreign asset positions and trade balance have the same sign in the data (e.g. the US has negative net foreign assets and runs a trade deficit). For these countries, our calibrated trade imbalances have the wrong sign. However, this has negligible effects on our quantitative results because trade imbalances are small relative to the size of the regions.

shocks. Hence, since labor shares do not move much, Proposition 4 implies there is limited reallocation in response to industry-level productivity shocks. Hence, consumption elasticities are almost equal to cost-based Domar weights.

Whereas cost-based Domar weights approximate consumption elasticities very well, sales shares are poor proxies, especially for investment industries and their upstream suppliers. For example, sales of the construction industry relative to consumption are just 0.133, yet the elasticity of long-run consumption to a permanent TFP shock is 0.367. In contrast, for downstream sectors not involved in investment, like health care, sales relative to consumption (0.101), is a good proxy for the consumption impact (0.102).

Table 4: Long-run global consumption effect of TFP shocks for a selection of industries.

Sector	**Elasticity** $\left(\frac{\partial \log C}{\partial \log A_i}\right)$	**Sales share** (λ_i)	**Cost weight** $(\tilde{\lambda}_i)$
Agriculture	0.111	0.099	0.111
Mining	0.074	0.042	0.072
Machinery	0.139	0.054	0.138
Energy, Gas, Water	0.070	0.055	0.070
Construction	0.367	0.133	0.364
Real Estate	0.180	0.158	0.180
Prof. Services	0.235	0.148	0.235
Health	0.102	0.101	0.102

The Role of Capital-Labor Substitutability. When the elasticity of substitution between capital and labor, σ_{KL}, deviates from the Cobb-Douglas benchmark of one, the impact of productivity shocks on consumption can diverge further from the simple cost shares ($\tilde{\lambda}_i$) due to reallocation effects. Panel A of Table 5 illustrates this with a shock to machinery productivity. The consumption elasticity falls from its baseline of 0.139 to 0.093 when capital and labor are complements ($\sigma_{KL} = 0.6$), and rises to 0.154 when they are substitutes ($\sigma_{KL} = 1.2$).

These deviations occur because the productivity shock alters the capital-labor price ratio, which in turn changes labor shares. Per Proposition 4, an increase in the labor share dampens the effects of a positive productivity shock, whereas a reduction in the labor share amplifies it. When $\sigma_{KL} < 1$, the labor share of income rises in response to positive productivity shocks, dampening the consumption response; when $\sigma_{KL} > 1$, the labor share falls, amplifying the effect. These reallocation effects are even more pronounced in a representative-agent model with infinitely elastic asset demand, as endogenous adjustments in the rate of return no longer dampen swings in the capital stock.

Given that reallocation effects depend on changes in the capital-labor price ratio, they are absent for productivity shocks that do not alter this ratio. For example, as shown in Panel B, the value of σ_{KL} is essentially irrelevant for shocks to sectors like health, which is not upstream from capital production.

Table 5: Industry TFP increase with different capital-labor elasticities.

Panel A: Machinery Sector			
	$\tilde{\lambda}$	Benchmark	Rep. agent
$\sigma_{KL} = 0.6$	0.138	0.093	0.083
$\sigma_{KL} = 1.0$	0.138	0.139	0.139
$\sigma_{KL} = 1.2$	0.138	0.154	0.167
Panel B: Health Sector			
	$\tilde{\lambda}$	Benchmark	Rep. agent
$\sigma_{KL} = 0.6$	0.102	0.102	0.102
$\sigma_{KL} = 1.0$	0.102	0.102	0.102
$\sigma_{KL} = 1.2$	0.102	0.102	0.102

Note: The $\tilde{\lambda}$ depend only on expenditure shares and do not change as we vary elasticities of capital demand σ_{KL} or asset demand.

5.6 Effect of a Trade War

In this section, we consider the long-run consumption effects of distortions, focusing on import tariffs. We use our analytical results to decompose the channels through which tariffs can affect long-run consumption. Our analytical framework helps to clarify why elasticities related to capital and asset demand are quantitatively as or even more important than Armington trade elasticities for determining how long-run consumption responds to tariffs. We begin by considering first-order approximations using Proposition 6, after which we provide nonlinear results by iterating the first-order results.

First-Order Approximation We consider a trade war where the U.S. imposes tariffs in line with the tariff schedule announced by the Trump administration on April 2, 2025, and where it faces symmetric retaliation from other countries.[21] We compare consumption under a trade war to those under free trade (no tariffs).

[21] The list of tariffs by country can be found in Table E.1 in the appendix.

To a first-order approximation, this trade war reduces global long-run consumption by about 1% (Table 6). Using the decomposition from Proposition 7 reveals that in every region, the decline is driven almost entirely by the interaction of the Golden Rule wedge with the tariff-induced reduction in the capital stock. The two traditional trade channels, changes in the terms-of-trade and changes in the current account, are comparatively unimportant in this long-run experiment.

Table 6: Decomposition of consumption changes via Proposition 7 for selected regions

Country	$d \log C_h$	**Capital adjustment**	**Terms of trade**	**Δ Current account**
United States	-0.022	-0.020	-0.002	-0.000
Canada	-0.022	-0.021	-0.001	-0.000
China	-0.018	-0.012	-0.007	0.001
Mexico	-0.046	-0.050	0.004	-0.001
European Union	-0.004	-0.004	0.000	-0.000
Global	-0.010	-0.010	-0.000	0.000

Table 7: Change in long-run consumption due to increase in tariffs

Selected regions	**Benchmark**	**Rep. agent**	**Static**	$\sigma_{KL} = 0.6$	$\sigma_{KL} = 1.2$	$\theta = 1$	$\delta = \infty$
United States	-0.022	-0.027	-0.002	-0.012	-0.025	-0.012	-0.003
Canada	-0.022	-0.026	-0.002	-0.012	-0.025	-0.018	-0.002
Mexico	-0.046	-0.071	0.003	-0.036	-0.049	-0.034	0.012
China	-0.018	-0.020	-0.008	-0.014	-0.020	-0.021	-0.008
European Union	-0.004	-0.004	0.000	-0.001	-0.005	-0.008	-0.000
Global	-0.010	-0.013	0.000	-0.005	-0.012	-0.010	-0.000

Intuitively, this dominant effect occurs because tariffs raise the price of investment goods relative to labor. This reduces capital demand and investment, and since there is a Golden Rule wedge, depresses long-run consumption. To further understand the economic forces, Table 7 displays the reduction in long-run consumption under alternative calibrations of the model. The first column is the benchmark model. The second column makes capital supply infinitely elastic, as in representative agent models. This makes the negative effects larger: global consumption now declines by 1.3% instead of 1.1%. The intuition here is familiar from the misallocation literature: if supply or demand curves are more elastic, a given wedge alters quantities by more, and hence reduces consumption by more.

The third column considers a static version of the model where capital is treated as an endowment (so that capital supply is inelastic). In this case, global consumption does not respond to a first-order change in tariffs, due to the envelope theorem. Instead, tariffs are purely redistributive to a first order. Since the trade war results in retaliatory off-setting tariffs, the redistribution effects are quite mild. The fourth and fifth column show that as we raise the elasticity of substitution between capital and labor, which makes capital demand more elastic, the losses from tariffs are greater. Once again, the intuition is similar to misallocation: raising σ_{KL} causes capital to adjust more to the wedge, which depresses long-run consumption by more since there was an initial wedge on investment to begin with. (See Example 4 for a pen-and-paper demonstration of this logic).

Lowering all trade elasticities to zero ($\theta - 1 = 0$) almost does not affect global consumption. This conflicts with intuition from static trade models, where losses from tariffs scale in the trade elasticity. The reason is that in static trade models, misallocation caused by tariffs is a second-order effect. In the dynamic model, tariffs cause long-run consumption to fall to a first order. This first-order effect works through capital-labor substitution rather than through domestic-foreign substitution. (Again, see Example 4 for an analytical example). Thus, what matters is how much tariffs raise investment prices relative to wages, which depends directly on capital goods' import content, but less on the trade elasticity. The trade elasticity does matter for the redistributive effects of the trade war, and lower trade elasticities spread out the losses more evenly across all countries. Finally, the last column shows the important role of the Golden Rule wedge by considering a case where capital depreciates instantaneously. In this case, the flow of capital services is literally an intermediate input and there are no capital wedges, hence global losses are, to a first order, equal to zero in response to a trade war exactly as in the static model (despite the fact that capital stocks fall in response to the trade war).

Nonlinear Results. Table 8 compares the linear and nonlinear effects on consumption from the increase in the tariff. The first-order approximation performs well for most regions as long as capital adjusts. This suggests that the capital adjustment logic continues to apply even in the nonlinear model with large shocks. The performance of the first-order approximation is much worse in the static model where capital is held constant. This is because when capital is held constant, the large first-order effects associated with capital adjustment are zero. The fact that first-order effects are muted when capital is held fixed means that higher order effects are relatively more important.

Table 8: Nonlinear and first-order log change in long-run consumption due to tariffs

Region	Capital adjusts		Capital fixed	
	Nonlinear	First-order	Nonlinear	First-order
United States	-0.022	-0.022	-0.007	-0.002
Canada	-0.023	-0.022	-0.005	-0.002
Mexico	-0.048	-0.046	-0.000	+0.003
China	-0.009	-0.018	-0.006	-0.008
EU	-0.004	-0.004	-0.001	+0.000
Global	-0.011	-0.010	-0.003	-0.000

Table 9: Log change in real wages in response to tariffs

Region	Capital adjusts	Capital fixed
United States	-0.030	-0.016
Canada	-0.039	-0.021
Mexico	-0.061	-0.014
China	-0.013	-0.007
European Union	-0.006	-0.003

Table 9 presents changes in real wages by country. The change in the real wage is always negative, and the reductions are larger than the ones in consumption. Real wage declines do not translate one-for-one into declines of consumption since rebated tariff revenue mitigates the effect on household income. Capital adjustment amplifies reductions in real wages since a lower capital stock lowers the marginal product of labor. These examples highlight the importance of accounting for capital accumulation when studying the long-run effects of tariffs.

6 Conclusion

In this paper, we provide a framework for analyzing long-run comparative statics of dynamic disaggregated economies by representing them as distorted static disaggregated economies. This representation allows us to use tools from the static literature to study these models, and provides intuition about which model features matter for understanding long-run comparative statics. The recurring theme is that long-run consumption responses can be understood through second-best principles, even in efficient economies.

While we focus on tangible capital accumulation in this paper, similar arguments and methods apply to the accumulation of non-physical and intangible capital as well, and this is an important avenue for future work.

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Appendix

A Appendix to Section 2

A.1 Proof of Proposition 1

Take any BGP of an economy in the class defined in Section 2.1, with common growth rate g for aggregate quantities, a common constant required return r on all assets, price vector $\{p_i, w_f\}_{i \in N, f \in F}$, and allocation

$$\{Y_i, C_i, X_i, K_i, \{K_{ij}\}_{j \in N}, \{L_{if}\}_{f \in F}, \{Y_{ij}\}_{j \in N}\}_{i \in N}$$

that satisfies (1)-(6). Define, for each $i \in N$,

$$R_i \equiv (r + \delta_i) p_i, \qquad A_{K_i} \equiv \frac{1}{g + \delta_i}, \qquad \mu_i \equiv \frac{r + \delta_i}{g + \delta_i}$$

We verify parts (1)-(2) of the proposition.

(1) BGP $\Rightarrow$ static equilibrium with linear capital technology and markups. *Production and profit maximization.* By assumption on the BGP, firms maximize

$$\pi_i = p_i Y_i - \sum_{f \in F} w_f L_{if} - \sum_{j \in N} p_j Y_{ij} - \sum_{j \in N} R_j K_{ij},$$

with R_j given by (4). Furthermore, since $K_i = A_{K_i} X_i$ with $A_{K_i} = 1/(g + \delta_i)$ we can write

$$R_j K_{ij} = \frac{r_j + \delta_j}{g + \delta_j} \frac{p_j}{A_{K_j}} K_{ij},$$

so the problem is exactly what firms face in a static economy where capital is produced with productivity A_{K_i} from X_i.

Resource feasibility and linear capital technology. On the BGP we have the accounting identities in (3), in particular

$$X_i = (g + \delta_i) K_i \quad \iff \quad K_i = A_{K_i} X_i,$$

with $A_{K_i} = 1/(g + \delta_i)$ as in item (ii) of the proposition. Market clearing for primary factors and capital services also holds by (3).

Profits and consumption. The profits associated with the capital-good sector are

$$\sum_i \Pi_i \equiv \sum_i (R_i K_i - p_i X_i) = p_i(r+\delta_i)K_i - p_i(g+\delta_i)K_i = \sum_i (r-g) p_i K_i,$$

Profits $\{\Pi_i\}$ are rebated to households. Aggregating household budgets then yields

$$\sum_{i\in N} p_i C_i \ \le\ \sum_{f\in F} w_f L_f \ +\ \sum_{i\in N} \Pi_i \ =\ \sum_{f\in F} w_f L_f \ +\ (r-g)\sum_{i\in N} p_i K_i \ =\ \sum_{f\in F} w_f L_f \ +\ (r-g)B,$$

where we used $B = \sum_i p_i K_i$ from (6). This is exactly the budget set in (5). Since (5) holds on the BGP by assumption (with the same U^h and prices), the BGP consumption vector $\{C_i\}$ is optimal in the static economy as well.

Combining these points, the BGP prices $\{p_i, w_f\}$ and quantities $\{Y_i, C_i, K_{ij}, L_{if}, Y_{ij}\}$ constitute a competitive equilibrium of the static economy with (ii) $K_i = A_{K_i} X_i$ and (iii) markups $\mu_i = (r+\delta_i)/(g+\delta_i)$ and profits rebated to households.

(2) Asset market clearing pins down r**.** On the BGP, (6) gives $B = \sum_i p_i K_i$ and

$$\mathcal{A}^d(r) \ =\ \frac{B}{\sum_{f\in F} w_f L_f} \ =\ \frac{\sum_{i\in N} p_i K_i}{\sum_{f\in F} w_f L_f}.$$

Thus the common return r that supports the BGP also clears the asset market.

A.2 Generalizations to case with uneven productivity growths

Below, we show a generalization of our result to the case with uneven productivity growths and Cobb-Douglas preferences and production functions. For simplicity, we consider the case with a single type of labor.

All prices and returns are expressed in *consumption units*. Let the Cobb–Douglas consumption price index be $P_C(t) = \prod_{i\in N} p_i(t)^{\beta_i}$ and normalize $P_C \equiv 1$; hence

$$\sum_{i\in N} \beta_i \, \pi_i \ =\ 0, \qquad \pi_i \ \equiv\ \frac{d}{dt} \ln p_i(t).$$

Technologies are given by

$$Y_i = A_i L_i^{\nu_i} \Big(\prod_{j\in N} Y_{ij}^{a_{ij}} \Big) \Big(\prod_{j\in N} K_{ij}^{\kappa_{ij}} \Big), \qquad \nu_i + \sum_j a_{ij} + \sum_j \kappa_{ij} = 1. \tag{26}$$

Hicks-neutral productivities grow at constant rates $g_{A_i} \equiv \dot{A}_i / A_i$. Define the *produced-*

input share matrix $M = [M_{ij}]$ by

$$M_{ij} \;\equiv\; a_{ij} + \kappa_{ij}.$$

Effective labor grows at exogenous rates g, and we write $w(t)$ be the wage/rental per efficiency unit in consumption units, and define its drift

$$g_w \;\equiv\; \frac{d}{dt} \ln w_f(t).$$

The user cost of capital goods (derived below) is

$$R_j \;=\; \big(r + \delta_j - \pi_j\big)\, p_j, \tag{27}$$

where r is the real interest rate expressed in terms of consumption goods. We also write $g_{K_j} \equiv \dot{K}_j / K_j$ for the growth rate of capital good K_j.

Lemma 1 (Relative price drifts)**.** *Cost minimization for the Cobb–Douglas technologies* (26) *implies the drift system*

$$(I - M)\,\pi \;=\; -\,g_A \;+\; \nu g_w, \qquad \pi \equiv (\pi_i)_{i \in N}, \quad g_A \equiv (g_{A_i})_{i \in N}, \quad \nu \equiv (\nu_i)_{i \in N}. \tag{28}$$

Because each row sum of M equals $1 - \nu_i \;<\; 1$, $\rho(M) \;<\; 1$ and $I - M$ is invertible, which implies a unique π for each g_w. The numeraire normalization $\sum_i \beta_i \pi_i = 0$ selects $g_w = \beta'(I - M)^{-1} g_A / \beta'(I - M)^{-1} \nu$.

Proof. The unit cost for sector i is $p_i \;\propto\; A_i^{-1}\big(\prod_f w_f^{\nu_{if}}\big)\big(\prod_j p_j^{a_{ij}}\big)\big(\prod_j R_j^{\kappa_{ij}}\big)$. Taking time derivatives and using $d \ln R_j / dt \;=\; d \ln p_j / dt \;=\; \pi_j$ (since r, δ_j, π_j are constant along the path) yields $\pi_i = -g_{A_i} + \nu_i g_w + \sum_j (a_{ij} + \kappa_{ij}) \pi_j$, i.e., (28). □

Input demands and growth rates (unit elasticity) Cobb–Douglas implies constant cost shares:

$$w L_i = \nu_i p_i Y_i, \qquad p_j Y_{ij} = a_{ij} p_i Y_i, \qquad R_j K_{ij} = \kappa_{ij} p_i Y_i. \tag{29}$$

Hence physical quantities satisfy

$$L_i = \nu_i \frac{p_i}{w} Y_i, \quad Y_{ij} = a_{ij} \frac{p_i}{p_j} Y_i, \quad K_{ij} = \kappa_{ij} \frac{p_i}{R_j} Y_i = \frac{\kappa_{ij}}{r + \delta_j - \pi_j} \frac{p_i}{p_j} Y_i. \tag{30}$$

Therefore their growth rates are

$$g_{L_i} = g_{Y_i} + \pi_i - g_w, \qquad g_{Y_{ij}} = g_{Y_i} + \pi_i - \pi_j, \qquad g_{K_{ij}} = g_{Y_i} + \pi_i - \pi_j. \tag{31}$$

Since $g_{L_i} = 0$ for all i, we have $g_{Y_i} + \pi_i = g_w$ for all i, which implies

$$g_{Y_j} = g_{Y_{ij}} = g_w - \pi_j \qquad g_{K_j} = g_{K_{ij}} = g_w - \pi_j.$$

Isomorphism

Proposition 9 (Equivalence under unbalanced growth and multiple primary factors)**.** *Consider any (generalized) balanced-growth path with constant drifts $(\pi_i)_i$, g_w, constant r, and constant δ_j. Let the price vector be $\{p_i, w\}$ in consumption units, and the allocation be*

$$\{Y_i, C_i, X_i, K_i, \{K_{ij}\}_j, L_i, \{Y_{ij}\}_j\}_{i \in N}$$

Define, for each capital type j,

$$A_{K_j} \equiv \frac{1}{g_{K_j} + \delta_j}, \qquad \mu_j \equiv \frac{R_j K_j}{p_j X_j} = \frac{r + \delta_j - \pi_j}{g_{K_j} + \delta_j}.$$

Then:

(i) ***Static production side.*** *The same prices $\{p_i, w_f\}$ and quantities $\{Y_i, C_i, K_{ij}, L_{if}, Y_{ij}\}$ constitute a competitive equilibrium of a static economy with the* same *technologies and preferences, except that each capital good j is produced from the investment good X_j by the linear technology*

$$K_j = A_{K_j} X_j = \frac{X_j}{g_{K_j} + \delta_j},$$

and is sold at an as-if *markup μ_j over its flow cost. Profits of the capital-good sectors equal*

$$\Pi_j = R_j K_j - p_j X_j = (r - g_{K_j} - \pi_j)\, p_j K_j,$$

and are rebated to households.

(ii) ***Household budget and asset market.*** *Let $B \equiv \sum_j p_j K_j$ and define the capital-gain-adjusted aggregates*

$$\bar{g}_K \equiv \sum_j \omega_j^K g_{K_j}, \quad \bar{\pi}_K \equiv \sum_j \omega_j^K \pi_j, \quad \omega_j^K \equiv \frac{p_j K_j}{B}.$$

Then along the path $\dot{B}/B = \bar{g}_K + \bar{\pi}_K$, and the integrated household budget (in consumption *units) is*

$$\sum_i p_i C_i \;\leq\; wL \;+\; rB - \dot{B} \;=\; \sum_f w_f L_f \;+\; \left(r - \bar{g}_K - \bar{\pi}_K\right) B.$$

If asset demand is summarized by a correspondence $\mathcal{A}^d(r)$ as in the baseline, the return r is pinned by $\mathcal{A}^d(r) = B/(wL)$.

A.3 Non-Physical Capital

To illustrate how the results can be extended to a model with non-physical capital, we analyze a model where firm entry costs operate as a form of capital. The model features one with a single homogeneous output good and a continuum of perfectly competitive firms. Firms with productivity z have production function $y(z) = z\ell^{\eta}$, where $\eta \in (0,1)$ captures diminishing returns and l is the number of workers hired for production. Firms produce a homogeneous good sold at price p. To start a firm, entrepreneurs pay a sunk entry cost, w/z_e, where z_e is the productivity of the entry technology. Firms exit at an exogenous rate δ and the discount rate is r. . We write ℓ, y, π for labor, output, and profit per firm, M for the steady-state measure of firms, and $Y \equiv My$, $M\ell = L_Y$, $M\delta/z_e = L_E$ for steady-state output, production labor, and entry labor. Aggregate production satisfies

$$Y = zM^{1-\eta}L_Y^{\eta} \equiv zF[M, L_Y],$$

with wages and per-firm per-period profits satisfying

$$w = zF_{L_Y}$$
$$\pi = zF_M.$$

Free entry implies that

$$w/z_e = \int_0^{\infty} e^{-(r+\delta)t}\pi = \frac{\pi}{r+\delta}.$$

Furthermore, the steady-state measure of firms need to satisfy

$$M\delta = z_e L_e,$$

where L_e is the amount of labor allocated to entry. This can be used to obtain the following set of steady-state equations

$$Y = zF[M, L_Y],$$

$$\begin{aligned} Y, M, L_Y &\in \arg\max Y - wL_Y - \pi M, \\ M\pi = L_e w \frac{r+\delta}{\delta} &\iff M\pi \left(1 - \frac{r}{r+\delta}\right) = L_e w \\ C &= wL + M\pi - L_e w \\ L_e + L_y &\leq L. \end{aligned}$$

We see that the equations are precisely as though there is an investment flow of value $L_e w$, a capital stock of value $\frac{L_e w}{\delta}$ rented out at $r + \delta$. We obtain the following proposition.

Proposition 10 (Isomorphism with firm entry and exit)**.** *Steady-state prices and quantities in the economy described above form an equilibrium of an equivalent static economy where the production functions of goods are the same as in the dynamic economy; entry costs in the static model are* $c_e^{static} = \delta / z_e$*; profits are taxed at rate* $\tau = \frac{r}{r+\delta}$ *with tax revenues distributed to households.*

A.4 Proof of Proposition 3

Fix a BGP of an economy satisfying (10)-(15) with growth rate g, prices $\{p_i, w_f\}$, wedges $\{\tau_i\}$, returns $\boldsymbol{r} = \{r_i\}_{i \in N}$, allocations

$$\left\{Y_i,\ X_i,\ K_i,\ \{K_{ij}\}_{j\in N},\ \{Y_{ij}\}_{j\in N},\ \{L_{if}\}_{f\in F},\ \{C_i^h\}_{h\in H}\right\}_{i\in N},$$

and household portfolios $\{B_i^h\}_{i\in N, h\in H}$ and wedge rebates $\{T^h\}_{h\in H}$. Define, for each i,

$$R_i \equiv (r_i + \delta_i) p_i, \qquad A_{K_i} \equiv \frac{1}{g + \delta_i}, \qquad \mu_i \equiv \frac{r_i + \delta_i}{g + \delta_i}.$$

(1) BGP ⇒ static equilibrium with linear capital technology, taxes, and markups. *Firms.* On the BGP, each i-producer solves

$$\max_{\{Y_i, L_{if}, Y_{ij}, K_{ij}\}} (1 - \tau_i) p_i Y_i - \sum_{f\in F} w_f L_{if} - \sum_{j\in N} p_j Y_{ij} - \sum_{j\in N} R_j K_{ij},$$

with technology (10) and $R_j = (r_j + \delta_j) p_j$. Since $R_j = \mu_j p_j / A_{K_j}$, this is exactly the firm problem in the static economy with the same $\{p_i, w_f, \tau_i\}$ and where capital goods are produced from X_i with productivity A_{K_i} and sold at a markup μ_i. Hence the BGP choices $\{Y_i, L_{if}, Y_{ij}, K_{ij}\}$ satisfy static profit maximization.

Resource feasibility and linear capital technology. The BGP resource constraints give

$$Y_i = \sum_{h\in H} C_i^h + X_i + \sum_{j\in N} Y_{ji}, \qquad X_i = (g+\delta_i)K_i, \qquad \sum_{i\in N} L_{if} \le \sum_{h\in H} L_f^h, \qquad \sum_{j\in N} K_{ij} \le K_i.$$

The second identity is equivalent to $K_i = A_{K_i} X_i$ with $A_{K_i} = 1/(g+\delta_i)$, i.e., the linear capital technology in the static economy.

Capital-good "as-if" markups and profits. The capital-good sector's profit is

$$\Pi_i \equiv R_i K_i - p_i X_i = p_i(r_i+\delta_i)K_i - p_i(g+\delta_i)K_i = (r_i - g)\, p_i K_i = \Big(1 - \frac{1}{\mu_i}\Big) R_i K_i.$$

Taxes and wedge rebates. For non-capital goods, CRS and price-taking imply zero pure profits net of wedges, so wedge revenue equals $\tau_i p_i Y_i$ per good. By assumption, $\{N_h\}_{h\in H}$ partitions N and $T^h = \sum_{i\in N_h} \tau_i p_i Y_i$, so aggregate wedge rebates match aggregate wedge payments.

Households and budgets. In the static economy, let aggregate (as-if) capital profits $\sum_i \Pi_i = \sum_i (r_i - g) p_i K_i$ be rebated across households with shares $\{\pi^h\}$. Household h's static budget set is then

$$\sum_{i\in N} p_i C_i^h \;\le\; \underbrace{\sum_{f\in F} w_f L_f^h}_{\text{labor income}} + \underbrace{T^h}_{\text{wedge rebate}} + \underbrace{\pi^h \sum_{i\in N} (r_i - g) p_i K_i}_{\text{profit rebate}}\,.$$

With the choice

$$\pi^h \equiv \frac{\sum_i (r_i - g) B_i^h}{\sum_{h'} \sum_i (r_i - g) B_i^{h'}}\,,$$

the profit rebate equals $\sum_i (r_i - g) B_i^h$; hence the static budget coincides exactly with the BGP budget

$$\sum_i p_i C_i^h \;\le\; \sum_f w_f L_f^h + \sum_i (r_i - g) B_i^h + T^h.$$

Since preferences U^h and prices are the same, the BGP consumption vector $\{C_i^h\}$ is optimal in the static economy. Market clearing conditions carry over from the BGP. Therefore, the BGP prices, wages, quantities, and wedge payments constitute a competitive equilibrium of the static economy with (i) the same technologies, wedges, and preferences, (ii) $K_i = A_{K_i} X_i$, (iii) the same sales wedges on non-capital goods with rebates T^h, and (iv) capital-good markups $\mu_i = (r_i+\delta_i)/(g+\delta_i)$ with profits rebated by $\{\pi^h\}$.

(2) Auxiliary conditions for $\boldsymbol{r}$ and $\boldsymbol{\pi}$. On the BGP, asset market clearing $\sum_{h\in H} B_i^h = p_i K_i$ holds asset-by-asset, and by definition of the asset-demand correspondences,

$$B_i^h \;=\; \Big(\sum_{f\in F} w_f L_f^h + T^h\Big)\,\mathcal{A}_i^{d,h}(\boldsymbol{r}).$$

In the static economy constructed above, define

$$\mathcal{L}_h^{static} \equiv \sum_{f\in F} w_f L_f^h + T^h, \qquad \mathcal{K}_i^{static} \equiv p_i K_i.$$

Then asset-by-asset market clearing rewrites as

$$\sum_{h\in H} \mathcal{L}_h^{static}\,\mathcal{A}_i^{d,h}(\boldsymbol{r}) = \mathcal{K}_i^{static}, \qquad \forall i \in N,$$

which is the first equation in part (2). Moreover,

$$\pi^h = \frac{\sum_i (r_i - g) B_i^h}{\sum_{h'} \sum_i (r_i - g) B_i^{h'}} = \frac{\mathcal{L}_h^{static} \sum_i (r_i - g)\mathcal{A}_i^{d,h}(\boldsymbol{r})}{\sum_{h'} \mathcal{L}_{h'}^{static} \sum_i (r_i - g)\mathcal{A}_i^{d,h'}(\boldsymbol{r})},$$

which is the second equation in part (2). This completes the proof. □

B Appendix to Section 3

B.1 Proof of Proposition 4

The comparative statics of productivity change are given by $\frac{d}{dA_i} X^{static}(A, \mu, \pi(A))$. This distorted static economy belongs to the class studied in Baqaee and Farhi (2024). Applying their Theorem 2 yields our proposition.

B.2 Proof of Proposition 5

Multiply the resource constraint by p_i, sum over i, and totally differentiate to obtain

$$\sum_i p_i\, dY_i \;=\; \underbrace{\sum_{h,i} p_i\, dC_{hi}}_{dC} \;+\; \sum_i p_i\, dX_i \;+\; \sum_{i,j} p_i\, dY_{ji}. \tag{32}$$

Totally differentiating firm output using the first-order conditions for inputs and summing over i gives us:

$$\sum_i p_i\, dY_i \;=\; \sum_i p_i Y_i\, d\log A_i \;+\; \sum_f w_f\, dL_f \;+\; \sum_{i,j} p_j\, dM_{\bullet j,i} \;+\; \sum_{j\in\mathcal{K}} R_j\, dK_j, \tag{33}$$

where $dL_f \equiv \sum_i dL_{if}$ and $dK_j \equiv \sum_i dK_{ij}$.

Intermediates and non-accumulable factors cancel in aggregate. Subtract (32) from (33). The intermediate sums cancel by relabeling $i \leftrightarrow j$: $\sum_{i,j} p_j\, dM_{ij} = \sum_{i,j} p_i\, dM_{ji}$. For non-accumulable primary factors $f \notin \mathcal{K}$, aggregate supplies are fixed, so $dL_f = 0$. Hence,

$$dC \;=\; \sum_i p_i Y_i\, d\log A_i \;+\; \sum_{j\in\mathcal{K}} \big(R_j\, dK_j - p_j\, dX_j\big). \tag{34}$$

Balanced growth for investment. On the BGP, $X_j = (g+\delta_j)K_j$ so $dX_j = (g+\delta_j)\, dK_j$. Substitute into (34):

$$dC \;=\; \sum_i p_i Y_i\, d\log A_i \;+\; \sum_{j\in\mathcal{K}} \big(R_j - (g+\delta_j)p_j\big)\, dK_j \;=\; \sum_i p_i Y_i\, d\log A_i \;+\; \sum_{j\in\mathcal{K}} \big(R_j K_j - p_j X_j\big)\, d\log K_j,$$

where the last equality uses $R_j = (r+\delta_j)p_j$ and $X_j = (g+\delta_j)K_j$.

Divide by total consumption expenditure. Let $C = \sum_{h,j} p_j C_{hj}$ and $\lambda_i = p_i Y_i / C$. Dividing by C yields the Divisia change in aggregate consumption:

$$d\log C \;=\; \sum_i \lambda_i\, d\log A_i \;+\; \sum_{j\in\mathcal{K}} \frac{R_j K_j - p_j X_j}{C}\, d\log K_j.$$

Taking the partial derivative w.r.t. $\log A_i$ gives

$$\frac{\partial \log C}{\partial \log A_i} \;=\; \lambda_i \;+\; \sum_{j\in\mathcal{K}} \frac{R_j K_j - p_j X_j}{C}\, \frac{\partial \log K_j}{\partial \log A_i},$$

which proves the claim. Under Cobb–Douglas, Proposition 4 implies $\partial \log C/\partial \log A_i = \tilde{\lambda}_i$, yielding the stated corollary. Since $R_j K_j - p_j X_j = (r-g)p_j K_j$, the "capital amplification" term is positive whenever the shock raises capital stocks ($\partial \log K_j/\partial \log A_i > 0$) and the economy is below the Golden Rule ($r > g$).

B.3 Empirical cost and revenue weights

We use the World Input Output Database augmented with investment flow data from Ding (2022) to calibrate the cost-share matrix $\tilde{\Omega}$, the capital wedges μ, and thus the revenue-share matrix Ω.

We calibrate $\tilde{\Omega}$ using two primary data sources: the World Input Output Database (WIOD) (and the associated Socio-Economic Accounts) for consumption spending, intermediate input use, and labor inputs, and investment flow data from Ding (2022) for investment spending. To ensure consistency, we aggregate WIOD sectors to match the sectoral classification in Ding (2022), yielding 27 sectors.

The matrix $\tilde{\Omega}$ consists of submatrices giving the cost shares for consumption goods $\tilde{\Omega}_{C,N}$ in terms of perishable goods from different countries, cost shares of perishable goods $\tilde{\Omega}_{C,N}, \tilde{\Omega}_{C,K}, \tilde{\Omega}_{C,F}$ in terms of intermediate inputs, capital goods, and labor inputs, and cost shares of investment goods $\tilde{\Omega}_{K,N}$. All other submatrices are zero.

Consumption shares $\tilde{\Omega}_{H,N}$. Since households only consume perishable goods, the consumption share submatrix $\tilde{\Omega}_{H,N}$ has dimensions $H \times N$, with each row h containing the shares of country h's consumption spending across all country-industry pairs (h', j') that produce perishable goods. The elements of this matrix are given by

$$\tilde{\Omega}_{h,h'j'} = \begin{cases} \frac{X^H_{h,h'j'}}{X^H_h} & \text{if } j' \in N \\ 0 & \text{otherwise} \end{cases}$$

where $X^H_{h,h'j'}$ is the dollar value spent by country h on consumption goods from country h', industry j', and $X^H_h = \sum_{h'j'} X^H_{h,h'j'}$ is aggregate consumption in country h. We define consumption as the sum of household, government, and non-profit consumption.

Intermediate input shares $\tilde{\Omega}_{N,N}$. For the submatrix $\tilde{\Omega}_{N,N}$ giving intermediate input shares of perishable goods on other perishable goods (h, i), we have

$$\tilde{\Omega}_{hi,h'j'} = \frac{X_{hi,h'j'}}{GO_{hi}} \quad h' \in H \quad j' \in N$$

The input coefficients on origin-industry pairs is the intermediate input spending taken from the WIOD on that pair, divided by gross output.

Capital cost shares $\tilde{\Omega}_{N,K}$. For capital cost shares $\tilde{\Omega}_{N,K}$, we have

$$\tilde{\Omega}_{hi,h'k'} = \begin{cases} \frac{GOS_{hi}}{GO_{hi}} & \text{if } h' = h \text{ and } k' = \hat{k}(i) \\ 0 & \text{otherwise} \end{cases}$$

where $\hat{k}(i)$ denotes the capital good associated with industry i, and $GOS_{hi} \equiv GO_{hi} - \sum_{h' \in H, j' \in N} X_{hi,h'j'} - \sum_{f \in F} X^L_{hi,f}$ is the gross operating surplus. This expression states that the capital cost share for (h, i) is only positive for the capital good associated with that industry, which is $(h, k(i))$. For this good, the cost share is the ratio of gross operating surplus of the industry relative to its gross output. This assumption captures that we assume no pure profits beyond capital rents, which means that the full operating surplus reflects rental payments on capital.

Labor cost shares $\tilde{\Omega}_{N,F}$. For labor, we have

$$\tilde{\Omega}_{hi,h'f} = \begin{cases} \frac{X^L_{hi,f}}{GO_{hi}} & h' = h, f \in F^h, \\ 0 & \text{Otherwise.} \end{cases}$$

The condition $h' = h, f \in F^h$ captures that countries only use labor inputs from their own country. For these labor inputs, the share are given by labor compensation of (h, i) on labor type f taken from the Socio-Economic Accounts, with labor types being low-skilled, medium-skilled, and high-skilled. For these, cost shares are defined relative to gross output.

Investment cost shares $\tilde{\Omega}_{K,N}$. The submatrix $\tilde{\Omega}_{K,N}$ gives the cost shares of different investment goods (h, k) on different inputs (h', i').

$$\tilde{\Omega}_{hk,h'i'} = \frac{X^{Inv}_{h\hat{i}(k),h'i'}}{X^{Inv}_{c\hat{i}(k)}},$$

where $\hat{i}(k)$ is the regular good associated with investment good k, $X^{Inv}_{h\hat{i}(k),h'i'}$ is the investment spending of industry $(h, \hat{i}(k))$ on (h', i') in the data of Ding (2022), and $X^{Inv}_{h\hat{i}(k)} = \sum_{h'i'} X^{Inv}_{h\hat{i}(k),h'i'}$ is the total investment of $(h, \hat{i}(k))$ in the database.

Aggregating $\tilde{\Omega}$ over time. The WIOD data is annual while the data in Ding (2022) is only from 1997. For all submatrices but $\tilde{\Omega}_{K,N}$, we take averages from 1995 to 2009. For $\tilde{\Omega}_{K,N}$, we take the 1997 values.

Calibrating μ. For each industry-country pair, we calculate $\mu_{h,\hat{k}(i)}$ as the ratio of gross fixed capital formation to gross output in industry (h,i), averaged over 1995-2009.

Consumption share by country Φ. The final component is the vector Φ, where each element Φ_h represents country h's share of total world spending. We set this to be consistent with observed net foreign asset (NFA) positions and the factor incomes implied by our input-output matrices. The specific procedure is detailed in Appendix D alongside the full quantitative model. While one could calculate these shares directly from final expenditure data in the WIOD, the resulting figures are not fully consistent with the balanced growth path conditions required by the structural model introduced later, since current net exports are not consistent with stable ratios of NFAs to world consumption. Therefore, to ensure consistency between the data presented here and the eventual model calibration, we use this inferred measure. The resulting shares are quantitatively very similar to those computed directly from the raw data.

B.4 Details on the Neoclassical Growth Model Example

Setup. The production and accumulation blocks of the economy are given by

$$Y = A_Y F[K, A_L L], \quad C = A_C Y_C, \quad X = A_X Y_X, \quad Y = Y_C + Y_X, \quad \dot{K} = -\delta K + X.$$

Final output is produced using capital and labor, which is used to produce consumption and investment goods. The terms A_Y and A_L are Hicks-neutral and labor-augmenting technology terms, with A_L assumed to assume grow at a constant rate g.[22] The productivity of consumption and investment goods is given by the productivity terms A_C and A_X. With a representative household, the long-run capital supply function is infinitely elastic, with long-run equilibrium in capital markets requiring that

$$r = \rho + g/\gamma + \tau^k,$$

where ρ is the discount rate, γ is the intertemporal elasticity of substitution, and τ^k is a wealth tax on capital.

Representation. Proposition 1 applies to this economy, so its balanced growth path is an equilibrium of a distorted static economy, which can be represented using the notation in Section 3.1. Writing $\alpha = \frac{\partial \log F}{\partial \log K}$ for the output elasticity with respect to capital, and order-

[22]For simplicity, we assume there is no population growth, but none of the arguments hinge on this assumption.

ing commodities as consumption, output, capital, and labor, we obtain the following:

$$\Phi' = (1 \quad 0 \quad 0 \quad 0), \tag{35}$$

$$\Omega = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 1-\alpha \\ 0 & \left(\frac{r+\delta}{g+\delta}\right)^{-1} (\equiv \mu^{-1}) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \tilde{\Omega} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 1-\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{36}$$

$$\Psi = \begin{bmatrix} 1 & \frac{1}{1-\alpha/\mu} & \frac{\alpha}{1-\alpha/\mu} & \frac{1-\alpha}{1-\alpha/\mu} \\ 0 & \frac{1}{1-\alpha/\mu} & \frac{\alpha}{1-\alpha/\mu} & \frac{1-\alpha}{1-\alpha/\mu} \\ 0 & \frac{1/\mu}{1-\alpha/\mu} & \frac{1}{1-\alpha/\mu} & \frac{(1-\alpha)/\mu}{1-\alpha/\mu} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \tilde{\Psi} = \begin{bmatrix} 1 & \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 1 \\ 0 & \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 1 \\ 0 & \frac{1}{1-\alpha} & \frac{1}{1-\alpha} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{37}$$

$$\lambda' = \Phi'\Psi = \left(1 \quad \frac{1}{1-\alpha/\mu} \quad \frac{\alpha}{1-\alpha/\mu} \quad \frac{1-\alpha}{1-\alpha/\mu} \right), \quad \tilde{\lambda}' = \Phi'\Psi = \left(1 \quad \frac{1}{1-\alpha} \quad \frac{\alpha}{1-\alpha} \quad 1 \right). \tag{38}$$

The vector Φ represents the share of final expenditure on different commodities, with all weight on the consumption good. The revenue-based and cost-based input-output matrices Ω and $\tilde{\Omega}$ have identical first two rows, reflecting that there are no markups in the consumption and raw output sectors. The first row shows that 100% of consumption good costs come from Y, while Y's cost shares are α for capital and $1-\alpha$ for labor. In $\tilde{\Omega}$, the third row indicates that all costs for producing capital goods come from sector Y, while the third row of Ω shows that only a fraction $\frac{g+\delta}{r+\delta}$ of capital sector rental income goes to the producer of investment goods, with the remainder being net capital income. The difference between $\tilde{\Omega}$ and Ω reflects the as-if markup μ – while raw inputs constitute 100% of investment costs, they do not account for 100% of rental revenue. The final row in both matrices represents labor, which requires no inputs.

The revenue-based Leontief inverse, Ψ, captures how much each dollar spent on that row's commodity contributes to the revenues of other commodities, while each row of the cost-based $\tilde{\Psi}$ captures how much of the costs of that row's commodity are accounted for by different inputs. Note that the term $1-\alpha/\mu$ that consistently shows up in Ψ represents the consumption share of GDP, since $\alpha/\mu = \frac{\alpha}{r+\delta}(g+\delta)$ is the investment share of GDP. Since all final demand is in the consumption good sector, our primary interest is in the first rows of each matrix, which equal the Domar weights λ and $\tilde{\lambda}$.

The revenue-based Domar weights λ are the sales of different sectors relative to final consumption. The first element shows that consumption sales are, unsurprisingly, 100% of final consumption. Since consumption-to-GDP is $1 - \alpha/\mu$, the subsequent three elements follow from dividing GDP, capital income, and labor income by total consumption. Moreover, when as-if markups are positive, the cost-based Domar weights $\tilde{\lambda}$ are weakly larger than revenue-based Domar weights. Consider, for example, the last element in λ and $\tilde{\lambda}$, which represents the Domar weight of labor. While the cost-based weight of labor is 1 (consistent with all costs scaling linearly with the price of labor), its revenue-based weight is less than one if $\mu > 1$, since consumption in this case is partly financed by net capital income (i.e., profits in the equivalent static economy).

C Appendix to Section 4

Proof of Proposition 6

Throughout, interpret bold variables (e.g. $\boldsymbol{r}$, $\boldsymbol{\delta}$, $\boldsymbol{\tau}$, $\boldsymbol{\mu}$) elementwise and let "$\cdot$" denote inner products over assets; all derivatives are taken at the reference BGP. By Proposition 3, any BGP $(\boldsymbol{A}, \boldsymbol{\tau})$ is represented by the (as-if) static equilibrium mapping

$$X^{BGP} = X^{static}(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi}), \qquad \boldsymbol{\mu} = \frac{\boldsymbol{r} + \boldsymbol{\delta}}{g + \boldsymbol{\delta}} \frac{1}{1 - \boldsymbol{\tau}},$$

with $\boldsymbol{r}$ and $\boldsymbol{\pi}$ determined by the auxiliary conditions

$$\sum_{h \in H} \mathcal{L}_h^{static}(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi}) \, \mathcal{A}^{d,h}(\boldsymbol{r}) = \mathcal{K}^{static}(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi}), \tag{39}$$

$$\pi^h = \frac{\mathcal{L}_h^{static}(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi}) \left[(\boldsymbol{r} - g) \cdot \mathcal{A}^{d,h}(\boldsymbol{r}) \right]}{\sum_{h'} \mathcal{L}_{h'}^{static}(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi}) \left[(\boldsymbol{r} - g) \cdot \mathcal{A}^{d,h'}(\boldsymbol{r}) \right]}. \tag{40}$$

We adopt the conventions from the statement of the proposition: $\delta_i = \infty$ for non-capital goods (so $\mu_i = (1 - \tau_i)^{-1}$ and $\partial \log \mu_i / \partial r_i = 0$) and $\tau_i \equiv d\tau_i \equiv 0$ for capital goods.

Step 1: Total differentiation of the equilibrium mapping. By the chain rule applied to $X^{BGP} = X^{static}(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi})$,

$$dX^{BGP} = \frac{\partial X^{static}}{\partial \boldsymbol{A}} \, d\boldsymbol{A} + \frac{\partial X^{static}}{\partial \boldsymbol{\mu}} \, d\boldsymbol{\mu} + \frac{\partial X^{static}}{\partial \boldsymbol{\pi}} \, d\boldsymbol{\pi},$$

which is the first display in the proposition.

Step 2: Differential of wedges $\boldsymbol{\mu}$. Elementwise,

$$\log \mu_i = \log(r_i + \delta_i) - \log(g + \delta_i) - \log(1 - \tau_i),$$

so

$$\frac{d\mu_i}{\mu_i} = d\log \mu_i = \frac{dr_i}{r_i + \delta_i} + \frac{d\tau_i}{1 - \tau_i}. \qquad \Longrightarrow \qquad \frac{d\boldsymbol{\mu}}{\boldsymbol{\mu}} = \frac{d\boldsymbol{r}}{\boldsymbol{r} + \boldsymbol{\delta}} + \frac{d\boldsymbol{\tau}}{1 - \boldsymbol{\tau}}. \tag{41}$$

This proves (19). (By convention, $dr_i/(r_i + \delta_i) \equiv 0$ for non-capital goods and $d\tau_i/(1 - \tau_i) \equiv 0$ for capital goods.)

Step 3: Solving for $d\boldsymbol{r}$ by implicit differentiation. Define the (vector-valued) market-clearing function

$$\boldsymbol{F}(\boldsymbol{r}; \boldsymbol{A}, \boldsymbol{\tau}, \boldsymbol{\pi}) \equiv \sum_{h \in H} \mathcal{L}_h^{static}(\boldsymbol{A}, \boldsymbol{\mu}(\boldsymbol{r}, \boldsymbol{\tau}), \boldsymbol{\pi})\, \mathcal{A}^{d,h}(\boldsymbol{r}) - \mathcal{K}^{static}(\boldsymbol{A}, \boldsymbol{\mu}(\boldsymbol{r}, \boldsymbol{\tau}), \boldsymbol{\pi}).$$

Equation (39) is $\boldsymbol{F}(\boldsymbol{r}; \boldsymbol{A}, \boldsymbol{\tau}, \boldsymbol{\pi}) = \boldsymbol{0}$. Totally differentiate and group $d\boldsymbol{r}$ terms:

$$\underbrace{\frac{\partial}{\partial \boldsymbol{r}}\left[\sum_h \mathcal{L}_h^{static}\, \mathcal{A}^{d,h}(\boldsymbol{r})\right]}_{\epsilon_{\boldsymbol{r}}^d} d\boldsymbol{r} - \underbrace{\frac{\partial \mathcal{K}^{static}}{\partial \boldsymbol{r}}}_{-\epsilon_{\boldsymbol{r}}^s} d\boldsymbol{r} + \sum_{x \in \{\boldsymbol{A}, \boldsymbol{\tau}, \boldsymbol{\pi}\}} \left[\sum_h \frac{\partial \mathcal{L}_h^{static}}{\partial x}\, \mathcal{A}^{d,h}(\boldsymbol{r}) - \frac{\partial \mathcal{K}^{static}}{\partial x}\right] dx = \boldsymbol{0},$$

where

$$\epsilon_{\boldsymbol{r}}^d \equiv \frac{\partial}{\partial \boldsymbol{r}}\left[\sum_h \mathcal{L}_h^{static}\, \mathcal{A}^{d,h}(\boldsymbol{r})\right], \qquad \epsilon_{\boldsymbol{r}}^s \equiv -\frac{\partial \mathcal{K}^{static}}{\partial \boldsymbol{r}}.$$

Both Jacobians incorporate the dependence of $\mathcal{L}_h^{static}$ and $\mathcal{K}^{static}$ on $\boldsymbol{r}$ through $\boldsymbol{\mu}(\boldsymbol{r}, \boldsymbol{\tau})$ (and, if present, through $\boldsymbol{\pi}$ when treated as a function of $\boldsymbol{r}$).[23] Assuming the matrix $\epsilon_{\boldsymbol{r}}^d + \epsilon_{\boldsymbol{r}}^s$ is nonsingular (local solvability), we solve for $d\boldsymbol{r}$:

$$d\boldsymbol{r} = (\epsilon_{\boldsymbol{r}}^d + \epsilon_{\boldsymbol{r}}^s)^{-1} \sum_{x \in \{\boldsymbol{\tau}, \boldsymbol{A}, \boldsymbol{\pi}\}} \left[\frac{\partial \mathcal{K}^{static}}{\partial x} - \sum_h \frac{\partial \mathcal{L}_h^{static}}{\partial x}\, \mathcal{A}^{d,h}(\boldsymbol{r})\right] dx, \tag{42}$$

which is (20).

Step 4: Differential of profit shares $\boldsymbol{\pi}$. Let

$$S_h \equiv \mathcal{L}_h^{static}\left[(\boldsymbol{r} - g) \cdot \mathcal{A}^{d,h}(\boldsymbol{r})\right], \qquad \Pi \equiv \sum_{h'} S_{h'}, \qquad \pi^h = \frac{S_h}{\Pi}.$$

[23] Formally, these are the Jacobians of the left- and right-hand sides of (39) w.r.t. $\boldsymbol{r}$ holding $(\boldsymbol{A}, \boldsymbol{\tau}, \boldsymbol{\pi})$ fixed.

Then

$$\begin{aligned}\frac{d\pi^h}{\pi^h} =& d\log S_h - \sum_{h'} \frac{S_{h'}}{\Pi}\, d\log S_{h'} \\ =& d\log\Big(\mathcal{L}_h^{static}\, \mathcal{A}^{d,h}(\boldsymbol{r})\cdot(\boldsymbol{r}-g)\Big) - \sum_{h'} \pi^{h'}\, d\log\Big(\mathcal{L}_{h'}^{static}\, \mathcal{A}^{d,h'}(\boldsymbol{r})\cdot(\boldsymbol{r}-g)\Big),\end{aligned}$$

i.e. (21). This completes the proof. □

Proof of Corollary 2

Consider an infinitesimal change in A_i, holding $\boldsymbol{\tau}$ fixed. Aggregate long-run consumption equals the static-equilibrium aggregate consumption with arguments $(\boldsymbol{A}, \boldsymbol{\mu}, \boldsymbol{\pi})$. Aggregate consumption depends on the level of aggregate non-capital income and profits but not on its distribution across households: $\partial \log C^{static}/\partial \boldsymbol{\pi} = \mathbf{0}$. By the chain rule,

$$\frac{\partial \log C}{\partial \log A_i} = \frac{\partial \log C^{static}}{\partial \log A_i} + \frac{\partial \log C^{static}}{\partial \log \boldsymbol{\mu}} \cdot \frac{\partial \log \boldsymbol{\mu}}{\partial \boldsymbol{r}}\, \frac{\partial \boldsymbol{r}}{\partial \log A_i}.$$

From (41) with $d\boldsymbol{\tau} = \mathbf{0}$,

$$\frac{\partial \log \boldsymbol{\mu}}{\partial \boldsymbol{r}} = \frac{1}{\boldsymbol{r}+\boldsymbol{\delta}} \quad \text{(elementwise)}.$$

Combining gives the stated expression:

$$\frac{\partial \log C}{\partial \log A_i} = \frac{\partial \log C^{static}}{\partial \log A_i} + \frac{\partial \log C^{static}}{\partial \log \boldsymbol{\mu}} \cdot \underbrace{\frac{1}{\boldsymbol{r}+\boldsymbol{\delta}}}_{\frac{d\log \boldsymbol{\mu}}{d\boldsymbol{r}}}\, \frac{\partial \boldsymbol{r}}{\partial \log A_i},$$

with $\partial \boldsymbol{r}/\partial \log A_i$ obtained by evaluating (20) at the A_i-shock (i.e. set $dx = d\log A_i$ and others zero).

Proof of Corollary 3

Fix i and consider a change in τ_i (with productivities fixed). As above, aggregate consumption C is independent of $\boldsymbol{\pi}$ holding aggregate income fixed, so

$$\frac{\partial \log C}{\partial \tau_i} = \frac{\partial \log C^{static}}{\partial \log \mu_i}\, \frac{\partial \log \mu_i}{\partial \tau_i} + \frac{\partial \log C^{static}}{\partial \log \boldsymbol{\mu}} \cdot \frac{\partial \log \boldsymbol{\mu}}{\partial \boldsymbol{r}}\, \frac{\partial \boldsymbol{r}}{\partial \tau_i}.$$

By (41), holding $\boldsymbol{r}$ fixed,

$$\frac{\partial \log \mu_i}{\partial \tau_i} = \frac{1}{1-\tau_i}, \qquad \frac{\partial \log \boldsymbol{\mu}}{\partial \boldsymbol{r}} = \frac{1}{\boldsymbol{r}+\boldsymbol{\delta}}.$$

Therefore,

$$\frac{\partial \log C}{\partial \tau_i} = \frac{\partial \log C^{static}}{\partial \log \mu_i} \underbrace{\frac{1}{1-\tau_i}}_{\frac{\partial \log \mu_i}{\partial \tau_i}} + \frac{\partial \log C^{static}}{\partial \log \boldsymbol{\mu}} \cdot \underbrace{\frac{1}{\boldsymbol{r}+\boldsymbol{\delta}}}_{\frac{d \log \boldsymbol{\mu}}{d\boldsymbol{r}}} \frac{\partial \boldsymbol{r}}{\partial \tau_i},$$

with $\partial \boldsymbol{r}/\partial \tau_i$ given by (20) applied to the τ_i-shock (set $dx = d\tau_i$ and others zero).

Remarks. (i) In a one-asset world, $\epsilon^d_{\boldsymbol{r}} + \epsilon^s_{\boldsymbol{r}}$ is a positive scalar under standard assumptions, so shocks to net capital demand shift r in the same direction. (ii) In the neoclassical-growth limit with infinitely elastic asset demand, $\epsilon^d_{\boldsymbol{r}} = \infty$ and $d\boldsymbol{r} = \boldsymbol{0}$; then $d\boldsymbol{\mu} = \boldsymbol{0}$ and $dX^{BGP} = (\partial X^{static}/\partial \boldsymbol{A})\, d\boldsymbol{A}$, recovering the results from the baseline section.

C.1 Proof of Proposition 7

Define permanent domestic product (PDP) along the balanced growth path to be GDP minus investment:

$$PDP_h = \underbrace{\sum_{i \in N_h} p_i Y_i - \sum_{i \in N_h} \sum_{j \in N} p_j Y_{ij}}_{\text{GDP of country } h} - \underbrace{\sum_{i \in K_h} p_i X_i}_{\text{investment of country } h}$$

Define net permanent output of each good $i \in N$ produced by country h to be

$$q_{ci} = Y_i \mathbf{1}\left[i \in \mathcal{N}_h\right] - \sum_{j \in \mathcal{N}_h} Y_{ji} - X_i$$

Note that

$$PDP_h = \sum_{i \in \mathcal{N}} p_i q_{ci}.$$

Define growth in real PDP, denoted by $d \log Y_h$, to be

$$d \log Y_h = \sum_{i \in \mathcal{N}} \frac{p_i}{PDP_h} dq_{ci}.$$

Similarly, define the implicit PDP deflator $d\log P_h^Y$ to be

$$d\log Y_h = \sum_{i\in\mathcal{N}} \frac{q_i}{PDP_h} dp_i.$$

Define net exports of each good i to be

$$NX_{ci} = p_i\left[\sum_{j\notin N_h} Y_{ji} + \sum_{h'\neq h} C_{h'i} + \sum_{j\notin K_h} Y_{ji}\right] - p_i\left[\sum_{j\in N_h} Y_{ji} + C_{ci} + \sum_{j\in K_h} Y_{ji}\right].$$

Define

$$NX_h = \sum_i NX_{ci}.$$

Note that

$$PDP_h = P_h C_h + NX_h.$$

Hence,

$$\begin{aligned}
d\log\left[P_h C_h\right] &= \frac{PDP_h}{P_h C_h} d\log\left[PDP_h\right] - \frac{1}{P_h C_h} d\left[NX_h\right] \\
d\log C_h &= \frac{PDP_h}{P_h C_h} d\log Y_h + \frac{PDP_h}{P_h C_h} d\log P^Y - d\log P_h - \frac{1}{P_h C_h} d\left[NX_h\right]
\end{aligned}$$

Writing this out gives

$$\begin{aligned}
d\log C_h &= \frac{PDP_h}{P_h C_h} d\log Y_h + \frac{PDP_h}{P_h C_h} \sum_{i\in\mathcal{N}} \frac{\left[q_i - c_{ci}\right]}{PDP_h} dp_i - \frac{1}{P_h C_h} d\left[NX_h\right] \\
&= \frac{PDP_h}{P_h C_h} d\log Y_h + \sum_{i\in\mathcal{N}} \frac{\left[p_i q_i - p_i c_{ci}\right]}{P_h C_h} d\log p_i - \frac{1}{P_h C_h} d\left[NX_h\right] \\
&= \frac{PDP_h}{P_h C_h} d\log Y_h + \sum_{i\in\mathcal{N}} \frac{NX_{ci}}{P_h C_h} d\log p_i - \frac{1}{P_h C_h} d\left[NX_h\right].
\end{aligned}$$

The result follows if we can show that

$$\frac{PDP_h}{P_h C_h} d\log Y_h = \sum_{i\in K_h} \frac{R_i K_i}{P_h C_h}\left[\frac{r_i - g}{r_i + \delta}\right] d\log K_i.$$

From the resource constraints:

$$\frac{dq_{ci}}{q_{ci}} = \frac{Y_i}{q_{ci}} d\log Y_i \mathbf{1}\left[i\in N_h\right] - \sum_{j\in N_h} \frac{Y_{ji}}{q_{ci}} d\log Y_{ji} - \frac{X_i}{q_{ci}} d\log X_i,$$

and from technologies (using that $\tilde{\Omega}_{n,\cdot} = \Omega_{n,\cdot}$ for $n \in N$ since the only initial wedge is on capital):

$$(d\log Y_j - \sum_{f\in F_h} \Omega_{jf} d\log L_{jf} - \Omega_{j\hat{k}(j)} d\log K_j) = \sum_{i\in N} \Omega_{ji} d\log Y_{ji},$$

where $\hat{k}(j)$ is the capital good associated with j. Last, along the balanced growth path, we have:

$$d\log K_j = d\log X_j.$$

Hence, unpacking the definition of $d\log Y$ and substituting, we get:

$$\begin{aligned}
d\log Y_h &= \sum_{i\in N} \frac{p_i}{PDP_h}\left[dq_{ci}\right] = \sum_{i\in N} \frac{p_i}{PDP_h}\left[dY_i \mathbf{1}\left[i\in\mathcal{N}_h\right] - \sum_{j\in\mathcal{N}_h} dY_{ji} - dX_i\right] \\
&= \left[\sum_{i\in N_h} \frac{p_i Y_i}{PDP_h} d\log Y_i - \sum_{i\in N} \frac{p_i}{PDP_h} \sum_{j\in\mathcal{N}_h} dY_{ji} - \sum_{i\in N_h} \frac{p_i X_i}{PDP_h} d\log X_i\right] \\
&= \left[\sum_{i\in N_h} \frac{p_i Y_i}{PDP_h} d\log Y_i - \sum_{i\in\mathcal{N}_h}\sum_{j\in N} \frac{p_j Y_{ij}}{PDP_h} d\log Y_{ij} - \sum_{i\in N_h} \frac{p_i X_i}{PDP_h} d\log X_i\right] \\
&= \left[\sum_{i\in N_h} \frac{p_i Y_i}{PDP_h} d\log Y_i - \sum_{i\in\mathcal{N}_h} \frac{p_i Y_i}{PDP_h} \sum_{j\in N} \Omega_{ij} d\log Y_{ij} - \sum_{i\in N_h} \frac{p_i X_i}{PDP_h} d\log K_i\right] \\
&= \left[\sum_{i\in N_h} \frac{p_i Y_i}{PDP_h} d\log Y_i - \sum_{i\in\mathcal{N}_h} \frac{p_i Y_i}{PDP_h}\left[d\log Y_i - \sum_{f\in F_h} \Omega_{if} d\log L_{if} - \Omega_{i\hat{k}(i)} d\log K_i\right]\right. \\
&\left. - \sum_{i\in N_h} \frac{p_i X_i}{PDP_h} d\log K_i\right] \\
&= \left[\sum_{i\in\mathcal{N}_h} \frac{p_i Y_i}{PDP_h}\left[\sum_{f\in F_h} \Omega_{jf} d\log L_{jf} + \Omega_{i\hat{k}(i)} d\log K_i\right] - \sum_{i\in N_h} \frac{p_i X_i}{PDP_h} d\log K_i\right] \\
&= \left[\sum_{i\in K_h} \frac{R_i K_i - p_i X_i}{PDP_h} d\log K_i\right] \\
&= \left(\frac{PDP_h}{P_h C_h}\right)^{-1}\left[\sum_{i\in K_h} \frac{R_i K_i}{P_h C_h}\left[\frac{r_i - g}{r_i + \delta}\right] d\log K_i\right]
\end{aligned}$$

which completes the proof.

D Appendix to Section 5

D.1 Model characterization

Here we present the full model without imposing the assumption of a balanced growth path, and also include the explicit aggregation decisions that link household decisions to aggregates. For generality, we also include a capital tax τ^h that is not part of the model in the main paper.

Indexing. We write H, N, K, and F for the set of countries, types of perishable industries, types of capital goods, and factors. There is one consumption industry per country. The full set of industries consists of a collection $\{(h,i) : h \in H, i \in N\}$ giving pairs of countries and industry types (we use the index (h,h) with $h \in H$ to index quantities associated with h's consumption good). There is one capital good (h,k) associated with each country-industry pair (h,i). We write $\hat{k}(i)$ for the element in K associated with $i \in N$. We also write F^h for the set of factors located in country h.

Production and profit maximization. The production functions for consumption goods, other perishable goods, and investment goods are given by

$$C_h(t) = A_{hh} \prod_{j \in N} Y_{hh,j}(t)^{\tilde{\Omega}_{hh,j}} \quad \forall h \in H \tag{43}$$

$$Y_{hi}(t) = A_{hi} \prod_{j \in N} Y_{hi,j}(t)^{\tilde{\Omega}_{hi,j}} \left[\alpha_{hi,L}^{\frac{1}{\sigma_{KL}}} L_{hi}(t)^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} + \alpha_{hi,K}^{\frac{1}{\sigma_{KL}}} \left(K_{hi}(t)^{\tilde{\Omega}_{hi,K}} \right)^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} \right]^{\frac{\sigma_{KL}}{\sigma_{KL}-1}} \forall h \in H, \forall i \in N \tag{44}$$

$$L_{hi}(t) = A_f e^{g_A t} \prod_{f \in F_h} [L_{hi,f}(t)]^{\frac{\tilde{\Omega}_{hi,f}}{\sum_{f' \in F_h} \tilde{\Omega}_{hi,f'}}} \qquad \forall h \in H, \forall i \in N \tag{45}$$

$$X_{hk}(t) = A_{hk} \prod_{j \in N} Y_{hk,j}(t)^{\tilde{\Omega}_{hk,j}} \quad \forall h \in H, \forall k \in K. \tag{46}$$

For all three goods, there is a Cobb-Douglas aggregates of intermediates from different types of industries. For regular goods, there is also a CES aggregator of labor inputs and capital from that industry's capital good, with labor being a Cobb-Douglas aggregate of different labor types, with factor-augmenting productivity growth on labor input that is common across labor types and countries. Note that the expression K_{hi} here refers to a quantity of a capital good and not the set of capital goods. For all three expressions, we have $\sum_j \tilde{\Omega}_{hi,j} + \sum_f \tilde{\Omega}_{hi,f} + \tilde{\Omega}_{hi,K} = 1$, with $\tilde{\Omega}_{hi,f}$ and $\tilde{\Omega}_{hi,K}$ interpreted as zeros in the case of consumption and investment good production. We also have $\alpha_{hi,L} + \alpha_{hi,K} = 1$ and $\alpha_{hi,L} = \sum_f \tilde{\Omega}_{hi,f}$

The terms $Y_{hh,j}(t), Y_{hi,j}(t), Y_{hk,j}(t)$ are Armington aggregates associated with the industry type j. They are given by

$$Y_{hi,j}(t) = \left[\sum_{h'} W_{hi,jh'}^{\frac{1}{\theta}} Y_{hi,jh'}(t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \qquad i \in \{h\} \cup N \cup K, j \in N \tag{47}$$

where $\sum_{h'} W_{hi,jh'} = 1$ and h' stands for origin countries. The indexing for i captures that the same aggregation is performed for consumption goods, regular goods, and capital goods. The indexing over j captures that there are no cross-country Armington aggregates of consumption and capital goods.

Producers allocate expenditure across different countries to minimize costs:

$$\{Y_{hi,h'j}(t)\} \in \arg\min \sum_{h'} (1 + t_{h,jh'}) p_{h'j}(t) Y_{hi,h'j}(t) \quad s.t. \quad (47) \qquad \forall h \in H \quad i,j \in N$$

subject to (47), where $t_{h,jh'}$ is an import tariff levied by country i on good j from country h'. Similarly, labor is chosen to minimize cost given the desired labor aggregate

$$\{L_{hi,f}(t)\} \in \arg\min \sum_{f \in F_h} w_f(t) L_{hi,f}(t) \quad s.t. \quad (45) \qquad \forall h \in H, i \in N.$$

Writing $p_{hi,j}(t)$ and $w_{hi}(t)$ for the unit cost for industry (h,i) associated with buying a unit of inputs from industry j and a unit of labor input, the choices of producers maximize profits

$$\{C_h(t), Y_{hh,j}(t)\} \in \arg\max p_h(t) C_h(t) - \sum_{j \in N} p_{hh,j}(t) Y_{hh,j}(t) \quad \forall h \in H \tag{48}$$

$$\begin{aligned} \{Y_{hi}(t), Y_{hi,j}(t), L_{hi}(t), K_{hi}(t)\} \in \arg\max & (1 - \tau_{hi}) p_{hi}(t) Y_{hi}(t) && (49) \\ & - \sum_{j \in N} p_{hi,j}(t) Y_{hi,j}(t) - w_{hi}(t) L_{hi}(t) - R_{hi}(t) K_{hi}(t) \quad \forall h \in H, \forall i \in N && (50) \end{aligned}$$

$$\{X_{hk}(t), Y_{hk,j}(t)\} \in \arg\max (1 - \tau_{hk}) p_{hk}(t) X_{hk}(t) - \sum_{j \in N} p_{hk,j}(t) Y_{hk,j}(t) \quad \forall h \in H, \forall k \in K, \tag{51}$$

where τ_{hi}, τ_{hk} are output taxes associated with regular goods and investment goods.

Households. A household born at time t_b chooses which capital good to enter, solving the problem:

$$V_h(t_b) = \max_{i \in N_h} V_i(t_b),$$

where $V_i(t_b)$ is the expected utility of operating capital good i, and it is determined by a lifecycle problem where households choose a path of net worth $n(t)$, consumption $c(t)$, capital $k(t)$, and bonds $b(t)$ to maximize expected utility

$$V_i(t_b) = \max_{n(t),c(t),k(t),b(t)} \mathbb{E} \int_{t_b}^{\infty} e^{-\nu_h(t-t_b)} e^{-\rho_h(t-t_b)} \frac{c(t_b,t)^{1-1/\gamma}}{1-1/\gamma} dt, \tag{52}$$

The households choose subject to the following constraints

$$dn(t) = \left\{ (1+T_h(t)) \sum_{f \in F} w_f(t)\ell_f + \nu_h n(t) + n(t)[r + \phi(t)(r_i(t) - r)](1-\tau^h) - p_h(t)c(t) \right\} dt \tag{53}$$

$$+ \sigma_i n(t)\phi(t) dZ_i, \tag{54}$$

$$\phi(t) = \frac{p_i(t)k(t)}{n(t)} \tag{55}$$

$$r_i(t) = R_i(t)/p_i(t) - \delta_i \tag{56}$$

$$n(t) = b(t) + p_i(t)k(t) \tag{57}$$

$$-w_h\bar{b} \leq n(t), \tag{58}$$

$$n(t_b) = 0. \tag{59}$$

In the evolution equation (54), $\sum_{f \in F} w_f(t)\ell_f$ is primary factor income, where ℓ_f is the households' endowment of factor f, and $T_h(t)$ is the transfers in country h as a share of labor income. We assume that labor ℓ_f whenever the household is alive. Households have access to an actuarially fair annuity and optimally choose to annuitize all their wealth. Thus, capital income consists of $r(t)b(t)$ in interest payments on bonds and $\nu_h n(t)$ of survival benefits from the annuity.[24] In addition, the household allocates $p_i(t)k(t)$ to their idiosyncratic capital line, earning a net return $R_i(t) - \delta_i$, where $R_i(t)$ is the rental rate of capital i and δ_i is its depreciation rate. All capital income is subject to a capital tax τ^h that

[24] Formally, the household contracts with a financial intermediary to obtain a flow payment conditional on survival in return for giving up all assets upon death, including their individual capital line, which can be resold at a price $p_i k_i(t)$.

is common across industries in a country. [25]

The net return on capital generally exceeds the risk-free rate r, but comes at the cost of exposure to idiosyncratic risk, given by $\sigma_i p_i(t)k(t)dZ_i$. Here, σ_i captures the volatility of idiosyncratic risk in industry i, and dZ_i is an increment of the standard Brownian motion. This formulation, following Di Tella (2017), can be microfounded in terms of information frictions between the entrepreneur and the investor. We assume that either all capital goods are risky, or that none are. We assume that the capital tax does not reduce the risk faced by the entrepreneur, consistent with an interpretation where the entrepreneur has to be exposed to a sufficient amount of risk in their post-tax income.

The choice across industries yields stochastic processes for the household variables. For any given variable x, we write $x_i(t_b, t)$ for the stochastic process associated with a household born at time t_b and operating in industry i. To streamline notation, we extend the stochastic processes to the full range of t by assuming that they take value 0 before birth and after death.

Financial intermediaries. We assume that annuities and claims on capital lines are intermediated by a financial institution. On the annuity side, the intermediary sells annuities that provide flow payments to living households in exchange for their assets upon death. In terms of claims of capital lines, the intermediary issues derivatives which promise payments to households after bad shocks in return for payments after good shocks. The intermediary does not hold any assets or make any profits. For annuities, this follows from free entry which implies that annuities are actuarially fair, making the intermediary a mere conduit for redistributing between surviving and dying households. On the capital side, zero profits follow from free entry and the fact that the intermediary is risk-neutral with respect to household shocks, meaning that the derivatives have no upfront value and yield no expected profits.

We assume that risk-free bonds are in zero net supply and that financial markets are integrated, implying a market clearing condition

$$\sum_{i \in \mathcal{N}} B_i(t) = 0, \tag{60}$$

where financial integration is captured by summing across all industries in the world.[26]

[25]The common rental rate is consistent with the setup of industry-specific capital stocks consisting of perfectly substitutable varieties.

[26]If there is financial autarky, this equilibrium condition is replaced by one for each country, with the sum running over $\mathcal{N}_h$ for each country h.

Demographics and aggregation. For each $t_b \in (-\infty, \infty)$, there is an exogenous number of births $L_{0,c}(t_b) = e^{g_L t_b} L_{0,c}$ in country h. To obtain aggregate variables, we integrate household outcomes across cohorts:

$$L_f(t) = \int_{-\infty}^{\infty} L_h^0(t_b) \mathbb{E} \ell_f dt_b, \qquad f \in F, \tag{61}$$

$$K_{ci}(t) = \int_{-\infty}^{\infty} L_h^0(t_b) \varphi_{ci}(t_b) \mathbb{E} k_{ci}(t_b, t) dt_b, \quad h \in H. i \in N, \tag{62}$$

$$B_i(t) = \int_{-\infty}^{\infty} L_h^0(t_b) \varphi_{ci}(t_b) \mathbb{E} b_i[t_b, t] dt_b, \quad i \in N, \tag{63}$$

$$C_h(t) = \int_{-\infty}^{\infty} L_h^0(t_b) \sum_{i \in K_h} \varphi_{ci}(t_b) \mathbb{E} c_i[t_b, t] dt_b, \quad h \in H. \tag{64}$$

The expectations are taken over realizations of cohort-t_b's processes, which are stochastic due to mortality risk, as well as due to idiosyncratic risk to capital accumulation. The terms $\varphi_{hi}(t_b)$ are the shares of households born at time t_b in country h that operates capital line i. Further, the convention of treating stochastic processes as zero before birth and after death implies that the survival probability is implicitly accounted for in the expectations. It also implies that there are no contributions to aggregates from cohorts born after time t.

Resource constraints and market clearing. In addition to the bond market clearing condition, the resource constraints are given by

$$\begin{aligned} C_h(t) &= Y_{hh}(t), \qquad h \in H \\ Y_{hi}(t) &= \sum_{h' \in H, j' \in H+N+K} Y_{h'j',hi}(t), \qquad h \in H, i \in N \\ \dot{K}_{h,i}(t) &= -\delta_{k(i)} K_{h,i}(t) + X_{h,k(i)}(t) \qquad h \in H, i \in N \\ L_f(t) &= \sum_{i \in N} L_{hi,f}(t), \qquad h \in H, f \in F^h. \end{aligned}$$

Furthermore, transfer income to households need to be consistent with tax income for the government:

$$T_h(t) = \frac{\sum_{i \in N_h} \tau_{hi} p_{hi}(t) Y_{hi}(t) + \sum_{i,j,h'} t_{i,jh'} p_{jh'}(t) Y_{hi,jh'}(t) + \tau^h [r(t) B_h(t) + \sum_{i \in K^h} p_{hi}(t) K_{hi}(t) r_i(t)]}{\sum_{f \in F_h} w_f(t) L_f(t)},$$

where the size of the transfer is expressed relative to factor income in that country.

Equilibrium. Given taxes and tariffs, an equilibrium of the model consists of prices, quantities, transfers, household values and decision functions, as well as the shares of households $\varphi_{hi}(t_b)$ that enter different capital lines. They satisfy the following properties.

1. Given prices, aggregate quantities are consistent with profit maximization, capital accumulation equations, resource constraints, and bond market clearing.

2. For each cohort t_b, the households' value and decision functions solve their optimization problem (52)-(59) given prices, with the implied stochastic processes being consistent with aggregate quantities, (61)-(64).

3. For each t_b, all industries with strictly positive entry offer the same expected utility to newborn households. That is, if $\varphi_{hi}(t_b), \varphi_{hi'}(t_b) > 0$, then

$$V_{hi}(t_b) = V_{hi'}(t_b).$$

Balanced growth path. Balanced growth paths are equilibria with the following properties:

1. Constant risk-free rate r, rental rates $\{R_i\}$, and good prices $\{p_i\}$.

2. Constant shares φ_{hi} of households in every country entering each industry.

3. Primary factor prices $w_f(t)$ grow at a constant common rate g_A.

4. Consumption, output, intermediate inputs, and capital stocks grow at a common constant rate $g \equiv g_L + g_A$.

D.2 Details on the household solution

Consider the household problem when they face a wage profile $w_f(t) = e^{g_A t} w_f$ and constant rates of returns $r(t) = r$, $r_i(t) = r_i$, and fixed prices $p_h(t) = p_h$, $p_i(t) = p_i$. Furthermore, write $y = \sum_f w_f \ell_f (1 + T_h)$ for total factor income. Define $\hat{n}(t) = e^{-g_A t} n(t)$ and $\hat{c}(t) = e^{-g_A t} c(t)$ to be normalized levels of net worth and consumption. Then, the problem can be expressed as

$$\max \int_0^\infty e^{-\tilde{\rho}_h t} \frac{\hat{c}(t)^{1-1/\gamma}}{1 - 1/\gamma}$$

where $\tilde{\rho}_h = \rho_h + \nu_h - g_A(1 - 1/\gamma)$, subject to

$$d\hat{n}(t) = \{y - p_h \hat{c}(t) + \nu_h \hat{n} + \hat{n}(t) [r - g_A + \phi(t)(r_i - r)]\} dt + \sigma_i \phi(t) \hat{n}(t) dZ_i$$

$$\hat{n}(t) \geq -\frac{y}{r+\nu_h-g_A},$$
$$\phi(t) \geq 0.$$

Defining effective wealth as

$$\hat{\omega}(t) = \hat{n}(t) + \frac{y}{r+\nu_h-g_A},$$

we obtain

$$d\hat{\omega}(t) = [(r+\nu_h-g_A)\hat{\omega}(t) + \phi(t)(r_i-r)\hat{\omega}(t) - p_h c(t)]dt + \sigma_i\phi(t)\hat{\omega}(t)dZ$$

This is a Merton portfolio problem with a risk-free return $r+\nu_h-g_A$ and a risky return $r_i+\nu_h-g_A$. The solution is given by allocating a constant share of effective wealth to the risky asset:

$$\phi_i \equiv \gamma \times \frac{r_i-r}{\sigma_i^2} = \gamma S_i \sigma_i^{-1},$$

where $S_i \equiv \frac{r_i-r}{\sigma_i}$ is the Sharpe ratio. The household also consumes a constant share of effective wealth

$$p_h c(t) = \xi\hat{\omega}(t) \qquad \xi \equiv \gamma \times \left(\tilde{\rho} - (1-1/\gamma)\left(\frac{\gamma(r_i-r)^2}{2\sigma_i^2} + r + \nu_h - g_A\right)\right)$$

Substituting this expression into $p_h c(t)$, and using that the expected growth of the Brownian term is 0, we obtain that the expected growth rate of $\hat{\omega}(t)$ is

$$\mathbb{E}d\hat{\omega}(t) = \hat{\omega}(t)\left[-g_A + \gamma(r-\rho_h) + \frac{\gamma(\gamma+1)}{2}S_i^2\right].$$

This mean that non-normalized wealth grows as $\gamma(r-\rho_h) + \frac{\gamma(\gamma+1)}{2}S_i^2$, as stated in the proposition.

To derive the expected utility at birth, we note that flow utility at time t is given by

$$\frac{\xi e^{-\tilde{\rho}t}\hat{\omega}(t)^{1-1/\gamma}}{1-1/\gamma}.$$

Using the linearity of expectation, the expectation of the integral over $\frac{\xi e^{-\tilde{\rho}t}\hat{\omega}(t)^{1-1/\gamma}}{1-1/\gamma}$ equals the integral over $\frac{\xi e^{-\tilde{\rho}t}\mathbb{E}\hat{\omega}(t)^{1-1/\gamma}}{1-1/\gamma}$. Using standard Ito algebra, we can derive a stochastic differential equation for $\hat{\omega}(t)^{1-1/\gamma}$ in terms of the drift and diffusion of $\hat{\omega}$, which lets us solve for the growth rate of expected utility $\mathbb{E}\hat{\omega}(t)^{1-1/\gamma}$, and thus for expected utility at birth.

D.3 Proof of Proposition 8

First, we note that since attained utility only depends on industry properties through S_i, these must be equalized within a country when all capital goods are active. This implies there exists an S_h such that

$$r_i = r + \sigma_i S_h \quad \forall i \in K^h$$

as stated in the proposition. Moreover, since the growth rate of effective wealth among survivors also depends only on the Sharpe ratio, there exists a country-specific growth rate $g_{\omega,c}$ of effective wealth as well.

To derive the remaining parts of the proposition, we first observe that the total flow $L_{0,f}$ of new units of factor f satisfies

$$L_{0,f} = L_f(g_L + \nu_h), \qquad f \in F_h$$

reflecting that the inflow of new factors must compensate for deaths and maintain growth at rate g_L. Furthermore, the normalized amount of effective wealth in the economy is the integral over historical cohorts, yielding

$$\begin{aligned} \mathcal{W}_h &= \int_0^\infty \sum_f \frac{[e^{-g_L t} L_{0,f}][e^{-g_A t} w_f](1+T_h)}{r + \nu_h - g_A} e^{(g_{\omega,c} - \nu_h)t} \\ &= \frac{\nu_h + g_L}{\nu_h + g_L + g_A - g_{\omega,c}} \frac{\sum_f L_f w_f (1+T_h)}{r + \nu_h - g_A}, \end{aligned}$$

where the term $e^{-g_A t}$ reflects that the nominal value of wages started out at a lower level in the past.

Using that the share of risky assets in industry i is $\phi_i = \gamma \frac{S_i}{\sigma_i}$, market clearing for capital implies that

$$\sigma_i p_i K_i = \varphi_{h,i} \gamma S_h \mathcal{W}_h \qquad \forall i \in K^h,$$

where $\varphi_{h,i}$ is the share of households in country h investing in industry i. Using $\sigma_i =$

$(r_i - r)/S_h$ and summing over $i \in K^h$ yields

$$\sum_{i \in K^h} \frac{(r_i - r)p_i K_i}{S_h} = \gamma S_h \mathcal{W}_h$$

Substituting in $\mathcal{W}_h$ and moving S_h to the right-hand side gives us

$$\sum_i (r_i - r)p_i K_i = \gamma S_h^2 \frac{\nu_h + g_L}{\nu_h + g_L + g_A - g_{\omega,c}} \frac{\sum_f w_f L_f (1 + T_h)}{r + \nu_h - g_A}$$

as required.

Finally, given that $\sum_h B_h(t) = 0$, the value of all capital assets must equal total effective wealth minus effective wealth from labor. This gives us

$$\begin{aligned}
\sum_{h \in H} \sum_{i \in K} p_{hi} K_{hi} &= \sum_h \left[\mathcal{W}_h - \sum_{f \in F^h} \frac{(1 + T_h) \sum_{f \in F_h} w_f L_f}{r + \nu - g_A} \right] \\
&= \sum_{h \in H} \left[\frac{\nu_h + g_L}{\nu_h + g_L + g_A - g_{\omega,c}} - 1 \right] \frac{\sum_f L_f w_f (1 + T_h)}{r + \nu_h - g_A} \\
&= \sum_{h \in H} \frac{g_{\omega,c} - g_A}{\nu_h + g_L + g_A - g_{\omega,c}} \frac{\sum_f L_f w_f (1 + T_h)}{r + \nu_h - g_A},
\end{aligned}$$

which concludes the proof.

D.4 Balanced growth equations

The following equations determine the BGP equilibrium using the input-output notation in Section 3. Without loss of generality, following Baqaee and Farhi (2019), we relabel the input-output matrix so that each CES aggregator is treated as a separate producer (this simplifies notation). This means that we also drop the notation that indexes producers in terms of countries and type of industry. Instead, we use a single index to denote every CES aggregate in the model. We assume that the Armington CES nests are located in the destination country. We write $t_{i,j}$ for a bilateral tax on nest i's purchases of j, $t_{i,j}$, with the assumption that tax revenues are rebated to destination households. Allowing for $t_{i,j}$ nests the tariffs in the main model. Furthermore, we allow for reduced-form output wedges, $\tilde{\mu}$, which behave like markups and whose revenues are rebated to origin households.[27] The allocational consequences of such wedges can be constructed from tariffs and output

[27] Allowing for such output wedges does not require extending the framework in Appendix D.1, because their effect is equivalent to that of output taxes.

taxes in our model description. CES share parameters are denoted using overlines, with shares being zero for goods not in that nest (for example, in an Armington nest, all the shares but the ones associated with that input bundle are zero).

Capital Supply Equations

- Wealth growth conditional on survival
  $g_{\omega,c} = \gamma \times \left[(1-\tau_h^k)r - \rho + \left(\frac{\gamma+1}{2}\right)(1-\tau_h^k)^2 S_h^2\right]$
- Ratio of total wealth to human wealth
  $\chi_h = \frac{\nu_h + g_L}{\nu_h + g - g_\omega^h}$
- Desired financial wealth of households in country h
  $W_h^{fin} = \frac{\left(\sum_{f\in F_h} \lambda_f (1+T_h)\right)}{(1-\tau_h^k)r + \nu_h - g_A}\left[\chi_h - 1\right]$

Production Block

- Growth rate
  $g = g_L + g_A$
- Tax and markup revenues relative to labor income
  $T_h(t) = \frac{\tau_h^k\left[rb_h(t) + \sum_{i\in K_h}\frac{r_i}{r_i+\delta_i}\lambda_i\right] + \sum_{i\in N_h}\left[1-\frac{1}{\tilde{\mu}_i}\right]\lambda_i + \sum_{j\in N_h+K_h}\sum_{(ih')\in N+K}\frac{t_{j,i}}{1+t_{j,i}}\lambda_{jh}\Omega_{jh,ih'}}{\sum_{f\in F_h}\lambda_f}$
- Country consumption
  $\Phi_h = \sum_{f\in F_h}\lambda_f(1+T_h) + \sum_{i\in K_h}\lambda_i\left[1-\frac{1}{\mu_i}\right] + (r-g)b_h$
- Goods prices
  $p_n = \frac{\tilde{\mu}_n}{A_n}\left(\sum_{j\in N}\bar{\Omega}_{nj}[(1+t_{nj})p_j]^{1-\theta_n} + \sum_{j\in F}\bar{\Omega}_{nj}w_j^{1-\theta_n} + \sum_{j\in K}\bar{\Omega}_{nj}R_j^{1-\theta_n}\right)^{\frac{1}{1-\theta_n}} \qquad n \in H+N$
- Capital user costs
  $R_n = \frac{\mu_n(\delta_n+g)}{A_n}\left(\sum_{j\in N}\bar{\Omega}_{nj}[(1+t_{nj})p_j]^{1-\theta_n}\right)^{\frac{1}{1-\theta_n}}, \qquad n \in K.$
- Effective markups
  $\mu_m = \frac{r_n+\delta_n}{g+\delta_n}\mathbf{1}(m\in K) + \tilde{\mu}_m\mathbf{1}(m\notin K).$
- Goods, labor, and capital services market clearing
  $\lambda_i = \sum_{h'\in H}\Phi_{h'}\frac{\Omega_{h',i}}{1+t_{h',i}} + \sum_{j\in N+K}\lambda_j\frac{1}{1+t_{j,i}}\Omega_{j,i}.$

Capital Market Clearing and Distribution of Free Cash Flows

- Physical capital market clearing by country[28]
  $\sum_{i \in K_h} \sigma_i \frac{\lambda_i}{r_i+\delta_i} = W_h^{fin} \frac{\chi_h}{\chi_h - 1} \gamma S_h$

- Bond market clearing
  $\sum_{i \in K} \frac{\lambda_i}{r_i+\delta_i} = \sum_{c \in C} W_h^{fin}$

- No arbitrage within country
  $r_i = r + \frac{\sigma_i S_h}{(1-\tau_k)}, \qquad i \in K_h$

- Net foreign assets
  $b_h = W_h^{fin} - \sum_{i \in K_h} \frac{\lambda_i}{r_i+\delta_i}$

Invertibility of BGP system. To see that there exists a set of share and productivity parameters such that our calibration targets constitute a balanced growth path, we consider the case when all TFPs are 1 except for investment goods that have productivity $\mu_n(g+\delta_n)$. Furthermore, we set the values of the share parameters $\bar{\Omega}$ to the observed cost shares for each nest. In that case, the equations for goods prices and capital costs are satisfied with prices $p_n = 1$ for all $n \in N + C$ and $R_n = 1$ for all $n \in K$.

D.5 Linearized solution

Here are the linearized equations. It is conducted around a balanced growth path with no initial taxes.

Wealth growth conditional on survival:

$$dg_{\omega,c} \equiv \gamma \times \left[dr + 2\left(\frac{\gamma+1}{2}\right) dS_h - d\tau_h^k \left(r + 2\left(\frac{\gamma+1}{2}\right) S_h^2 \right) \right]$$

Ratio of total wealth to human wealth:

$$d\chi_h = \frac{d\nu_h}{\nu_h + g - g_\omega^h} + \chi_h \frac{dg_{\omega,c} - d\nu_h}{\nu_h + g - g_\omega^h}$$

Desired financial wealth of households in country h:

$$dW_h^{fin} = \frac{\sum_{f \in F_h} \lambda_f}{r + \nu_h - g_A} \left[\frac{d\left(\sum_{f \in F_h} \lambda_f\right)}{\sum_{f \in F_h} \lambda_f} + d\chi_h + \frac{r\, d\tau_h^k}{r + \nu_h - g_A} - \frac{dr}{r + \nu_h - g_A} + dT_h \right]$$

[28]Recall that if i is a capital good, then λ_i is the compensation of capital i relative to world consumption, hence $\lambda_i/(r_i+\delta_i)$ is the value of the capital stock.

Tax revenues relative to labor income:

$$dT_h = \frac{d\tau_h^k \left[rb_h + \sum_{i\in K_h} \frac{r_i}{r_i+\delta_i}\lambda_i\right] + \sum_{i\in N_h} \frac{\lambda_i}{\tilde{\mu}_i^2} d\tilde{\mu}_i + \sum_{j\in N_h+K_h} \sum_{i\in N+K} \left[dt_{j,i}\lambda_j\Omega_{j,i}\right]}{\sum_{f\in F_h} \lambda_f}$$

Country consumption:

$$d\Phi_h = \sum_{f\in F_h} d\lambda_f + dT_h \sum_{f\in F_h} \lambda_f + \sum_{i\in K_h} \left[d\lambda_i \left(1 - \frac{1}{\mu_i}\right) + \lambda_i \frac{d\log \mu_i}{\mu_i}\right] + d[(r-g)b_h]$$

Goods prices:

$$d\log p_n = d\log \frac{\tilde{\mu}_n}{A_n} + \sum_{j\in N} \tilde{\Omega}_{n,j}[dt_{nj} + d\log p_j] + \sum_{m\in K} \tilde{\Omega}_{n,m} d\log R_m + \sum_{f\in F_h} \tilde{\Omega}_{n,f} d\log w_f$$

Capital user costs:

$$d\log R_k = d\log\left(\frac{\mu_{k'}}{A_{k'}(\delta_{k'}+g)}\right) + \sum_{j\in N} \tilde{\Omega}_{k,j}[d\log p_j + dt_{n,j}] \quad k \in K$$

Effective markups:

$$d\mu_m = \left[\frac{dr_m + d\delta_m}{g+\delta_m} - \mu_m \frac{d\delta_m}{g+\delta_m}\right] \mathbf{1}(m\in K) + d\tilde{\mu}_m \mathbf{1}(m \notin K)$$

Goods, labor, and capital services demand:

$$d\lambda_i = \sum_{h'\in H} \left[d\Phi_{h'}\Omega_{h',i} + \Phi_{h'}\left(d\Omega_{h',i} - dt_{h',i}\Omega_{h',i}\right)\right] + \sum_{j\in N+K} \left[d\lambda_j\Omega_{j,i} + \lambda_j(d\Omega_{j,i} - dt_{ji}\Omega_{j,i})\right]$$

Change in cost shares:

$$d\tilde{\Omega}_{ij} = (1-\theta_i)\tilde{\Omega}_{ij} \left[d\log p_j + dt_{ij} - \sum_k \tilde{\Omega}_{ik}\left(d\log p_k + dt_{ik}\right)\right]$$

$$d\Omega_{ij} = -d\log \mu_i \Omega_{ij} + \mu_i^{-1} d\tilde{\Omega}_{ij}$$

Labor market clearing:

$$d\log w_f = d\log \lambda_f \qquad f \in F$$

Asset market clearing:

$$\sum_{i\in K_h} \left[d\sigma_i \frac{\lambda_i}{r_i+\delta_i} + \sigma_i d\left(\frac{\lambda_i}{r_i+\delta_i}\right)\right] = \gamma W_h^{fin} \frac{\chi_h}{\chi_h - 1} S_h \times \left[d\log W_h^{fin} + d\log\left(\frac{\chi_h}{\chi_h-1}\right) + d\log S_h\right]$$

$$\sum_{i\in K} \left[\frac{d\lambda_i}{r_i+\delta_i} - \frac{\lambda_i}{r_i+\delta_i}\frac{dr_i + d\delta_i}{r_i+\delta_i}\right] = \sum_{c\in C} dW_h^{fin}$$

No arbitrage within country:

$$dr_i = dr + d\sigma_i S_h + \sigma_i dS_h + (r_i - r)d\tau_k, \qquad i \in K_h$$

Net foreign assets:

$$db_h = dW_h^{fin} - \sum_{i \in K_h} \left[\frac{d\lambda_i}{r_i + \delta_i} - \frac{\lambda_i}{r_i + \delta_i} \frac{dr_i + d\delta_i}{r_i + \delta_i} \right]$$

D.6 Details on calibration

The calibration of $\tilde{\Omega}$, Ω, and μ is described above in the Appendix to Section 3.5.

Depreciation rates We map BEA industry-specific depreciation rates to WIOD sectors. For each sector and year, we calculate the depreciation rate as the total value of depreciation divided by the total value of the capital stock of the sector. The resulting depreciation rate δ_i for $i \in N$ is used for all countries.

Revenue-based input output matrix. From $\tilde{\Omega}$ and μ, we obtain the revenue-based input output matrix from $\Omega_{ij} = \frac{1}{\mu_i} \tilde{\Omega}_{ij}$. This matrix is identical to $\tilde{\Omega}$, apart from the rows associated with investment goods being deflated by $\frac{r_i + \delta_k}{g + \delta_k}$. In particular, this means that the input-output matrix rows associated with investment generally sum to less than 1, reflecting that not all rental payments to capital goods end up as investment good spending.

Revenue shares. From Ω, we define the total requirement matrix $\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \cdots$, with Ψ_{ij} capturing the share of spending on i that ends up in j, directly and indirectly through the input-output network. The revenue of each i relative to world consumption satisfies

$$\lambda_i = \sum_{h' \in H} \Phi_{h'} \Psi_{h',i}, \tag{65}$$

where $\Phi_{h'}$ is the share of world consumption in country h', where i indexes perishable goods, capital goods, and labor.

Consumption weights by country. The previous results express λ_i up to consumption shares Φ_h. To solve for these shares, we note that they need to satisfy

$$\Phi_h = \sum_{f \in F_h} \lambda_f + \sum_{k \in K_h} \lambda_k \left(1 - \frac{1}{\mu_k} \right) + (r - g) b_h, \tag{66}$$

where b_h is the ratio of net foreign assets (bonds in our model) relative to world consumption. This equation states that consumption in a country equals its factor income plus net income from domestic capital plus earnings on net foreign assets. Since labor and net capital income can be expressed in terms of Φ_h using (65), and $\sum_h \Phi_h = 1$, equation (66) can be solved for Φ_h as a function of net factor payments $(r - g) b_h$. To calibrate the latter, we use our earlier calibration for r and g, and set b_h equal to net foreign asset positions relative to global consumption, with net foreign asset positions taken from the External Wealth of Nations Database (Lane and Milesi-Ferretti, 2018), and global consumption from WIOD.[29]

E Appendix to Section 5.6

E.1 Table of Tariffs

Table E.1: US tariffs imposed on imports from each region during the trade war

Country	Tariff Rate
Canada	10%
India	26%
China	54%
United Kingdom	10%
Japan	24%
Mexico	10%
European Union	20%
Rest of the World	25.4%

E.2 Effect of Markups

Table E.2 shows the first-order effects on long-run global consumption from a uniform increase in markups of goods-producing industries in every country. These markups are equivalent to output wedges, and we assume the profits they generate are rebated lump sum to local households in proportion to households' labor income. All results are expressed as semi-elasticities of long-run consumption with respect to markups, so that a value of -1.0 means that a net markup of 1% lowers log long-run consumption by 0.01.

The first row shows that consumption effects are very powerful: a 10% increase in markups reduces long-run global consumption by 7.3%. As Proposition 7 illustrates, the reduction in global consumption is driven by reductions in capital stocks. Intuitively, the increase in markups raise the price of investment goods relative to labor, which reduces capital demand and investment. Since capital is below its long-run consumption maximizing level, this reduction in investment depresses long-run consumption.

[29] We do not calibrate Φ_h according to country-level consumptions from WIOD because this would result in an inconsistency between trade balance and net foreign assets. Nevertheless, the Φ_h we do calibrate to are similar to those implied by the WIOD. See footnote 20 for more information.

Table E.2: Change in long-run consumption due to increase in markups

Scenario	Description	Global Consumption
Benchmark	Baseline calibration in Section 5.4	-0.727
Rep. agent	Baseline calibration holding returns and current accounts constant	-1.145
Static	Investment treated as a final expenditure and capital treated as an endowment	-0.000
$\sigma_{KL} = 1.2$	Higher elasticity of substitution between capital and labor	-0.844
$\sigma_{KL} = 0.6$	Lower elasticity of substitution between capital and labor	-0.371
$\theta = 1$	Benchmark calibration, but trade elasticities equal to zero ($\theta - 1 = 0$)	-0.726
$\delta = \infty$	All depreciation rates set to infinity. Implies that all as-if markups equal 1, and that capital is treated as an intermediate	0.000

To explore the economic forces, Table E.2 also displays how this semi-elasticity changes as we vary some of our modeling choices. When the capital supply elasticity is raised to infinity, as it would be with an infinitely-lived representative agent, then long-run consumption losses are approximately 60% larger. This is because infinitely elastic capital supply eliminates a mitigating force: in the benchmark, falling capital demand reduces the rates of return, which lowers the user cost of capital, and partially offsets higher investment prices. Without this offset, capital, and hence consumption, falls by more. This effect is consistent with a general intuition from the misallocation literature that reallocation effects are more important when quantities are highly elastic with respect to distortions. By contrast, when we lower the capital supply elasticity to zero, as it would be in a static model, losses from markup increases are zero. This is also consistent with Proposition 7, since in this case, the quantity of capital does not adjust and there are no losses.

Similarly, altering the elasticity of capital demand affects losses. For example, when the elasticity of substitution between capital and labor is 1.2, as in Karabarbounis and Neiman (2013), instead of 1.0, losses from markups are magnified. In contrast, when the elasticity of substitution between capital and labor is lower, for example 0.6 as in Antras (2004), the losses are substantially smaller. The losses are roughly linear in the elasticity of substitution between labor and capital. This also mirrors the intuition from the misallocation literature where losses from wedges are proportional to elasticities of substitution (see, e.g. Baqaee and Farhi, 2020).

In contrast, the trade elasticity does not have an important effect on how either capital supply or capital demand responds to higher investment prices. Hence, its value is relatively unimportant for the overall consumption losses from markups.

Finally, holding elasticities constant, losses also decline if the Golden Rule wedge were smaller. For example, if we set $\delta = \infty$, capital becomes non-durable and acts like a regular intermediate input. In this case, the Golden Rule wedge is zero and hence long-run aggregate consumption is maximized in the initial equilibrium. Accordingly, changes in markups have no first-order effect due to the envelope theorem.