Aggregate Efficiency with Heterogeneous Agents

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Abstract

We study aggregate efficiency when households have heterogeneous preferences and outcomes. We generalize the consumption-equivalent variation of Lucas (1987) to a multi-agent setting, asking: how much can the consumption-possibility set shrink while keeping every agent at least as well off as in their status-quo allocation? The resulting scalar — resources left over after compensating everyone — is our measure of aggregate efficiency. Efficiency rises whenever the same status-quo welfare can be achieved with fewer resources. We show how to convert this problem into an equivalent utility-maximization problem, enabling the use of tools and results normally applicable only in representative agent settings. We characterize changes in aggregate efficiency in terms of observables, like expenditures and price elasticities, and apply our results to study, among other things, the effects of productivity shocks, the costs of misallocation, and the gains from trade, both with and without costly redistribution.

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1 Introduction

Two central themes in economics are efficiency—how large the economic "pie" is and what determines its size—and equity—how that pie is, or ought to be, shared. The normative question of how resources should be divided among people is controversial and subjective. For this reason, it is useful to study efficiency in isolation from equity. With a single agent, this separation is trivial and automatic because distributional issues never arise. With heterogeneous consumers, however — especially when they differ in tastes or face different prices — the two questions are harder to separate.

In policy circles, changes in aggregate efficiency are typically measured by cost-benefit analysis. This may be done explicitly, by estimating and summing compensating variations, or implicitly, by using market-based quantity indices like real GDP (which, under appropriate assumptions, are equivalent to summing compensating variations). These metrics measure the change in aggregate efficiency by the amount of money (at constant prices) left-over after winners compensate losers. Such measures are popular in policy contexts because they are objective, taking the view that a dollar is a dollar, regardless of who earns it.

However, it is well-known that these measures have theoretical problems. For one, the transfers implied by summing up compensating variations may be infeasible. Either because there is no mechanism by which winners can compensate the losers (e.g. Antràs et al., 2012 and Schulz et al., 2023) or because, even if transfers can be made, relative prices may change in response to the transfers making compensations impossible (e.g. Boadway, 1974). At a more abstract level, in some settings, for example when decentralized markets are incomplete or absent, the notion of a compensating variation, or even prices themselves, may be undefined.

In academic settings, aggregation across heterogeneous agents is often achieved via a social welfare function. This approach, pioneered by Bergson (1938) and Samuelson (1947), avoids the paradoxes and shortcomings of cost-benefit analysis, but does so at the cost of entangling efficiency and equity considerations. Social welfare functions automatically take a stance not just on the size of the economic pie, but also on its optimal division. This means that social welfare functions depend on subjective Pareto-weights and are not invariant to monotone transformations of utility functions. This makes them unattractive for use in practical policy design, especially if heterogeneity in tastes is important (i.e. in cases where we cannot use the same utility function to represent every agent's preferences).

In this paper, we provide an alternative approach to study aggregate efficiency in isolation from the question of distribution, building on ideas from Allais (1979), Debreu (1951), and Luenberger (1996). Our measure generalizes the popular Lucas (1987) consumption-equivalent beyond the one-agent setting, without taking a stance on how resources should be divided across people. To measure changes in aggregate efficiency we ask: *by how much can the consumption-possibility set shrink while keeping every agent at least indifferent to their status-quo allocation?* We call the resulting contraction factor the *aggregate consumption-equivalent* variation. It measures the resources left over after everyone has been compensated; efficiency rises whenever the same welfare as in the status-quo can be achieved with less resources. This is a measure of efficiency since it summarizes the amount of resources that can be saved while attaining indifference. We do not refer to it as a measure of aggregate welfare to distinguish it from a social welfare function that embeds normative judgements about interpersonal utility comparisons.

Our measure answers a counterfactual question in terms of observables. Unlike social welfare functions, it does not introduce free parameters such as Pareto weights and is invariant to monotone transformations of utility (i.e. it depends only on ordinal properties of preference relations). Moreover, the measure does not take a normative stance on the optimal distribution of resources across individuals beyond compensating every agent relative to the status-quo. For example, if aggregate efficiency increases and there are extra resources left over after everyone has been compensated, our measure takes no stance on who should get those resources.

The numerical value of our efficiency measure is interpretable and, in the special case of a single agent, collapses to the familiar consumption-equivalent measure of Lucas. Because its definition neither presupposes the existence of markets nor depends on prices, it avoids the limitations and paradoxes of traditional cost-benefit analysis. The measure accommodates a wide range of decentralization mechanisms — such as competitive markets, search-and-matching, bargaining, and imperfect competition — as well as constraints on redistribution — including limited taxation or costly transfers. Moreover, our measure remains tractable across a broad class of models.

We characterize our measure of aggregate efficiency in terms of observables and generalize well-known representative-agent results to settings with heterogeneous-agents. This paper has two stand-alone companions —Baqaee and Burstein (2025a) and Baqaee and Burstein (2025b) — where we apply our framework to answer two different questions: the cost of misallocation due to financial market incompleteness (in both closed and open-economies) and changes in aggregate efficiency in random utility models with discrete choice. In both applications, household heterogeneity is a central feature of the problem.

The structure of the paper is as follows. In Section 2, we define our measure in abstract terms and present a key result: Theorem 1 converts the problem of calculating aggregate consumption-equivalents into an equivalent fictional utility-maximization problem for an agent with homothetic preferences. This forms the basis for all other results in the paper because it allows us to port tools used to study the welfare of representative-agents with homothetic preferences, like Hulten (1978), Harberger (1964), Arkolakis et al. (2012), Petrin and Levinsohn (2012), and Baqaee and Farhi (2019b, 2020) to economies with heterogeneous agents.

The abstract environment in Section 2 does not impose much structure on how consumption possibility sets come about (for example, general equilibrium is a special case). In Section 3, we specialize the environment to a general equilibrium setting with potentially distorting wedges and lump-sum transfers. We also define some popular alternative measures of efficiency from the literature to facilitate comparison with our approach: real output (using a Divisia or chain-weighted index), Kaldor-Hicks efficiency (which compares total income to the sum of compensating incomes), and the welfare of a positive representative agent (if such an agent exists). We establish an important equivalence result: if all households have identical homothetic preferences and face the same relative prices, our measure of aggregate efficiency (with lump-sum transfers) coincides with those alternative measures. Outside of these common but restrictive assumptions, however, the measures generally differ.

A key result in Section 3 is that our aggregate efficiency measure can be calculated using a *compensated* equilibrium. The compensated equilibrium is the general equilibrium of an economy with the same technologies and distortions as the real economy, but populated by a fictional representative agent with homothetic preferences. We show that the utility of this fictional representative agent measures the change in aggregate efficiency. This result makes it straightforward to use tools and methods from representative-agent economies to analyze aggregate efficiency with heterogeneous agents.

In Section 4 we restrict attention to perfectly competitive economies where both the first and second welfare theorem hold. We show that in such settings and to a first-order approximation, Hulten (1978) applies to our measure of aggregate efficiency unaltered. That is, to a first-order, our measure of changes in aggregate efficiency coincide with total factor productivity as measured by the Solow (1957) residual. We then derive a nonlinear version of Hulten (1978) that applies to aggregate consumption-equivalents, extending the nonlinear characterizations in Baqaee and Farhi (2019b) to economies with heterogeneous agents and potentially non-homothetic household preferences. We show that changes in aggregate efficiency depend only on expenditure shares and price elasticities. We also generalize the sufficient-statistics of Arkolakis et al. (2012), developed for single-agent economies, to quantify the gains from trade in economies with heterogeneous agents.

In Section 5 we consider distorted economies, and derive versions of Hsieh and Klenow (2009), Petrin and Levinsohn (2012), Harberger (1954, 1964), and Baqaee and Farhi (2020) that apply to economies with heterogeneous agents. In particular, we derive a version of the famous Harberger triangles formula that can be used to quantify misallocation with heterogeneous agents, and show that there is a sense in which misallocation losses in the heterogeneous agent model are lower than in representative agent models. This is because our Harberger triangles formula discards dispersion in wedges due to difference in average wedges paid by each household. This is because differences in the average wedge by household are equivalent to lump-sum transfers and do not imply (Pareto) inefficiency.

In Section 6 we consider economies with costly redistribution (i.e. without lump-sum transfers). We discuss how Theorem 1 can be used to analyze changes in aggregate efficiency when redistributive instruments are limited (e.g. revenues used for redistribution are raised via distortionary taxes). We show that, starting in perfect competition, the change in aggregate efficiency due to a change in primitives is, to a first-order, the same as Hulten (1978). To a second-order, the change in efficiency is equal to what would have happened with lump-sum transfers (characterized in Sections 4 and 5) minus the additional Harberger triangles caused by inefficient redistribution (which are zero if lump-sum transfers are available). We end the section with a quantitative example: how the rise of China affected the United States, as measured by the aggregate consumption equivalent. We show that the answer depends on the ease with which workers can move across sectors, and the range of redistributive tools available. Whereas the aggregate consumption equivalent change is positive when workers can move across sectors or if lump-sum transfers are available, it is negative if workers are restricted to working within narrow industries and redistribution is impossible or difficult.

Related literature. Our approach to measuring aggregate efficiency is related to willingness-to-pay based measures, which have a very long history dating all the way back to at least Dupuit (1844). For example, the compensating variation, and sum of compensating variations, in Hicks (1939) and Kaldor (1939), are special cases. Furthermore, the notion of social surplus in Allais (1979), the coefficient of resource utilization in Debreu (1951, 1954), the measure of efficiency in Farrell (1957), and the benefit function in Luenberger (1996) are all related to our measure. Our contribution relative to these works is to provide a characterization without assuming either Pareto efficiency or markets, explicitly allowing for limited or costly redistribution, and applying our measure to modern models.

Our paper is also related to cost-benefit analysis, typically performed by using the sum of compensating variations, as in Harberger (1971), and related ideas like the marginal value of public funds (Hendren and Sprung-Keyser, 2020). The idea behind these measures is to ask: "after the winners compensate the losers using lump-sum transfers, is there still money left on the table?" Our measure of efficiency coincides with these measures when both welfare theorems hold and the consumption-possibility set is linear. However, outside of these cases, the two measures are different. First, if the consumption-possibility set is nonlinear, then as shown by Boadway (1974), a pure transfer between agents can cause the sum of compensating variations to exceed zero. Intuitively, the transfer lowers prices for goods that are relatively more valued by losers than winners. Hence, it is possible to compensate the losers using the post-transfer prices and still have money left-over. Our measure, which can be defined even when prices do not exist, does not have this property.

Second, unlike the sum of compensating variations, our measure does not presuppose that lump-sum transfers are feasible. In this sense, our approach has similarities to Schulz et al. (2023), who generalize the sum of compensating variations to allow for limited redistribution.[1] Our paper complements and differs from Schulz et al. (2023) in many ways, the most important being a difference in focus. They consider economies with a single consumption good, focusing their attention on a mechanism design problem where lump-sum taxes are unavailable because of asymmetric information. Although our formalism and definitions can be applied to such economies, we do not focus on these issues. Instead, we focus on allowing for multiple goods and heterogeneity in preferences and relative prices faced by consumers. This means that even with perfect information and lump-sum transfers, there are interesting questions about how to aggregate across consumers that consume and value different goods.

As mentioned above, a different approach to aggregation is to use a social welfare function to evaluate outcomes. A prominent example is the behind the veil-of-ignorance measure of Harsanyi (1955). Social welfare functions are by far the most common approach in the modern literature to aggregating across heterogeneous agents.[2] Our paper,

[1]In response to a shock, they consider a tax reform that makes households indifferent to the status-quo and then measure the monetary value of aggregate welfare gains or losses by the fiscal surplus from this reform.

[2]There is a branch of the literature that assumes observed allocations can be rationalized by maximizing some social welfare function within some parametric class, estimates this function, and uses it to conduct policy analysis (see Heathcote and Tsujiyama, 2021 and the references therein). This is equivalent to assuming there exists a normative representative agent: a hypothetical single decision-maker whose utility

which instead looks for and quantifies the potential for Pareto improvements (i.e. compensating everyone and looking to see if resources are left over), studies an alternative question that the one analyzed by this methodology.

Following in the social-welfare-function tradition, a recent set of papers, including Bhandari et al. (2021), Dávila and Schaab (2022, 2023), and Donald et al. (2023) provide approximate decompositions of changes in social welfare functions. Our goal in this paper is different: we do not provide decompositions of social welfare functions, but instead, define and characterize aggregate efficiency directly as an answer to a counterfactual question. The decompositions in the papers mentioned above contain components the authors refer to as capturing efficiency. However, since our objective is different, our notion of efficiency is also generically different to the efficiency components in these papers. Defining efficiency directly, instead of as part of an approximate decomposition, is useful because it means that we can also study large changes.[3]

In terms of the tools and methods, our paper is closely related to the literature that studies the macroeconomic consequences of microeconomic productivity changes and wedges. For productivity changes, this includes Gabaix (2011), Acemoglu et al. (2012), Baqaee and Farhi (2019b) and others. For wedges, this includes Harberger (1954), and more recently, Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Bigio and La'O (2016), Liu (2017), Baqaee and Farhi (2020), among others. We relax the assumption typically maintained in both of these literatures that households have common preferences and face common prices.[4]

Finally, because we use the gains from trade as one of our examples, our paper is also related to the gains from trade with heterogeneous agents. Much of the work on international trade with heterogeneous agents focuses on the distributional effects of trade. Some examples of papers that also calculate aggregate welfare are Antras et al. (2017) and Galle et al. (2023) (using an Atkinson (1970)-style social welfare function with inequality aversion), Kim and Vogel (2020) (using the sum of compensating variations),

function is maximized by observed allocations (Chapter 4 Mas-Colell et al., 1995). Our approach is different since we do not need to assume the existence of either a positive nor normative representative agent. Furthermore, even if a normative representative agent exists, there is nothing to say that its preferences should be privileged over any other social welfare function (see Example 6 below).

[3]Whereas infinitesimal changes in our measure of efficiency can be integrated to study large changes, integrals of components in a decomposition of social welfare are path-dependent. To see this point, suppose we approximately decompose changes in some function $y = f(x_1, x_2)$ into $dy \approx (\partial f/\partial x_1)dx_1 + (\partial f/\partial x_2)dx_2$. Then we can write non-infinitesimal changes as $\Delta y = \int(\partial f/\partial x_1)dx_1 + \int(\partial f/\partial x_1)dx_1$ but, unless $f(x_1, x_2)$ is linear in x_1 and x_2, the size of each component of this nonlinear decomposition depends on the arbitrary path of integration.

[4]One exception is Bornstein and Peter (2024), who study misallocation with differences in tastes and markups across households. In their setting, symmetry and the law-of-large numbers implies that every households' problem is identical despite the fact that households have different preferences.

and Rodríguez-Clare et al. (2022) (using a population-weighted average of welfare gains across regions), all of which differ from our measure of aggregate efficiency for reasons already discussed.

2 Abstract Definition and Characterization

Consider an economy populated by agents indexed by $h \in \{1, ..., H\}$. Agent h has ordinal preferences $\succeq_h$ over commodity vectors $\boldsymbol{c}_h \in \mathbb{R}^N$, where N is the number of goods.[5] Assume preferences are represented by utility functions $u_h(\boldsymbol{c}_h)$.[6] A *consumption allocation* is a matrix $\boldsymbol{c} \in \mathbb{R}^{H \times N}$ whose hth row, denoted by $\boldsymbol{c}_h$, equals the consumption vector of agent h.

Fix some consumption allocation, denoted by $\boldsymbol{c}^0$, as the *status-quo*. We think of $\boldsymbol{c}^0$ as the data generated by some initial equilibrium outcome. Define $\mathcal{C} \subset \mathbb{R}^{H \times N}$ to be some set of feasible consumption allocations, determined by deeper primitives like technologies, policies, and so on. We think of $\mathcal{C}$ as the counterfactual we are interested in studying.

Definition 1 (Aggregate Consumption Equivalent Variation)**.** The *Aggregate Consumption-Equivalent Variation* of the set $\mathcal{C}$ relative to the status-quo $\boldsymbol{c}^0$ is the maximum proportional contraction of $\mathcal{C}$ such that every agent can be kept at least indifferent to the status-quo allocation. Formally,

$$A(\boldsymbol{c}^0, \mathcal{C}) \equiv \max \left\{ \phi \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \phi^{-1}\mathcal{C} \text{ and } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h \right\}. \quad (1)$$

We refer to A as aggregate efficiency throughout the paper. The *change* in aggregate efficiency relative to the status-quo is

$$\Delta \log A(\boldsymbol{c}^0, \mathcal{C}) = \log A(\boldsymbol{c}^0, \mathcal{C}) - \log A(\boldsymbol{c}^0, \boldsymbol{c}^0) = \log A(\boldsymbol{c}^0, \mathcal{C}).$$

The cardinal value of A is interpretable. For concreteness, say, $A = 1.01$, then this means that it is possible to make everyone at least as well off as in the status-quo and discard 1% of every good (or more precisely, $(1 - 1/A)\%$). Agents may not be consuming the same bundle as in the status-quo after they are compensated — we only require that they be indifferent to the status-quo. If there is a single household and $\mathcal{C}$ is a consumption allocation, then A is the same as the consumption-equivalent variation of Lucas (1987). As in both Lucas (1987) and Debreu (1951), the definition of A treats all commodities

[5]We assume that preferences are continuous and locally nonsatiated.

[6]That is, for each household $u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}'_h)$ if, and only if, $\boldsymbol{c}_h \succeq \boldsymbol{c}'_h$.

symmetrically by shifting the consumption possibility set, $\mathcal{C}$, proportionately in every dimension.[7]

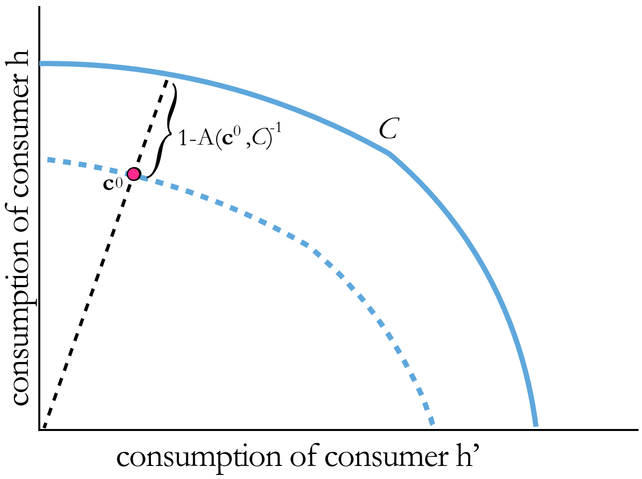

Figure 1: Aggregate efficiency is measured by the maximal radial contraction of the feasible set necessary to achieve indifference.

Figure 1 illustrates the change in aggregate efficiency for a simple economy with two households, indexed by h and h', and two consumption goods, one consumed only by h and the other only by h'. The set $\mathcal{C}$ is the set of feasible consumption possibilities. As the figure shows, if $\Delta \log A > 0$, then it must be the case that there are feasible allocations in $\mathcal{C}$ that Pareto-dominate the status-quo $\boldsymbol{c}^0$.

The measure in Definition 1 has some desirable properties: (1) it answers a counterfactual question about observable phenomena with interpretable units (i.e. how much of every good is left-over after everyone is compensated?). The answer to this question is invariant to monotone transformations of utility functions, and only relies on ordinal

[7]Note that the choice of how to scale the set $\mathcal{C}$ is present even with a single agent. For example, do we shrink or expand every element in a consumption vector to reach indifference, as in the consumption-equivalent variation of Lucas (1987), or only some components. Similarly, do we scale the whole budget set proportionally, as in the compensating variation of Hicks (1939), or do we change individual prices of certain commodities to reach indifference. In this sense, this choice is not special to having heterogeneous agents or not. One reason to focus on a radial contraction is because in general equilibrium, expanding and contracting the feasible consumption set proportionally is isomorphic to scaling the vector of factor endowments (including quasi-fixed factors capturing decreasing returns to scale). This is an intuitive way to measure efficiency — efficiency is measured by the reduction in factor endowments necessary to reach indifference to status-quo. The reason our measure of aggregate efficiency differs from standard approaches like using social welfare functions or chained real consumption in multi-agent settings is not due to the fact that we scale the consumption possibility set proportionally. For example, our measure differs even if there is only a single consumption good (so there is no choice of how to expand or shrink the consumption set because there is only one consumption good). In some settings, it may be interesting to expand or shrink $\mathcal{C}$ in a particular direction, rather than radially. This requires generalizing our definition to allow for non-radial expansions, as in the transferable surplus notion in Allais (1979). We do not pursue this generalization in this paper.

properties of preference relations. (2) Our measure does not take a stance on how social surplus or losses should be divided among agents. That is, while we can assign a numerical efficiency value to every feasible set of consumption allocations, $\mathcal{C}$, we do not attempt to pick a specific allocation among the possibilities as being socially "optimal." (3) Our definition places no restrictions on the set of redistributive tools that are available or the mechanism by which allocations are decentralized (e.g. spot markets, search, bargaining, etc.). The instruments of redistribution are implicitly embedded into the shape of the feasible set $\mathcal{C}$.[8] (4) This abstract definition is applicable to a wide variety of models and circumstances, allowing for, among other things, non-homothetic preferences, discrete choice, risk, and dynamics.[9]

To characterize $\Delta \log A$, we prove a useful theorem, which we repeatedly use in the rest of the paper. This theorem proves that calculating $\Delta \log A$ is equivalent to solving a utility-maximization problem for some fictitious agent. To state this result, we first define *homothetized* transformations of individual preferences.[10]

Definition 2. Let $u_h(\boldsymbol{c}_h)$ denote a utility representation for agent h. The *homothteized utility function* $\tilde{u}_h(\boldsymbol{c}_h)$ is implicitly defined by

$$u_h(\frac{\boldsymbol{c}_h}{\tilde{u}_h}) = u_h(\boldsymbol{c}_h^0).$$

The homothetized utility function, $\tilde{u}_h$, is homogenous of degree one in consumption by construction. If the preference relation $\succeq_h$ is homothetic, then $\tilde{u}$ is a cardinalization of $\succeq_h$ — in this case, $\tilde{u}_h$ ranks consumption bundles in the same order as $\succeq_h$. By construction, $\tilde{u}_h$ is homogenous of degree one and normalized to equal to 1 at $\boldsymbol{c}_h^0$. The magnitude of $\tilde{u}_h(\boldsymbol{c}_h)$ is interpretable — it measures the amount the consumption bundle $\boldsymbol{c}_h$ has to be scaled to make the household exactly indifferent to the status-quo. In this sense, $\tilde{u}_h(\boldsymbol{c}_h)$ is the household's consumption-equivalent variation relative to the status-quo.

Example 1 (Single good). Suppose there is a single consumption good, so $u_h(c_h)$ is some

[8]For example, for convex competitive economies, if lump-sum transfers are available, then the second welfare theorem implies that $\mathcal{C}$ is the set of all Pareto-efficient allocations. However, the definition can also be applied to economies where such transfers are not available.

[9]Our measure of aggregate efficiency can be used to order feasible sets under a given status-quo. The ordering of two feasible sets may flip for different status-quos, similar to Scitovszky (1941). For a fixed status-quo, our measure of aggregate efficiency gives a unique ordering of feasible sets. For example, if $\mathcal{C}' \subseteq \mathcal{C}$, then $A(\boldsymbol{c}^0, \mathcal{C}') \leq A(\boldsymbol{c}^0, \mathcal{C})$.

[10]The homothetized utility function is also called the *distance* function in the duality literature on optimization (see, for example, Cornes, 1992).

increasing function. In this case,

$$\tilde{u}_h(\boldsymbol{c}_h) = \frac{c_h}{c_h^0},$$

regardless of the functional form of u_h. See Appendix D for a more involved example using non-homothetic CES preferences.

If $\succeq_h$ is non-homothetic, then $\tilde{u}_h$ is *not* a cardinalization of $\succeq_h$ (i.e. $\tilde{u}_h$ does not rank consumption allocations according to $\succeq_h$). Figure 2 graphically depicts indifference curves of $\tilde{u}_h$ — they are radial expansions of the status-quo indifference curve defined by $u_h(\boldsymbol{c}_h) = u_h(\boldsymbol{c}_h^0)$. When $\succeq_h$ is homothetic, all indifference curves are radial expansions, so that the ranking produced by $\tilde{u}_h$ coincides with the one produced by $\succeq_h$.

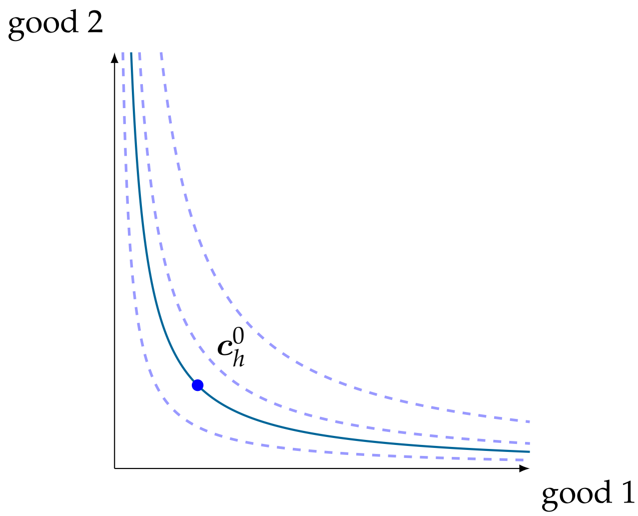

Figure 2: The solid blue line is the indifference curve $u_h(\boldsymbol{c}_h) = u_h(\boldsymbol{c}_h^0)$ and the dashed lines are the indifference curves of $\tilde{u}_h$.

Define a fictitious compensated agent as follows.

Definition 3. The *compensated agent* is an agent whose preferences are represented by

$$U(\boldsymbol{c}) = \min_h \{\tilde{u}_h(\boldsymbol{c}_h)\},$$

where $\tilde{u}_h$ are homothetized utility functions.

The utility function $U(\boldsymbol{c})$ is homogeneous of degree one by construction. We call this agent compensated because, as we show in Appendix A, the budget shares of this agent are the average of the compensated budget shares of all the agents weighted by each agent's compensating income.

Note that, the function $U(\boldsymbol{c})$ is not a social welfare function. There are two reasons for this. First, U does not depend on households' true utility functions, instead it depends on the "homothetized" utility functions. Second, U depends on the minimum growth in homothetized utility relative to status-quo, rather than the level of utility (i.e. $U(\boldsymbol{c})$ is not a Rawlsian social welfare function).

We can now state the main result of this section.

Theorem 1 (Aggregate Efficiency by Utility Maximization)**.** *The aggregate consumption equivalent is equal to the value of $\mathcal{C}$ to the compensated agent:*

$$A(\mathcal{C}, \boldsymbol{c}^0) = \max_{\boldsymbol{c} \in \mathcal{C}} U(\boldsymbol{c}).$$

Figure 3 graphically illustrates the content of Theorem 1. Rather than proportionally shifting $\mathcal{C}$ to reach the indifference point, Theorem 1 states that we can instead maximize $U(\boldsymbol{c})$ — by shifting out the indifference curves of the compensated agent — until we reach the boundary of $\mathcal{C}$. The utility of this fictional agent is numerically identical to the maximal reduction in $\mathcal{C}$ needed to reach indifference.

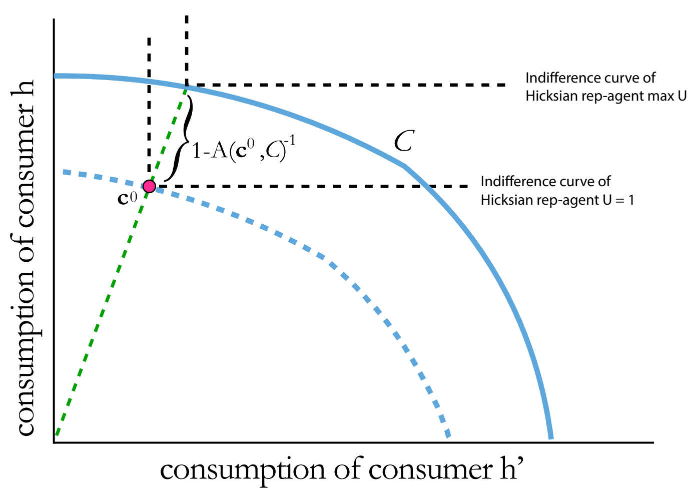

Figure 3: The increase in the utility of the compensated agent also measures the amount by which $\mathcal{C}$ needs to shrink to ensure indifference.

Theorem 1 is crucial because it converts the problem of calculating aggregate efficiency in (1) into an equivalent utility-maximization problem. Since utility-maximization problems are common in economics, this means that Theorem 1 allows us to easily convert results about representative agent problems into ones about aggregate efficiency with heterogeneous agents.

Theorem 1 guarantees that maximizing $U(\boldsymbol{c})$ yields the same number as the maximization problem that defines A. However, unlike a social welfare maximization problem, the specific allocation in $\mathcal{C}$ that maximizes $U(\boldsymbol{c})$ has no special significance, since the primitive problem defining $\Delta \log A$ is stated in terms of shrinking the possibility frontier, not choosing an allocation inside it.

The rest of the paper uses Theorem 1 in different contexts to characterize changes in aggregate efficiency in terms of observables. Although Theorem 1 applies very generally, we focus on cases where the consumption possibility set is determined by general equilibrium (possibly with distortions and limited transfers).

3 Decentralized Equilibrium

In this section we set up a general-equilibrium framework that admits distortive wedges. We assume the consumption possibility set is the set of equilibrium consumption allocations achievable by lump-sum transfers (Section 6 considers when transfers are restricted). We show, via Theorem 1, how to compute the aggregate consumption-equivalent variation in these settings. For comparison, we also define three alternative metrics that are popular in the literature: (1) chain-weighted real output, (2) the sum of compensating variations (Kaldor-Hicks/Cost-Benefit), and (3) the welfare of a positive representative agent. We establish the restrictive conditions under which all four measures coincide. The remainder of the paper then examines the more general cases in which these conditions fail, and where our measure no longer aligns with the conventional ones.

3.1 Environment and Equilibrium

Each household maximizes utility $u_h(\boldsymbol{c}_h)$ subject to the budget constraint

$$\sum_i p_i c_{hi} \leq \sum_f \omega_{hf} w_f L_f + T_h,$$

where the left-hand side is total expenditures and the right-hand side is total income. As in Arrow-Debreu, commodities could be indexed by time and state of nature. On the left-hand side, p_i is the price of i and c_{hi} is the quantity of good i purchased by household h. On the right-hand side, households derive income from factors and lump-sum transfers. Households h owns a share ω_{hf} of factor f, where w_f is the wage and L_f is the total quantity of factor f. Lump-sum transfers are T_h.

Producer i chooses its inputs to minimizes costs

$$\sum_j p_j y_{ij} + \sum_f w_f l_{if},$$

subject to production technology

$$y_i = z_i F_i \left(\{ y_{ij} \}, \{ l_{if} \} \right),$$

where y_i is the quantity of output, F_i is a constant-returns production function, y_{ij} are intermediate inputs used by i produced by j, and l_{if} are primary factors used by i. The assumption that F_i has constant-returns is without loss of generality, since we can capture decreasing returns using producer-specific factors. The parameter z_i is a Hicks neutral productivity shifter. The price of i is equal to an exogenous markup or tax, $\mu_i > 0$, times i's marginal cost of production. That is, the price of i is inclusive of the wedge on i's output.

The resource constraint for goods and factors is

$$\sum_j y_{ji} + \sum_h c_{hi} \leq y_i, \qquad \text{and} \qquad \sum_i l_{if} \leq z_f L_f,$$

where z_f, when f indexes a factor, controls the endowment of efficiency units of factor f. Finally, net transfers across households are equal to the revenues generated by the wedges:

$$\sum_h T_h = \sum_i p_i y_i \left(1 - \frac{1}{\mu_i} \right). \tag{2}$$

Buyer-seller-specific productivity and wedges. Although we assume that z_i is Hicks neutral and wedges are on gross output only, both of these assumptions are made without loss of generality. This is because we can recreate buyer-seller productivity changes and wedges by relabeling. Specifically, we can treat firm or household i's purchases of an input from j as a distinct good (made linearly using j's output). A productivity shock or a wedge on this good is then isomorphic to a buyer-seller specific productivity shock or wedge. We make the assumption that z_i is Hicks neutral and assume all wedges take the form of taxes on gross output to simplify the notation.

We now define a general equilibrium with wedges.[11]

[11] This notion of general equilibrium is the same one used by Baqaee and Farhi (2020), extended to allow for multiple households.

Definition 4 (Decentralized Equilibrium with Wedges)**.** A *decentralized equilibrium with wedges* is the collection of prices and quantities such that: (1) the price of each good i equals its marginal cost times a wedge μ_i; (2) each producer chooses quantities to minimize costs taking prices as given; (3) each household chooses consumption quantities to maximize utility taking prices, consumption taxes, and income as given; (4) net transfers across households are equal to wedge revenues; (5) all resource constraints are satisfied.

3.2 Different Aggregate Measures in General Equilibrium

We specialize the definition of aggregate-consumption equivalents to this environment. We also define some other popular measures of aggregate economic activity used in the literature. To do so, we index all exogenous parameters (productivities, wedges, transfers) by a scalar t and let $t = 0$ denote the status-quo allocation. For any equilibrium price or quantity X, we write $X(t)$ to denote its dependence on the exogenous parameters.[12]

We begin by defining our measure of efficiency.

Aggregate Consumption-Equivalent Variation. Denote the equilibrium consumption allocations given productivity parameters, $\boldsymbol{z}$, wedges, $\boldsymbol{\mu}$, and lump-sum transfers, $\boldsymbol{T}$ by $c(\boldsymbol{z}, \boldsymbol{\mu}, \boldsymbol{T})$. If the equilibrium is unique, then this is a singleton. The consumption possibility set, given lump-sum transfers, is

$$\mathcal{C}(t) = \{\boldsymbol{c}(\boldsymbol{z}(t), \boldsymbol{\mu}(t), \boldsymbol{T}) \text{ for some transfers } \boldsymbol{T} \text{ satisfying (2)}\}.$$

In words, $\mathcal{C}(t)$ is the set of equilibrium consumption allocations that can be attained by varying lump-sum transfers.

Applying Definition 1 to $\mathcal{C}(t)$ yields the following:

$$A(t) = \max \left\{ \phi \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \phi^{-1}\mathcal{C}(t) \text{ and } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h \right\}. \tag{3}$$

In words, $A(t)$ is the maximum contraction of the consumption possibility set, scaling every feasible allocation by $1/A(t)$, that allows every agent to be kept at least indifferent to the status-quo. Note that the consumption possibility set may be distorted, in the sense that it is not the Pareto frontier defined by technologies. Furthermore, by construction, aggregate efficiency at the status-quo is equal to one: $A(0) = 1$.

[12] In the case of multiple equilibria, we assume there is an equilibrium selection mechanism. The nature of this equilibrium selection mechanism is not relevant for $A(t)$, because $A(t)$ is unique given t and the status-quo.

We now define some other commonly used measures of aggregate efficiency for comparison.

Real Output. In national income accounting, real output is measured using approximations to the Divisia (1925) index. Aggregate real output, defined using the Divisia index, is

$$\log Y(t) = \int_0^t \sum_i \frac{p_i(s)c_i(s)}{\sum_{i'} p_{i'}(s)c_{i'}(s)} \frac{d \log c_i(s)}{ds} ds,$$

where $c_i(s) = \sum_{h \in H} c_{hi}(s)$ denotes aggregate consumption of good i at $s \in [0,t]$.[13] In words, this is the cumulative change in the quantity of every final good, weighted by the contemporaneous expenditure share on that final good. By construction, real output in the status-quo is one: $Y(0) = 1$.[14]

Kaldor-Hicks/Cost-Benefit Efficiency. Another popular aggregate measure is the Kaldor-Hicks efficiency measure. This measure compares the sum of compensating incomes to aggregate income at t. If the sum of compensating incomes is less than aggregate income, then the winners can hypothetically compensate the losers and there can still be money left-over. The amount of money left over is a measure of the increase in efficiency. This method is the foundation of most of cost-benefit style analyses in applied welfare economics and policy evaluation in public finance and industrial organization.

Let $e_h(\boldsymbol{p}, u_h)$ be an expenditure function representing preferences $\succeq_h$. The Kaldor-Hicks measure of efficiency at t is

$$A^{KH}(t) = \frac{\sum_h e_h(\boldsymbol{p}(t), u_h(t))}{\sum_h e_h(\boldsymbol{p}(t), u_h(0))}. \tag{4}$$

Note that, by construction, Kaldor-Hicks efficiency at the status-quo is equal to one: $A(0) = 1$.

Consumption-equivalent of Representative Agent. Another well-known aggregate measure, when a representative agent exists, is the consumption-equivalent variation used by

[13] In practice, statistical agencies apply this formula in a static way, period-by-period, to define real output and use chain-weighted discretized approximations to the true (integral) Divisia index.

[14] As we show below, and is well-understood, in the absence of a representative agent with homothetic preferences, chain-weighted real output does not necessarily measure anything welfare relevant. See, for example, Hulten (1973) and, more recently, Baqaee and Burstein (2023). Nevertheless, $\Delta \log Y$ is a useful, and commonly relied upon, statistic which, under the assumptions of Proposition 1, is a welfare-relevant measure.

Lucas (1987). A *representative agent* is a hypothetical single consumer such that the demand of the representative agent for each good, given prices and total income, coincides with equilibrium quantity of that good, given the same prices and aggregate income.[15]

If a representative agent exists, define the consumption-equivalent for the representative agent, $A^{RA}(t)$, to be

$$u^{RA}\left(\boldsymbol{c}^{RA}(t)/A^{RA}(t)\right) = u^{RA}\left(\boldsymbol{c}^{RA}(0)\right),$$

where u^{RA} is the utility function of the representative agent. In words, $A^{RA}(t)$ is the amount by which the aggregate consumption bundle in t must be contracted to make the positive representative agent exactly indifferent to the status-quo. As with all the other measures, $A^{RA}(0) = 1$ by construction.

3.3 Characterizing $A(t)$ Using Theorem 1

We characterize aggregate efficiency via Theorem 1. To do so, we define a compensated equilibrium, which is a useful fictional construct for proving results and constructing sufficient statistics formulas.[16]

Definition 5 (Compensated Equilibrium)**.** A *compensated equilibrium* is the general equilibrium of an economy with the same technologies, resource constraints, and wedges as the original economy but where there is a representative agent with preferences as in Definition 3. For any equilibrium variable $X(t)$, denote the same variable in the compensated equilibrium by $X^{\text{comp}}(t)$.

It is important to note that compensated equilibrium prices and quantities are not of direct interest themselves, but are instead a useful stepping-stone to calculating changes in aggregate efficiency.

The following result, which is a consequence of Theorem 1, shows that aggregate efficiency is as simple as calculating welfare in an economy with a representative agent.

[15] For a formal definition, see Appendix B.

[16] This notion of the compensated equilibrium has many antecedents in prior work, for example it nests the concept in Jones (2002) and Johansson et al. (2022), the Hicksian equilibrium in Baqaee and Burstein (2023), the synthetic equilibrium in Debreu (1951), and is closely related to the adjusted price function in Luenberger (1996). A major difference relative to these notions is that our compensated equilibrium need not be efficient.

Theorem 2 (Aggregate Efficiency Using Compensated Equilibrium). *The aggregate consumption equivalent variation can be calculated using the compensated equilibrium:*[17]

$$A(t) = U(\boldsymbol{c}^{comp}(t)) = Y^{comp}(t) = A^{KH,comp}(t) = A^{RA,comp}(t).$$

In words, aggregate efficiency, $A(t)$, can be computed by solving for changes in the utility of the compensated representative agent in a Walrasian equilibrium where the representative agent takes prices as given. The first equality, $A(t) = U(\boldsymbol{c}^{\text{comp}}(t))$, is the core new insight; subsequent identities follow mechanically from the fact that the compensated economy has a single representative agent.

The importance of Theorem 2 lies in the fact that it allows every tool and result used to calculate welfare in homothetic representative agent economies to be used to calculate aggregate efficiency with heterogeneous (and non-homothetic) preferences.

Theorem 2 is expressed in terms of endogenous variables in the compensated equilibrium. Solving that equilibrium is essentially the same as solving a representative–agent model, which is well-understood problem. For this reason, we present the full characterization of variables in the compensated equilibrium in Appendix Section E.[18] However, we do note the following useful fact about the compensated equilibrium in this section.

Lemma 1 (Compensated equilibrium at the status quo). *At the status-quo $t = 0$, prices and quantities in the compensated equilibrium coincide with those in the decentralized equilibrium.*

This lemma, which guarantees that compensated and decentralized variables coincide at the status-quo, is important for calibrating the model when solving for the compensated equilibrium — expenditure shares at the compensated equilibrium must coincide with observations in the absence of any shock (i.e. at the status-quo).

Before deploying Theorem 2 to construct heterogeneous-agent generalizations of well-known results, we first point out an important, but highly restrictive, special case, where our measure of aggregate efficiency coincides with the other popular alternatives.

3.4 A Miraculous Consensus

To set aside household heterogeneity, a standard benchmark is to assume every household has identical homothetic preferences and faces the same relative prices (i.e. there are no household-specific wedges). Under this condition we obtain the following.

[17] If there are multiple compensated equilibria, pick the one with the highest $U(\boldsymbol{c}^{\text{comp}})$.

[18] In Appendix A we also provide the expenditure function of the Hicksian representative agent in the compensated equilibrium, since the expenditure function is a useful way to solve general equilibrium models.

Proposition 1 (Miraculous Consensus)**.** *If households have identical homothetic preferences, and face the same relative prices, then a positive representative agent exists and*

$$A(t) = Y(t) = A^{KH}(t) = A^{RA}(t).$$

The first equality can break down if lump-sum transfers are not available.

In words, the change in aggregate efficiency, measured by the consumption-equivalent variation, matches the change in chain-weighted index of real output, Kaldor-Hicks (cost-benefit) efficiency, and the consumption-equivalent of the positive representative agent all in the decentralized equilibrium. That is, under these assumptions, one can compute $A(t)$ without relying on the compensated equilibrium. Each of the underlying assumptions is individually essential: relaxing any one breaks the equivalence. The remainder of the paper explores those more general settings and illustrates, through examples, why our efficiency measure avoids the paradoxes of the alternatives when consensus fails. We consider non-identical and non-homothetic preferences in Section 4, allow for different households to pay different relative prices for the same goods (due to wedges) in Section 5, and consider limits to redistribution (e.g. no lump-sum taxes) in Section 6.[19]

Proposition 1 is a consequence of Theorem 2. The reason is that, under the stated assumptions, the price and quantity of each good in the compensated equilibrium coincides with those in the decentralized equilibrium. This implies that $A(t) = Y^{\text{comp}}(t) = Y(t)$, since real output in the compensated and decentralized equilibrium only depend on the prices and quantities of goods. The remaining two equalities are then standard.[20]

4 Competitive Economies with Lump-Sum Transfers

In this section, we characterize how aggregate efficiency responds to changes in productivity when both welfare theorems hold. This means that, for the remainder of this sec-

[19]In this paper, we are focused on household heterogeneity, but even with a single household, the miraculous consensus breaks down when preferences are non-homothetic. This point is discussed in detail by Baqaee and Burstein (2023). Intuitively, when preferences are non-homothetic, even for a single agent, scaling the production possibility set ($A(t)$), the budget constraint ($A^{KH}(t)$), and the equilibrium consumption allocation ($A^{RA}(t)$) do not coincide with one another since, as we shrink resources, the household would want to change the bundle of goods they consume.

[20]Under the stated assumptions, there is a positive representative agent with homothetic preferences, and it follows from Shephard's lemma that $Y(t) = A^{RA}(t)$ (see, e.g., Baqaee and Burstein, 2023). The final equality in Proposition 1 follows from the fact that the indirect utility function of each agent, $v(\boldsymbol{p}, I_h)$ can be written as $I_h/P(\boldsymbol{p})$, where $P(\boldsymbol{p})$ is an ideal price index. It then follows that, $v^{RA}(\boldsymbol{p}, \sum_h I_h)$, can be written as $(\sum_h I_h)/P^{RA}(\boldsymbol{p})$. Hence, $A^{RA}(t) = v^{RA}(\boldsymbol{p}(t), \sum_h I_h(t))/v^{RA}(\boldsymbol{p}(0), \sum_h I_h(0)) = \sum_h I_h(t)/(\sum_h I_h(0)) \times P(\boldsymbol{p}(0))/P(\boldsymbol{p}(t)) = A^{KH}(t)$.

tion, we assume that all wedges, $\boldsymbol{\mu}(t)$, are all equal to one (first welfare theorem holds) and lump-sum transfers are available (second welfare theorem holds). We consider distorted economies in Section 5 and economies without lump-sum transfers in Section 6.

We begin this section by providing some general comparative static results. We then apply these results to some analytical examples to build intuition.

4.1 Comparative Statics for Changes in Technologies

Denote the *Domar* weight of each producer or factor i by

$$\lambda_i(t) = \frac{p_i(t)y_i(t)}{\sum_{i'} p_i(t)c_i(t)}\mathbf{1}\{i \text{ is a producer}\} + \frac{w_i(t)z_i(t)}{\sum_{i'} p_i(t)c_i(t)}\mathbf{1}\{i \text{ is a factor}\}.$$

This the sales of i divided by total final expenditures. Recall that for factor i, the quantity of the factor is z_i. The following is a well-known result characterizing changes in real output in the decentralized equilibrium.

Proposition 2 (Hulten's Theorem)**.** *The change in chain-weighted real output is*

$$\log Y(t) = \int_0^t \sum_i \lambda_i(s) \frac{d \log z_i}{ds} ds. \tag{5}$$

The most well-known consequence of this result, given by differentiating with respect to t, is that $d \log Y/dt = \sum_i \lambda_i d \log z_i/dt$. This formula, which generalizes Solow (1957), shows that the elasticity of real output to the productivity of producer i or the quantity of factor i is just the Domar weight of i. Using Theorem 2, we can easily state a version of Hulten's theorem that applies to our measure of aggregate efficiency instead.

Proposition 3 (Compensated Hulten's Theorem)**.** *The change in aggregate efficiency at t is*

$$\log A(t) = \int_0^t \sum_i \lambda_i^{comp}(s) \frac{d \log z_i}{ds} ds. \tag{6}$$

In Appendix E, we characterize $\lambda_i^{comp}(s)$ explicitly as a function of the productivity changes $\Delta \log z$, elasticities of substitution, and expenditure shares.

Differentiating (6) with respect to t and evaluating at $t = 0$ shows that, to a first-order approximation, the change in aggregate efficiency, $\Delta \log A$, coincides with the change in real output in the competitive equilibrium $\Delta \log Y$.

Corollary 1 (First Order Changes in Aggregate Efficiency)**.** *To a first-order approximation, the change in aggregate efficiency is*

$$\Delta \log A \approx \sum_i \lambda_i^{comp}(0) \Delta \log z_i = \sum_i \lambda_i(0) \Delta \log z_i \approx \Delta \log Y.$$

The second equality follows from Lemma 1, which states that prices and quantities in the compensated equilibrium $t = 0$ are equal to those in decentralized economy. In other words, the first-order version of Hulten's theorem applies unaltered to aggregate efficiency.

Baqaee and Farhi (2019b) show that, to a second-order approximation, changes in real output are given by

$$\Delta \log Y \approx \sum_i \lambda_i \Delta \log z_i + \frac{1}{2} \sum_i \Delta \lambda_i \Delta \log z_i.$$

Differentiating (6) twice with respect to t and evaluating at $t = 0$, gives the following extension of Baqaee and Farhi (2019b) to multi-agent settings.

Proposition 4 (Second Order Changes in Aggregate Efficiency)**.** *To a second-order approximation, the change in aggregate efficiency due to changes in primitives is*

$$\Delta \log A \approx \sum_i \lambda_i \Delta \log z_i + \frac{1}{2} \sum_i \Delta \lambda_i^{comp} \Delta \log z_i,$$

where λ_i and $\Delta \lambda_i^{comp}$ are evaluated at status-quo. In Appendix E, we write $\Delta \lambda_i^{comp}$ explicitly as a function of the productivity changes $\Delta \log z$, microeconomic elasticities of substitution, and expenditure shares in the status-quo.

Proposition 4 shows that discrepancies between aggregate efficiency $\Delta \log A$ and real output $\Delta \log Y$ start at the second-order, since, generically $\Delta \lambda \neq \Delta \lambda^{\text{comp}}$ (explicit formulas are in Appendix E).[21] The gap between $\Delta \lambda_i$ and $\Delta \lambda_i^{\text{comp}}$ arises because the compensated agent's demand responds differently to shocks than aggregate demand in the decentralized economy. The compensated agent's demand is constructed to preserve indifference with status-quo for all households, whereas the decentralized aggregate demand comes from utility maximization by households whose incomes evolve according to equilibrium changes in relative factor prices.

[21] To derive an expression for $\Delta \lambda^{\text{comp}}$ in terms of microeconomic primitives, we use the fact that $\Delta \lambda^{\text{comp}}$ is the change in Domar weights in a special case of the environment considered by Baqaee and Farhi (2019b) where the consumption growth of each agent is treated as-if it is a final good, and there is a Leontief final demand aggregator over final goods.

4.2 Some Paradoxes of Real Output and Kaldor-Hicks

Having characterized $A(t)$ in economies where the first and second welfare theorem hold, we now contrast $A(t)$ with real output, $Y(t)$, and Kaldor-Hicks efficiency $A^{KH}(t)$.

The following proposition is a well-known paradox of Divisia-based indices, like real output.

Proposition 5 (Chain-Drift Paradox for Real Output)**.** *Unless households have identical and homothetic preferences, for $t > 0$, the value of $Y(t)$ can be any positive number, regardless of the technology parameters $\boldsymbol{z}(t)$ (outside of knife-edge cases).*

See Hulten (1973) or more recently Baqaee and Burstein (2023) for more information. Hence, although $A(t)$ and $Y(t)$ coincide up to a first-order approximation at $t = 0$, they are not the same nonlinearly. The technical reason is that $\log Y(t)$ is a line integral, and unless preferences are identical and homothetic, the vector field defined by Domar weights is not conservative, making $\log Y(t)$ dependent on the path of integration. Whereas $\boldsymbol{\lambda}(t)$ is not a conservative vector field, $\boldsymbol{\lambda}^{\text{comp}}(t)$ is a conservative vector field, so $\log A(t)$ only depends on $\boldsymbol{z}(t)$ (this is also obvious from the way $A(t)$ is defined).

We now turn our attention to Kaldor-Hicks efficiency. Of course, if lump-sum transfers are not available, then $A(t)$ and $A^{KH}(t)$ are different, and we explore the case with limited redistribution in Section 6. However, even when lump-sum transfers are available, our measure need not coincide with $A^{KH}(t)$. The following proposition illustrates this fact.

Proposition 6 (Paradox for Kaldor-Hicks Efficiency)**.** *For any change in technologies (movements of the Pareto efficient frontier), the change in aggregate efficiency, measured using aggregate consumption-equivalent variation, is weakly less than Kaldor-Hicks efficiency:*

$$\Delta \log A \leq \Delta \log A^{KH}.$$

For pure redistributions (movements along the Pareto efficient frontier), the change in aggregate efficiency, measured using aggregate consumption-equivalent variation, is zero, whereas the change in Kaldor-Hicks efficiency can be positive:

$$\Delta \log A = 0 \leq \Delta \log A^{KH}.$$

These inequalities are strict outside of knife-edge cases. The final inequality is a restatement of the so-called Boadway (1974) paradox. It states that $A^{KH}(t)$ assigns strictly positive value to pure redistributions when relative prices respond to transfers.[22]

[22]See also Blackorby and Donaldson (1990) for a related critique of the sum of compensating variations

Figure 4 illustrates the Boadway paradox using a two-good, two-consumer economy. Intuitively, redistributions lower the relative price of those goods that are more valued by the losers. Hence, in the new equilibrium, it is relatively cheap to compensate these households. Of course, such compensations are, in practice, infeasible because if they were to occur, then relative prices would rise for those households that need compensation.[23] Graphically, A^{KH} evaluates efficiency by shifting the line tangent to the frontier at $\boldsymbol{c}_1$ (the dashed line) until it reaches the status-quo $\boldsymbol{c}_0$. The increase in A^{KH} depends on how much the tangent line must be shifted inwards to reach the status-quo. In contrast, A uses the frontier itself. Since both $\boldsymbol{c}_0$ and $\boldsymbol{c}_1$ are on the same frontier, $\Delta \log A = 0$ in this case.

In fact, it is possible to construct examples where the production possibility set of the economy shrinks, $\mathcal{C}(t) \subset \mathcal{C}(0)$, so that $\Delta \log A < 0$, but $\Delta \log A^{KH} > 0$. This happens if the decentralized equilibrium associated with $\mathcal{C}(t)$ has very different relative prices to $\mathcal{C}(0)$. In this case, it is relatively cheap for the winners to compensate the losers under the new prevailing prices.

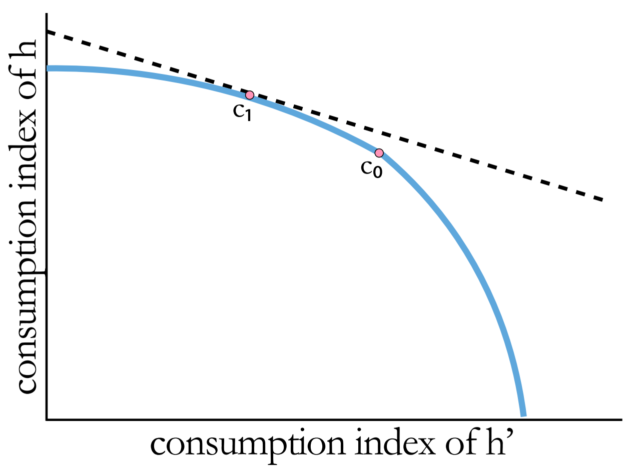

Figure 4: The sum of compensating variations is less than aggregate income since $\boldsymbol{c}^0$ is below the dashed straight line.

There are some special cases where $A^{KH}(t)$ and $A(t)$ coincide. First, $A(t)$ and $A^{KH}(t)$ coincide to a first-order approximation at $t = 0$ (assuming lump-sum transfers are available). Second, $A^{KH}(t)$ and $A(t)$ coincide nonlinearly if relative prices do not depend on final demand. This happens when the economy's production possibility frontier is a

as a measure of efficiency.

[23]See Jones (2002) for a detailed discussion.

hyperplane, so the marginal rate of transformation, and hence relative prices, are determined by technology only.[24] Under these conditions, the change in aggregate efficiency, $\Delta \log A$, coincides with the change in Kaldor-Hicks efficiency as defined in Equation (4). Given our assumption that production functions are all constant-returns to scale, the production possibility frontier becomes a hyperplane whenever there is only one primary factor of production.

Of course, there is another important reason (besides endogeneity of prices) why our measure of efficiency can differ from the Kaldor-Hicks measure. The Kaldor-Hicks measure, by summing up compensating variations, implicitly assumes that lump-sum transfers are available, so that winners can costlessly compensate the losers (assuming relative prices are constant). By contrast, our definition of aggregate efficiency naturally extends to allow for limited redistribution, as we discuss further in Section 6.

4.3 Analytical Examples

To build some intuition, we work through some analytical examples for how $\Delta \log A$ responds to changes in technologies. Appendix D provides more detailed derivations.

Example 2 (Regional Productivity Shocks). Consider households in different regions, indexed by h, with preferences over tradeable goods and locally produced nontradeable services:

$$u_h(\boldsymbol{c}_h) = c_{hg}^{\alpha} c_{hs}^{1-\alpha}, \qquad \sum_h c_{hg} = z_g, \qquad c_{hs} = z_{hs}.$$

The first equation shows that utility in each region depends on goods and services with the expenditure share on goods equal to α. The second equation is the resource constraint for goods, which clear at the aggregate level, since goods are traded. The third equation is the resource constraint for services, which clear region-by-region, since services are not traded. The parameters z_g and z_{hs} control the endowments of goods and services.

To calculate the status-quo (as well real output), we must make assumptions on factor ownership. Suppose that households in region h own the local endowment of services and own a share χ_h of the aggregate endowment of the traded good. This implies that, in equilibrium, χ_h is the expenditures of each household as a share of total consumption expenditures. The Domar weight on goods is $\lambda_g = \sum_h \chi_h \alpha = \alpha$ and the Domar weight on services in region h is $\lambda_{hs} = \chi_h(1-\alpha)$.

[24]See Proposition 11 in Appendix C for a formal statement.

When productivities change, according to Proposition 2, the change in real output is

$$\Delta \log Y = \alpha \Delta \log z_g + (1-\alpha)\mathbb{E}_\chi \left[\Delta \log z_s\right],$$

where $\mathbb{E}_\chi \left[\Delta \log z_s\right]$ is the average productivity shock to services weighted by the vector $\boldsymbol{\chi}$. This expression is exact because in the decentralized equilibrium Domar weights do not change ($\Delta\lambda = 0$). Furthermore, since the Domar weights are constant in the competitive equilibrium, there is a positive representative agent with Cobb-Douglas preferences over goods and services in all regions:

$$u^{RA}(\boldsymbol{c}) = c_g^\alpha \prod_h c_{hs}^{\chi_h(1-\alpha)}.$$

Since the positive representative agent has homothetic preferences, we also have that $\Delta \log Y = \Delta \log A^{RA}$.

According to Proposition 4, the change in aggregate efficiency, to a second-order, is

$$\Delta \log A \approx \alpha \Delta \log z_g + (1-\alpha)\mathbb{E}_\chi \left[\Delta \log z_s\right] - \frac{1}{2}\frac{(1-\alpha)^2}{\alpha} Var_\chi \left[\Delta \log z_s\right] \leq \Delta \log Y = \Delta \log A^{RA}.$$

The miraculous consensus of Proposition 1 fails because the agents do not have the same preferences. As predicted by Corollary 1, $\Delta \log Y$ and $\Delta \log A$ do coincide to a first-order, since the discrepancy scales in the square of $\Delta \log z_s$. The second-order approximation shows that $\Delta \log A$ is a concave envelope of $\Delta \log Y$ around the status-quo $\Delta \log \boldsymbol{z} = 0$ — amplifying negative shocks and mitigating positive shocks to services relative to real output. Intuitively, negative shocks to services are "more costly" since the losses cannot be shared across regions, whereas the positive representative agent is "willing" to substitute between regions with a unit elasticity.

Our next example uses Theorem 1 to apply a version of the popular Arkolakis et al. (2012) (ACR) formula to economies with heterogeneous agents with non-homothetic preferences.

Example 3 (Gains from Trade with Heterogeneous Non-Homothetic Preferences). Consider a country with different consumers, h, that value domestic and foreign goods differently:

$$u_h(\boldsymbol{c}_h) = \left[(\alpha_h)^{\frac{1}{\theta_h}} \left(u_h(\boldsymbol{c}_h)\right)^{\zeta_h} c_{hd}^{\frac{\theta_h - 1}{\theta_h}} + (1-\alpha_h)^{\frac{1}{\theta_h}} c_{hf}^{\frac{\theta_h - 1}{\theta_h}} \right]^{\frac{\theta_h}{\theta_h - 1}}.$$

The parameter α_h controls home bias for household h, $\theta_h > 1$ is the compensated Armington elasticity, and the parameter ζ_h controls the degree of non-homotheticity for agent h (preferences are homothetic if $\zeta_h = 0$). The domestic good is produced linearly from a labor endowment and trade is balanced. We consider the gains from trade relative to autarky by raising iceberg trade costs to infinity.

The country trades with the rest of the world in the status-quo. We measure the efficiency loss from autarky via the increase in the autarky consumption possibility set needed to keep every consumer indifferent to the status-quo. With one domestic good, this is simply the increase in the aggregate quantity of the domestic good.

Let s_{hd} denote household h's budget share on the domestic good in the status-quo. Replicating the argument from ACR, for the compensated representative agent in the compensated equilibrium, losses from autarky are

$$\Delta \log A = -\log \mathbb{E}_\chi \left[(s_{hd})^{\frac{1}{1-\theta_h}} \right] \leq 0. \tag{7}$$

Note that $(s_{hd})^{\frac{1}{1-\theta_h}}$ is the ACR formula for the gains from trade for a single agent. The equation above shows that aggregate efficiency losses are the average of these individual losses weighted by expenditures in the status-quo (denoted by χ).[25] Interestingly, the non-homotheticity plays no role in the sense that the utility elasticities ξ_i are not needed to calculate $\Delta \log A$. In particular, if there is a single agent, the equation above shows that ACR holds without change even if preferences are non-homothetic as long as we use the compensated Armington trade elasticity.[26]

To get more intuition, consider a second-order approximation of $\Delta \log A$ around autarky:[27]

$$\Delta \log A \approx \mathbb{E}_\chi \left[\frac{\log s_{hd}}{\theta_h - 1} \right] - \frac{1}{2} Var_\chi \left[\frac{\log s_{hd}}{\theta_h - 1} \right]. \tag{8}$$

The first term is just an "average" version of the ACR formula in logs — the log ACR formula is applied household-by-household and then averaged using households' share

[25]Compare (7) to the representative-agent ACR formula. Suppose that agents have common homothetic CES preferences with Armington elasticity θ. Then the losses from autarky are $\Delta \log A^{RA} = \log \left(\mathbb{E}_\chi \left[s_{hd} \right]^{\frac{1}{1-\theta}} \right)$. If the Armington elasticities are the same, $\theta_h = \theta$, then the losses are larger with heterogeneous agents due to Jensen's inequality. For intuition, consider the case where some household h consumes no home goods, i.e., $s_{hd} = 0$ for some h. In this case, $\Delta \log A = -\infty < \Delta \log A^{RA}$ because it is impossible to compensate h in autarky.

[26]If preferences are non-homothetic, then there is a distinction between the compensated and uncompensated trade elasticities. If we have estimates of the latter, one must use Slutsky's equation to first convert them into the compensated elasticities (see, e.g. Auer et al., 2024).

[27]This is an approximation in $\frac{\log s_{hd}}{\theta_h - 1}$ around $s_{hd} = 1$. To derive this, we follow the strategy in Theorem 3 of Baqaee and Farhi (2019a) who consider the gains from trade with a homothetic representative agent.

of aggregate income χ_h. The second term is the Jensen's inequality term, and it lowers aggregate efficiency if there is any heterogeneity in households' exposure to traded goods either due to variance in expenditure shares, s_{hd}, or trade elasticities, θ_h. In this sense, heterogeneity raises the costs of autarky since some households are more exposed to trade than average.

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5 Distorted Economies with Lump-Sum Transfers

We now relax the assumption in the previous section that there are no wedges, and we characterize how aggregate efficiency responds to changes in productivities and wedges. One important comparative static we focus on is the efficiency losses from misallocation — the increase in aggregate efficiency caused by the elimination of all wedges.

5.1 Comparative Statics for Changes in Technologies and Wedges

Theorem 2 means that we can convert results about real output into results about aggregate consumption-equivalent variation by applying them to variables in the compensated equilibrium. For example, consider the following generalization of Petrin and Levinsohn (2012).

Proposition 7 (Changes in Aggregate Efficiency with Wedges)**.** *In response to changes in wedges and productivities, the change in aggregate efficiency is*

$$\Delta \log A = \int_0^t \sum_i \lambda_i^{comp}(s) \left[\left(1 - \frac{1}{\mu_i(s)} \right) \frac{d \log y_i^{comp}}{ds} + \frac{1}{\mu_i(s)} \frac{d \log z_i}{ds} \right] ds.$$

In Appendix E, we characterize $\lambda_i^{comp}(s)$ and $d \log y_i^{comp}/ds$ explicitly as a function of the productivity changes $\Delta \log z$, elasticities of substitution, and expenditure shares using the results in Baqaee and Farhi (2020).

The compensated Domar weights, λ^{comp}, and quantities, $d \log y^{\text{comp}}$, can be computed using standard methods for inefficient economies with homothetic representative agents (see Appendix E).

We can contrast Proposition 7 with Harberger's social welfare formula. In his classic paper, Harberger (1971) argued that the welfare effect of a policy that changes quantities $\{y_i\}$ over time should be computed as

$$\Delta \log Y(t) = \int_0^t \sum_i [p_i(s) - mc_i(s)] \frac{dy_i}{ds} ds \;=\; \int_0^t \sum_i \lambda_i(s) \left(1 - \frac{1}{\mu_i(s)} \right) \frac{d \log y_i}{ds} ds, \qquad (9)$$

where the equality uses the fact that final expenditure is the numeraire ($\sum_i p_i(s)c_i(s) = 1$ for every s). In words, he argued that whenever a good's marginal benefit, $p_i(s)$, exceeds its marginal cost, $mc_i(s)$, then expanding its quantity (holding others fixed) raises aggregate output. Proposition 7 extends this expression to measure the change in aggregate efficiency.

5.2 Misallocation and the Distance to Pareto Frontier

We now focus on a particular counterfactual: we apply Proposition 7 to compute the economic waste caused by distortions. Let $\boldsymbol{\mu}$ be a vector of wedges. We measure economic waste by how far the Pareto frontier can be contracted while keeping every agent at least as well off as under the status quo.

Formally, denote the status-quo allocation by $\boldsymbol{c}^0(\boldsymbol{\mu})$ (we omit dependence on the transfers that decentralize the status-quo) and the consumption possibility set by $\mathcal{C}(\boldsymbol{\mu})$ (we suppress productivity parameters since we hold them fixed). By the second welfare theorem, the Pareto frontier is $\mathcal{C}(\mathbf{1})$. The economic waste caused by distortions is measured by

$$A\big(\boldsymbol{c}^0(\boldsymbol{\mu}),\, \mathcal{C}(\mathbf{1})\big).$$

With a complete structural model this term can be computed using Theorem 2. However, below we derive an approximation that is more intuitive and requires less information to be applied.

Proposition 8 (Harberger Triangles). *To a second-order approximation in* $\log \mu$*, the change in aggregate efficiency is*

$$\Delta \log A \approx -\frac{1}{2} \sum_i \lambda_i d \log y_i^{comp} \log \mu_i, \tag{10}$$

where $d \log y_i^{comp} \equiv \sum_j \frac{\partial \log y_i^{comp}}{\partial \log \mu_j} \log \mu_j$ *is the change in the quantity of i caused by the wedges in the compensated equilibrium. The approximation error is order* $\log \mu^3$*. The derivatives and expenditure shares in* (10) *are evaluated at the status-quo.*[28] *We provide an explicit formula for* $d \log y_i^{comp}$ *in terms of microeconomic primitives in Appendix E.*

This proposition generalizes deadweight loss triangles to measure aggregate efficiency losses from wedges in general equilibrium economies with heterogeneous agents and

[28]Usually, such quadratic expansions are evaluated at the undistorted point. However, since λ_i and $d \log y_i^{\text{comp}}$ are multiplied by one power of $\log \mu$, evaluating these terms at status-quo wedges changes the expression only at the third order. Hence, the stated approximation remains valid to a second-order.

non-homothetic preferences. The proof relies on translating results from Baqaee and Farhi (2024) using Theorem 2.

There are two advantages to using Proposition 8 over and above simply applying Proposition 7 using a fully-spelled out structural model. First, the Harberger triangles formula can be used to get analytical intuition for misallocation costs through the use of loglinearized expressions, as demonstrated below. Second, it is possible to populate the terms in (8) with considerably fewer assumptions about the primitives of the economy — e.g. the drivers of distortions, productivity processes, and so on.

The intuition for (10) is familiar — a wedge on i is more costly the higher is the Domar weight and the more elastic is the quantity of i relative to the wedge. However, compared to a representative agent model with homothetic preferences, the relevant notion of elasticity here is the one in the compensated equilibrium, not the decentralized one.

5.3 Analytical Examples

We provide some pen-and-paper examples to build intuition.

Example 4 (Misallocation when Markups Vary by Household). Consider the misallocation problem studied by Hsieh and Klenow (2009), but suppose there are multiple agents. Each agent h has CES preferences over consumption goods with elasticity of substitution θ_h. We consider a situation in which each agent pays potentially a different markup μ_{hi} on each good i.[29] Suppose that all consumption goods are ultimately produced linearly from a single common primary factor called labor, which is inelastically supplied.

We can apply Proposition 8 to write the aggregate efficiency losses, up to a second order approximation as

$$\Delta \log A \approx \frac{1}{2}\mathbb{E}_{\chi}\left[\theta_h Var_{\boldsymbol{b}_h}\left[\log \boldsymbol{\mu}_h | h\right]\right], \tag{11}$$

where the expectation uses the vector of household income shares, $\boldsymbol{\chi}$, and the variance uses household budget shares over goods, $\boldsymbol{b}_h$, as weights.[30] Since $A > 1$ means the efficient consumption possibility set can be contracted (scaled by $1/A$) while holding everyone indifferent, the larger $\Delta \log A$, the greater the losses. If all agents have the same preferences and face the same wedges, then the expectation in (11) disappears, and the equation collapses to the single agent case, equation (19), in Baqaee and Farhi (2020).

[29] Formally, hi indexes the intermediary between good i and household h. We assume that this intermediary charges a markup of μ_{hi} on its marginal cost. The intermediary's marginal cost is just the price of good i.

[30] Formally written out, (11) is $= \frac{1}{2}\sum_h \chi_h \theta_h \sum_n b_{hn}\left[\log \mu_{hn} - \sum_{n'} b_{hn'} \log \mu_{hn'}\right]^2$.

In words, the reduction in efficiency caused by the markups depends on the average variance in markups paid by each household multiplied by that household's elasticity of substitution. Intuitively, if θ_h is very high, then dispersion in markups faced by h causes a greater reduction in aggregate efficiency. Furthermore, aggregate efficiency falls by more if richer households (those with higher χ_h) are exposed to more markup dispersion. Importantly, this expression does not depend on the *level* of markups paid by each household. A proportional scaling of all markups paid by any household would leave this expression unchanged because increasing all markups on a single household is equivalent to a lump-sum tax on that household, and has no effect on aggregate efficiency.

The next example applies equation (11) to study the efficiency losses due to imperfect insurance.

Example 5 (Misallocation Due to Financial Market Incompleteness). Consider agents with expected utility

$$u_h(\boldsymbol{c}_h) = \sum_s \frac{c_h(s)^{1-1/\theta}}{1-1/\theta}.$$

States of nature, indexed by s, are all equally likely. The coefficient of relative risk aversion is $1/\theta$ (or equivalently, the elasticity of substitution across states is θ).

Each agent h has income $y_h(s) = a_h + \epsilon_h(s)$, where $\epsilon_h(s)$ is an idiosyncratic shock that sums to zero across agents, $\sum_h \epsilon_h(s) = 0$ for every s, with mean zero for each agent $\mathbb{E}[\epsilon_h(s)|h] = 0$. The status-quo allocation is financial autarky, so h's consumption in state s is $c_h^0(s) = y_h(s)$. The aggregate resource constraint for the economy is $\sum_h c_h(s) = \sum_h a_h$, because, by assumption, $\sum_h \epsilon_h(s) = 0$ for every s.

To decentralize this allocation using wedges, suppose that there are complete state-contingent markets with household-by-state wedges $\mu_h(s)$. Household h's budget constraint can be written as

$$\sum \mu_h(s) c_h(s) = a_h.$$

The wedges that decentralize the status-quo allocation must satisfy

$$\frac{c_h^0(s)}{c_h^0(s')} = \left[\frac{\mu_h(s)}{\mu_h(s')}\right]^{-\theta}.$$

Substituting these wedges into equation (11) implies that the gains from completing financial markets are approximately given by:

$$\Delta \log A \approx \frac{1}{2}\theta \mathbb{E}_\chi \left[Var\left[\log \mu_h(s)|h\right]\right] = \frac{1}{2}\frac{1}{\theta}\mathbb{E}_\chi \left[Var\left[\log c_h(s)|h\right]\right],$$

where χ_h is household h's expected share of consumption in the status-quo, $a_h / \sum_{h'} a_{h'}$. This formula disregards inequality due to dispersion in the persistent component of income (dispersion in consumption caused by a_h) because the variance is conditional on household h. Instead, misallocation depends on the average conditional consumption variance, weighted by household income. Holding the consumption process fixed, the gains from completing financial markets are larger, the higher is the risk aversion $1/\theta$ parameter.

The example above is simple, but hints at a much more general idea. Baqaee and Burstein (2025b) build on this basic idea to analyze and quantify the losses from financial market imperfections in both open and closed economies. In that paper, we discuss how to allow for dynamics, labor-leisure choice, capital accumulation, borrowing constraints, and international trade.

The final example in this section shows that misallocation, as measured by $\Delta \log A$, need not equal to changes in real output, $\Delta \log Y$, or changes in the welfare of a representative agent, $\Delta \log A^{RA}$, even in cases where a representative agent exists.

Example 6 (Real Output and Positive Representative Agent Losses from Markups). Suppose each agent h's has CES preferences over consumption goods with elasticity of substitution θ_h. Suppose each agent consumes a different selection of goods but all goods are produced linearly from the labor endowment. The markup on the ith good consumed by household h is denoted by μ_{hi}. Suppose that we eliminate markups, each households' share of income, χ_h, stays constant. Since the distribution of income is constant, there is a positive (and in this case, also normative) representative agent with Cobb-Douglas preferences across each households' consumption bundle (i.e. an agent whose utility is maximized by observed allocations). The change in the welfare of this representative agent, in consumption-equivalent terms, is equal to the change in chain-weighted real output, and both are equal to a second-order approximation to

$$\Delta \log Y = \Delta \log A^{RA} \approx \underbrace{\frac{1}{2}\mathbb{E}_{\chi}\left[\theta_h Var_{\boldsymbol{b}_h}\left[\log \boldsymbol{\mu}_h | h\right]\right]}_{\approx \Delta \log A} + \frac{1}{2} Var_{\boldsymbol{\chi}}\left[\mathbb{E}_{\boldsymbol{b}_h}\left[\log \boldsymbol{\mu}_h | h\right]\right],$$

where we use (11). The change in real output and the welfare of the representative agent are (weakly) larger than the change in aggregate efficiency. One limiting case of this is where all markup-variation is at the household level. In this case, $\Delta \log A = 0$ because the economy is already on the efficient frontier and eliminating markups is purely redistributive. However, in this example, $\Delta \log Y = \Delta \log A^{RA} > 0$.

We can push this example even farther: suppose again that all markup-variation is at the household level, and as we eliminate all markups, we also change the productivity of labor by $\Delta \log z < 0$ at the same time. In this case, the change in chain-weighted real output is

$$\Delta \log Y \approx \Delta \log z + \frac{1}{2} Var_{\boldsymbol{\chi}} \left[\mathbb{E}_{\boldsymbol{b}_h} \left[\log \boldsymbol{\mu}_h | h \right] \right],$$

whereas $\Delta \log A = \Delta \log z$. Hence, if the physical productivity shock is small enough, then real output rises, even though the Pareto frontier shifts inwards.

To summarize, in this example, $\Delta \log Y$ and $\Delta \log A^{RA}$ can assign a positive value to a pure transfer, and a positive value to a strictly smaller production possibility set.

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6 Aggregate Efficiency with Limits to Redistribution

In this section, we extend the analyses in Sections 4 and 5 to allow for imperfect redistributive tools. This is another advantage of our approach relative to measures based on adding up willingness-to-pay across all households (e.g. as in Kaldor-Hicks). Intuitively, when we add up willingness-to-pay, we implicitly assume that winners can compensate losers. In Section 4, we illustrated one issue with this approach: monetary compensations can change relative prices so that, in practice, the necessary compensations are infeasible. In this section, we focus on a second issue — monetary compensations may be infeasible because lump-sum transfers are not available.

Theorem 1 applies regardless of what redistributive tools are available. In this section, we apply Theorem 1 to the case where redistribution can only be achieved via linear taxation in general equilibrium with wedges.

6.1 General Solution with Linear Taxes

Consider a decentralized equilibrium with technologies $\boldsymbol{z}$ and wedges $\boldsymbol{\mu}$. We allow a vector of linear taxes $\boldsymbol{\tau}$ on different goods, and let the vector $\boldsymbol{T}$ dictate the amount of tax revenues sent to each household. We require budget-balance, so that total tax revenues must equal total transfers to households. Index the equilibrium consumption allocation $\boldsymbol{c}(\boldsymbol{z}, \boldsymbol{\mu}, \boldsymbol{\tau}, \boldsymbol{T})$ by technologies, $\boldsymbol{z}$, wedges, $\boldsymbol{\mu}$, and the tax-and-transfer scheme, $(\boldsymbol{\tau}, \boldsymbol{T})$.

Let the set of all feasible tax-and-transfer schemes be $\mathcal{T}(\boldsymbol{z}, \boldsymbol{\mu})$. The case with lump-sum transfers studied in Section 4 and Section 5 is the special case that places no non-negativity constraint on the vector $\boldsymbol{T}$ so that redistribution is accomplished without linear taxes. In this section, we allow for the possibility that this set has other restrictions. For example, the feasible set $\mathcal{T}$, may allow for only distortionary linear taxes on some subset of goods, limit lump-sum transfers to be non-negative, and so on.

Corollary 2 (Aggregate Efficiency with Restricted Tax-and-Transfer Instruments)**.** *Theorem 1 implies that aggregate efficiency satisfies:*[31]

$$A(\boldsymbol{z},\boldsymbol{\mu}) = \max\left\{U\left(\boldsymbol{c}(\boldsymbol{z},\boldsymbol{\mu},\boldsymbol{\tau},\boldsymbol{T})\right) : (\boldsymbol{\tau},\boldsymbol{T}) \in \mathcal{T}(\boldsymbol{z},\boldsymbol{\mu})\right\}. \tag{12}$$

That is, aggregate efficiency is given by the highest utility $U(\boldsymbol{c})$ that can be achieved by choosing feasible taxes and transfers, $(\boldsymbol{\tau},\boldsymbol{T}) \in \mathcal{T}(\boldsymbol{z},\boldsymbol{\mu})$, taking into account how those choices affect consumption, $\boldsymbol{c}$, and in turn utility, U.

In words, $A(\boldsymbol{z},\boldsymbol{\mu})$ measures the maximum contraction (or minimum expansion) of the set of feasible equilibrium consumption allocations, given technologies, $\boldsymbol{z}$, wedges, $\boldsymbol{\mu}$, and tax-and-transfer instruments, $(\boldsymbol{\tau},\boldsymbol{T})$. Corollary 2 applies to the special cases considered in Section 4 and 5, where we assume that the feasible set of instruments, $\mathcal{T}$, consists only of unrestricted lump-sum transfers.

As before, we index technologies and wedges by a scalar t and let $t = 0$ denote the status-quo. Figure 5 illustrates $A(t)$ using a two household example. In the figure, the status-quo allocation, $\boldsymbol{c}(0)$, and the decentralized allocation without transfers, $\boldsymbol{c}(t)$, are denoted by red circles. The solid blue line shows the feasible consumption possibility frontier at t given distortionary taxation, and the dashed line indicates the frontier at t given unrestricted lump-sum taxes. The two frontiers touch at the decentralized point, since the decentralized point does not engender any distortionary redistributive taxation. However, the solid blue set is strictly smaller than the dashed line since distortionary taxation limits the set of feasible redistributions. The change in efficiency, $\Delta \log A$, is still the largest radial contraction of $\mathcal{C}(t)$ that allows every household to be made at least indifferent to the status-quo. Since the larger is the possibility set $\mathcal{C}(t)$, the more it must be contracted to reach indifference, aggregate efficiency gains are larger with better redistributive tools.

If we substitute the definition of $U(\boldsymbol{c})$ into (12), we get

$$A(\boldsymbol{z},\boldsymbol{\mu}) = \max\left\{\min_h \tilde{u}_h\left(\boldsymbol{c}_h(\boldsymbol{z},\boldsymbol{\mu},\boldsymbol{\tau},\boldsymbol{T})\right) : (\boldsymbol{\tau},\boldsymbol{T}) \in \mathcal{T}(\boldsymbol{z},\boldsymbol{\mu})\right\}.$$

We say that *outcomes are interior* if the solution to the optimization problem above features

[31] If there are multiple equilibria, then $\boldsymbol{c}(\boldsymbol{z},\boldsymbol{\mu},\boldsymbol{\tau},\boldsymbol{T})$ is a correspondence and the maximization is applied to the set of potential equilibrium allocations. We could equivalently write $A(\boldsymbol{z},\boldsymbol{\mu}) = \max_{\boldsymbol{c}\in\mathcal{C}(\boldsymbol{z},\boldsymbol{\mu})} U(\boldsymbol{c})$, where the consumption possibility set is $\mathcal{C}(\boldsymbol{z},\boldsymbol{\mu}) = \{\boldsymbol{c}(\boldsymbol{z},\boldsymbol{\mu},\boldsymbol{\tau},\boldsymbol{T}) : (\boldsymbol{\tau},\boldsymbol{T}) \in \mathcal{T}(\boldsymbol{z},\boldsymbol{\mu})\}$.

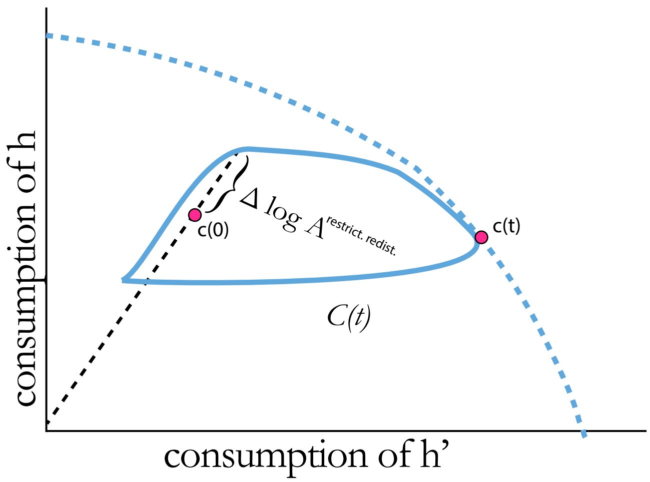


Figure 5: Aggregate efficiency is measured by the maximal radial expansion of the feasible set necessary to achieve indifference.

$\tilde{u}_h = \tilde{u}_{h'}$ for all h and h'. It is straightforward to show that if outcomes are interior, then in the primitive problem (1), where we solve for the maximum contraction of the consumption possibility set $\mathcal{C}$, every agent is *exactly* kept indifferent to the status-quo after compensation.[32] This typically requires having a sufficiently rich set of redistributive instruments.

To make this max-min problem more computationally tractable, it is useful to note that it is equivalent to

$$A(\boldsymbol{z},\boldsymbol{\mu}) = \max\left\{x : (\boldsymbol{\tau},\boldsymbol{T}) \in \mathcal{T}(\boldsymbol{z},\boldsymbol{\mu}), \tilde{u}_h\left(\boldsymbol{c}(\boldsymbol{z},\boldsymbol{\mu},\boldsymbol{\tau},\boldsymbol{T})\right) \geq x\right\},$$

where we replace the inner minimum with a series of constraints instead. This reduces the max-min problem to a simpler constrained maximization problem.

The following is a generalization of Proposition 4 to this setting.

Proposition 9 (Productivity Shocks with Limited Redistribution)**.** *Consider a perfectly competitive status-quo without linear taxes and wedges. If outcomes are interior, the response of aggregate productivity to a productivity shock, $\Delta \log \boldsymbol{z}$, to a second-order approximation, is given by*

$$\Delta \log A = \sum_i \left(\lambda_i + \frac{1}{2}\sum_j \frac{\partial \lambda_i^{comp}}{\partial \log z_j}\Delta \log z_j\right)\Delta \log z_i + \frac{1}{2}\sum_i \lambda_i \left(\sum_j \frac{\partial \log y_i^{comp}}{\partial \log \tau_j}\Delta \log \tau_j^*\right)\Delta \log \tau_i^*, \tag{13}$$

[32]Formally, let $\mathcal{B} = \left\{\boldsymbol{c} \in \mathcal{C}/A(\boldsymbol{z},\boldsymbol{\mu}) : u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h\right\}$. If outcomes are interior, then there is a $\boldsymbol{c} \in B$ such that $u_h(\boldsymbol{c}_h) = u_h(\boldsymbol{c}_h^0)$ for every h.

where $\boldsymbol{\tau}^*(t)$ *are the maximizers in* (12).

The first set of summands are exactly as in Proposition 4. The second set of summands, which are new and non-positive, capture the inefficiency caused by imperfect redistribution. These are the sum of Harberger triangles associated with the linear taxes in $\tau^*(t)$. If lump-sum taxation is not feasible, then linear taxes must be used, $\Delta \log \tau^* \neq 0$, and aggregate efficiency with limited redistribution is lower than with lump-sum taxation by exactly the sum of deadweight loss triangles. That is, the response of aggregate efficiency to productivity shocks is the same as it would be if lump-sum transfers were possible minus the deadweight loss triangles associated with distortionary taxes. The simplicity of Equation (13) follows from the fact that the status-quo is undistorted. This ensures that (1) there are no interactions of taxes with pre-existing distortions, (2) the cross-partials between $d \log \tau^*$ and $d \log z$ are all zero due to the envelope theorem. In contrast, Corollary 2 does not require that the status-quo be undistorted.

The following simple corollary, obtained by ignoring the second order terms, shows that Hulten's theorem holds, without change, even with limited redistribution, as long as outcomes are interior.

Corollary 3 (Hulten's Theorem with Limited Redistribution)**.** *Consider a perfectly competitive status-quo without linear taxes and wedges. If outcomes are interior, the response of aggregate productivity to a productivity shock,* $\Delta \log \boldsymbol{z}$*, to a first-order approximation, is given by*

$$\Delta \log A = \sum_i \lambda_i \Delta \log z_i.$$

Intuitively, the losses from costly-redistribution are second-order, and hence to a first-order approximation, only the direct effects of the productivity shock matter (assuming we start at a competitive equilibrium).

6.2 Analytical Example: Losses from Autarky

We now study the efficiency gains from international trade, accounting for limited redistribution. We use the approximation in Proposition 9 to provide intuition. We check the numerical performance of the second-order approximation by computing exact results using (12).

Example 7 (Gains from Trade with Limited Redistribution). We revisit Example 3, which studied the gains from trade, but this time we incorporate limits to redistribution. We add a labor-leisure margin, and assume redistribution can only be done by taxing consumption.

Suppose there are two households, and household h has nested-CES preferences over domestic consumption goods, c_{hd}, imported consumption goods, c_{hf}, and leisure l_h:

$$u_h(\boldsymbol{c}_h) = \left[(1-\gamma_h)^{\frac{1}{\rho}} \left[(1-\alpha_h)^{\frac{1}{\theta}} c_{hd}^{\frac{\theta-1}{\theta}} + \alpha_h{}^{\frac{1}{\theta}} c_{hf}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}\frac{\rho-1}{\rho}} + \gamma_h^{\frac{1}{\rho}} l_h^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}.$$

The model in Example 3 did not feature the leisure good. The inner nest combines domestic and foreign consumption goods with Armington elasticity θ and home bias controlled by the parameter α_h. The outer nest combines the goods bundle with leisure with elasticity of substitution ρ and share parameter γ_h.[33] The parameter ρ controls the Frisch elasticity of labor supply.

Household h is endowed with one unit of time and a_h efficiency units of labor and faces a budget constraint:

$$\tau p_d c_{hd} + \tau p_f c_{hf} = w a_h (1 - l_h) + T_h,$$

where p_d and p_f denote the price of each consumption good, w_h is the wage per efficiency unit, τ is the gross-tax rate on consumption, and T_h is a lump sum transfer. Budget balance requires $(\tau - 1)\sum(p_d c_{hd} + p_f c_{hf}) = \sum T_h$. The domestic consumption good is produced linearly with labor, so the resource constraint for domestic consumption is $\sum_h c_{hd} = \sum_h a_h(1 - l_h)$, with $p_d = w$. The resource constraint for leisure is $l_h \leq 1$.

The status-quo is a competitive equilibrium without taxes in which the country trades with the rest of the world. We consider the gains from trade relative to autarky, by raising iceberg trade costs to infinity. The efficiency loss from autarky, $\Delta \log A$, measures the increase in the autarky consumption possibility set needed to keep every consumer indifferent to the status-quo. The consumption possibility set encodes the potentially distortionary impact of taxes required to transfer income between households. We compare two cases: (1) lump-sum taxation is available and the second welfare theorem holds; (2) lump-sum taxation is not available, $T_h \geq 0$, and linear consumption taxes must be used.

Let

$$\Omega_{hd} = \frac{p_d^0 c_{hd}^0 + p_f^0 c_{hf}^0}{w^0 a_h}$$

[33] For simplicity of exposition, we abstract from non-homotheticities and differences in elasticity parameters across households. It is simple to extend the model in this way.

denote household h's budget share on consumption in the status-quo as a share of the value of h's total time endowment (the remainder is implicit expenditures on leisure). For simplicity of exposition, and since it is fairly realistic, we assume that both households work the same number of hours in the status-quo, which implies that $\Omega_{hd} = \Omega_d$ does not vary by household. Let s_{hd} denote household h's share of expenditures on the domestic consumption good relative to all consumption goods:

$$s_{hd} = \frac{p_d^0 c_{hd}^0}{p_d^0 c_{hd}^0 + p_f^0 c_{hf}^0}.$$

Lump-Sum Taxation. With lump-sum taxation, using Proposition 4, we can write the losses from autarky to a second-order approximation as

$$\Delta \log A^{\text{lump-sum}} \approx \underbrace{\Omega_d \mathbb{E}_\chi \left[\frac{\log s_h}{\theta - 1}\right]}_{\text{1st order}} \underbrace{- \frac{1}{2}\Omega_d^2 Var_\chi \left(\frac{\log s_h}{\theta - 1}\right) + \frac{1}{2}(\rho - 1)\Omega_d(1 - \Omega_d)\mathbb{E}_\chi \left[\left[\frac{\log s_h}{\theta - 1}\right]^2\right]}_{\text{2nd order with lump-sum taxation}}.$$

This expression is identical to Equation (8) in Example 3 when there is no leisure, $\Omega_d = 1$. The first and second summands are the same as in (8) but are now scaled by Ω_d to account for the fact that households also consume leisure. The final summand, which is absent in (8), accounts for complementarities/substitutabilities between consumption and leisure. If consumption and leisure are complements, $\rho < 1$, then a negative shock to consumption caused by autarky reduces the value of leisure through complementarity, raising the gains from autarky.

Linear Taxation. Now consider the case where lump-sum taxation is unavailable so that lump-sum transfers must be non-negative: $\boldsymbol{T} \geq 0$, financed by a uniform consumption tax. Proposition 9 now implies that, to a second-order approximation,

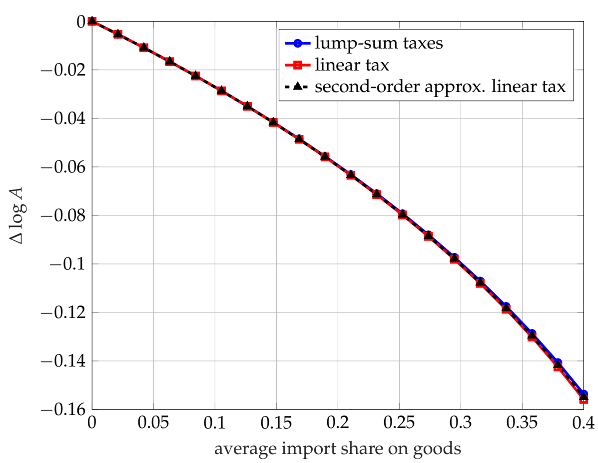
$$\Delta \log A^{\text{linear tax}} \approx \Delta \log A^{\text{lump-sum}} - \underbrace{\frac{1}{2}\rho\Omega_d(1 - \Omega_d)(d \log \tau^*)^2}_{\text{2nd order losses from distorting taxes}},$$

where τ^* is the optimal consumption tax in Equation (12).

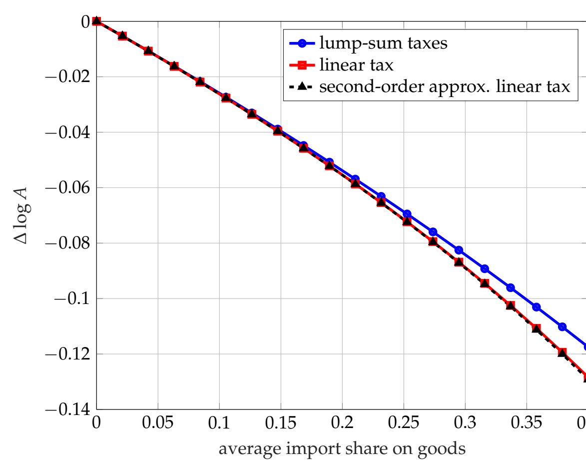
Index the two households by h and h' and suppose that h is more exposed to foreign goods: $s_{hd} < s_{h'd}$. This means that, in the decentralized equilibrium, household h is more negatively affected by the trade shock than h'. In this case, the optimal feasible tax-and-transfer from (12) sends all collected tax revenues to h. Furthermore, to a first-order, the tax required for the compensation is $d\log\tau^* = \frac{\chi_h}{\theta-1}\left[\log s_{h'd} - \log s_{hd}\right] > 0$, where χ_h is h's share of aggregate income.

The required consumption tax is larger the bigger is the heterogeneity in exposure to the trade shock, and the larger is household h's share of aggregate income. A given tax is more distorting the higher is ρ, which controls substitution between consumption and leisure (ρ can be interpreted as the Frisch elasticity of labor supply), and the closer is Ω_d to 1/2. If Ω_d is equal to either one (households do not value leisure) or zero (households do not value consumption), then there is no distortion from the tax.

Example 7 numerically illustrates the performance of the second-order approximation to the exact solution with distortionary linear taxes, and compares them both to the solution with lump-sum taxes. The second-order approximation performs well even for large shocks. Panel 6a uses $\rho = 0.5$, so consumption and leisure are complements and the Frisch elasticity of labor supply is a reasonable 0.5. Since ρ is low, distortionary taxes are able to achieve an outcome that is roughly as good as lump-sum taxes. Panel 6b uses a much higher $\rho = 3$. In this case, the gap between the lump-sum and linear taxation scenarios is larger since consumption taxes reduces labor and increase leisure, which causes efficiency to fall.



(a) $\rho = 0.5$



(b) $\rho = 3.0$

Figure 6: A numerical example of the losses from autarky with and without distortionary redistribution. The other parameter values are $\Omega_d = 0.5$, $\chi_h = 0.5$, $\theta_h = 3$, $s_{hd} = 3s_{h'd}$.

To summarize: losses from autarky are larger if some households are more badly affected than others, especially if efficient redistributive tools are not available to compensate the households that are more badly affected.

In Appendix D, we consider another example, where skill-biased technical change affects different workers differently, and show how the second order approximation in Proposition 9 performs.

6.3 Quantitative Example: the China Shock

Our final example uses an off-the-shelf quantitative trade model to quantify the aggregate efficiency effects of international shocks. We consider the effect of the rise of China on the United States using the Armington model in Baqaee and Farhi (2024). Each country has different factor endowments, and we treat owners of different factor endowments as different agents. The rise of China, which we model by raising Chinese productivity, changes relative wages amongst the different domestic factors, with different consequences for different agents. We study how aggregate efficiency for the US changes as China becomes more productive, as measured by the aggregate consumption equivalent, and we study how this number changes depending on assumptions about factoral mobility (across sectors) and the availability of tax and transfer tools.

Summary of Calibration. The calibration follows Baqaee and Farhi (2024). The model has 40 countries and 30 industries in each country. Production by each industry is a nested CES aggregator combining four domestic primary factors (low-, medium-, high-skill labor, and capital) with intermediate inputs. The intermediate input bundle used by each industry is a nested CES aggregator cover all industries and origin countries. All households in each country consume the same domestic consumption bundle, which is a nested CES aggregator over all industries and origin countries. (We abstract from heterogeneity in preferences within countries). The initial expenditure shares are all calibrated according to the World Input-Output Database in 2008. Since tariffs in 2008 were quite low, the model is calibrated assuming there are no import tariffs.[34]

China Shock. In 2008 (the calibration year), China's GDP is roughly 5% of the world's. By 2023, this number had risen to 18%. We model the rise of China through an increase in Chinese factor-augmenting productivity growth (roughly tripling the efficiency units of all Chinese factor endowments) to ensure that China's share of world GDP rises to 18%.

[34] The elasticity of substitution between primary factors is set to one. The elasticity of substitution between value-added and intermediates is 0.5. Each country-industry pair has a unique bundle of intermediate inputs sourced from different industries with elasticity of substitution 0.2. Each country also has a unique consumption bundle, with elasticity of substitution across industries of 0.9. Every destination country-industry pair purchases a unique bundle of inputs from other industries sourced from different origin countries. The Armington trade elasticity is equal to 5.

We consider how this shock affects the United States, measured using the aggregate consumption equivalent change, under four different scenarios. Each scenario defines a different consumption possibility set $\mathcal{C}$ for US consumers, depending on the tools available to the government. These scenarios are:

i. *Laissez-faire*: there are no redistributive tools available. The consumption possibility set is a single point corresponding to the decentralized equilibrium in the US.

ii. *Tariffs with non-targeted rebates:* the government can raise a uniform tariff on all imports but any revenues generated by tariffs are rebated back to domestic households in proportion to their pre-shock initial share of aggregate income.

iii. *Tariffs with targeted rebates:* the government can raise a uniform tariff on imports and has full discretion on how to rebate any additional tariff revenues (i.e. if tariff revenues rise after the shock, in units of world GDP, then the government can choose who to rebate that additional revenue to).

iv. *Tariffs with lump-sum transfers:* the government can raise a uniform tariff on imports, and also has access to unrestricted lump-sum transfers (i.e. lump-sum transfers can be positive or negative).

In scenarios ii., iii. and iv., the consumption possibility set is the set of equilibrium consumption allocations for US consumers given different levels of uniform import tariffs and feasible transfers.

We now turn to the choice of the status quo. We assume that the 2008 data correspond to laissez-faire in which all taxes are zero. If we were to treat this as the status quo, then even without the China shock our efficiency measure would detect $A > 1$, reflecting the gains from optimal tariffs. Our focus is not on quantifying these tariff gains, but to illustrate how an imperfect redistributive tool — here, an import tariff —influences the aggregate efficiency implications of the China shock.

We therefore assume that, if an import tariff is available (scenarios ii, iii, iv), then that import tariff is already set to maximize the quantity of the US consumption good prior to the China shock. This ensures that the status-quo allocation is not Pareto dominated by other allocations in the feasible status-quo consumption possibility set (i.e. in the absence of the China shock, aggregate efficiency is 1 by construction in every scenario). Nevertheless, to keep the status-quo allocation similar to the data in 2008 — which we calibrate the lessaiz faire model to — we assume that US tariffs provoke symmetric retaliation from the rest of the world. As a result, the optimal US tariff in the status-quo is small anyhow

(2.26%), ensuring that expenditure shares in the status-quo are close to the 2008 data prior to the shock.

The change in aggregate efficiency due to the China shock is shown in Table 1. We consider two specifications of the model, labelled "immobile" and "mobile" factors. When factors are "immobile", primary factors (labor and capital) in each country-industry pair cannot move across industries. When factors are mobile, there is one national market for each factor type (low-, medium-, high-skill labor and capital) and all industries in a country hire from the national market. Comparing these two specifications reveals the importance of reallocation for determining aggregate efficiency.

Table 1: Effect of China Shock on the United States

Scenario	Immobile Factors		Mobile Factors	
	$\Delta\log A$	Δ Tariff (p.p.)	$\Delta\log A$	Δ Tariff (p.p.)
Tariffs & lump-sum transfers	0.010	+0.1	0.010	0.0
Tariffs & targeted rebates	-0.010	+7.4	0.010	+0.6
Tariffs & non-targeted rebates	-0.179	+11.4	0.008	+0.4
Laissez-faire	-0.235	–	0.008	–

Tariff changes are expressed in percentage points.

The first row shows the change in aggregate efficiency for the US, assuming there are lump-sum transfers. In this case, Proposition 1 holds, and the response of $\Delta \log A$ coincides with the response of aggregate real consumption in the US. This also coincides with real consumption by the representative US household, as well as the change in the Kaldor-Hicks measure of efficiency. In this case, $\Delta \log A$ rises by around 1 log point in response to the China shock.

The second row considers the case where import tariffs are available, but redistribution can only draw on excess tariff revenues. In this case, $\Delta \log A$ falls by 1 log point if factors cannot move across sector. This is because the China shock causes real wages in some sectors to fall. To compensate these households, we must raise tariff revenues by around 7%. These tariffs, which trigger retaliatory tariffs from the rest of the world, cause overall efficiency to decline. In other words, in order to make every US household at least as well off after the China shock requires expanding the consumption possibility set by 1 log point. The picture is very different if factors are mobile, since in this case, real wages do not decline, and hence large tariffs are not needed to offset real income losses suffered by some subset of households.

The third row considers the case where tariff revenues can only be distributed according to a fixed distribution (pre-shock initial income shares). In this case, since the redistributive tools are much more severely restricted, it is much harder to compensate losers (it may not be possible to equalize the change in real consumption across consumers). Accordingly, aggregate efficiency falls by 17.9 log points, since compensating workers in losing industries requires instituting very large tariffs (of around 11.4%), which then trigger a large trade war. Once again, these restrictions on redistributive policy are much less important if factors are mobile across sectors, with the overall efficiency gains being barely affected.

The last and final row considers the Laissez-faire case, where $\Delta \log A$ simplifies to being simply the change in real consumption for the workers whose real wages decline the most in Lessaiz-faire. When factors are immobile, this is "Textile and Leather Products," where real wages decline by around 23 log points in response to the China shock. The contrast with the mobile factor case is very stark: when factors are mobile, heterogeneous effects among factors is substantially attenuated, and so the change in aggregate efficiency closely approximates that under full redistribution.

The results in Table 1 show that the change in aggregate efficiency depends strongly on (a) the extent to which shocks have asymmetric effects on households, and (b) the redistributive tools available. Note that although the redistributive tools are very important for quantifying the change in aggregate consumption equivalents in Table 1, we do not take a stance on how these redistributive tools should be used in practice. For example, if lump-sum transfers are available, then the consumption possibility set can be contracted by 1% and everyone can be kept at least indifferent. Hence, after the shock and compensations, there is a 1% surplus of the consumption good. We measure this surplus without taking a stance on how it should be distributed.

7 Conclusion

This paper defines a measure of aggregate efficiency using the aggregate consumption-equivalent variation. We establish that this measure can be computed by maximizing utility of some fictional agent. This provides a method for translating theorems and tools about representative-agent economies to study aggregate efficiency in economies with heterogeneous agents. This includes Hulten (1978), Harberger (1964), Petrin and Levinsohn (2012), Arkolakis et al. (2012), and Baqaee and Farhi (2019b) and Baqaee and Farhi (2020).

In two stand-alone companion papers, we apply the theoretical results of this paper to other contexts where household heterogeneity is central. Baqaee and Burstein (2025b) consider losses in aggregate efficiency from financial market incompleteness, within and across borders. Baqaee and Burstein (2025a) characterize aggregate efficiency in random utility models with discrete choice, focusing on spatial economies, where households make different choices due to differences in their preferences. An interesting extension, which we do not pursue in this paper but pursue in ongoing work, is to study policy problems where maximizing aggregate efficiency is the objective of the policymaker.[35]

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Appendix

Appendix A Expenditure Function of Compensated Agent

The following proposition characterizes the expenditure function of the compensated agent.

Proposition 10 (Dual Representation of Hicksian Representative Agent)**.** *The expenditure function associated with $U(\boldsymbol{c})$, in Definition 2, denoted by $E(p, U)$ is*

$$E(\boldsymbol{p}, U) = \left(\sum_{h \in H} e_h(\boldsymbol{p}; u_h(\boldsymbol{c}_h^0)) \right) U.$$

By Shephard's lemma, the budget share of the Hicksian representative agent on good i, denoted b_i^{comp}, is

$$b_i^{comp}(\boldsymbol{p}) = \frac{\partial \log E(\boldsymbol{p}, U)}{\partial \log p_i} = \sum_h \frac{e_h(\boldsymbol{p}, u_h^0)}{\sum_{h'} e_{h'}(\boldsymbol{p}, u_{h'}^0)} b_{hi}(\boldsymbol{p}, u_h^0),$$

where $b_{hi}(\boldsymbol{p}, u_h^0)$ is the compensated budget share of household i at the status-quo indifference curve $u_h^0 \equiv u_h(\boldsymbol{c}_h^0)$.

In words, the compensated agent's budget share on each good i is the average compensated budget share of all households, where each household is weighted according to its compensating income, $e_h(\boldsymbol{p}, u_h^0)$.

Given compensated aggregate budget shares, $b_i^{\text{comp}}(\boldsymbol{p})$, we can solve for equilibrium variables in the compensated equilibrium including prices $\boldsymbol{p}^{\text{comp}}$. Setting aggregate spending to be the numeraire in the compensated equilibrium, and using Theorem 2, we know that

$$A(t) = U(t) = \frac{1}{\left(\sum_{h\in H} e_h(\boldsymbol{p}^{\text{comp}}(t); u_h(\boldsymbol{c}_h^0))\right)}.$$

Appendix B Definition of Positive & Normative Representative Agent

We follow the definitions in Mas-Colell et al. (1995). We say that $u^{RA} : \mathbb{R}^N \to \mathbb{R}$ is a *positive representative agent* if the Marshallian demand curves generated by u^{RA}, given prices and total income, coincide with equilibrium allocations given the same prices and aggregate income:

$$\arg\max_{\boldsymbol{c}}\{u^{RA}(\boldsymbol{c}) : \sum_i p_i(t)c_i \leq I(t)\} = \sum_h \arg\max_{\boldsymbol{c}_h}\{u_h(\boldsymbol{c}_h) : \sum_i p_i(t)c_{hi} \leq I_h(t)\}.$$

The positive representative agent, $u^{RA} : \mathbb{R}^N \to \mathbb{R}$, is a *normative representative agent* relative to the social welfare function W if for every $(\boldsymbol{p}(t), I(t))$, the distribution of wealth across households, denoted by $\{I_h(t)\}$, also maximizes

$$W(v_1(\boldsymbol{p}(t), I_1(t)), \ldots, v_H(\boldsymbol{p}(t), I_H(t)))$$

subject to $\sum_{h=1}^H I_h(t) = I(t)$, where v_h is the indirect utility function of agent h.

Appendix C Proofs

Proof of Theorem 1. Denote the solution to (1) by ϕ^*, and an allocation that attains this solution (it does not need to be unique) by $\boldsymbol{c}^* \in (\phi^*)^{-1}\mathcal{C}$. By the definition of ϕ^* and $U(\cdot)$, and given local nonsatiaton, $U(\boldsymbol{c}^*) = U(\boldsymbol{c}^0) = 1$. Define the value to the compensated representative agent of the set $\mathcal{C}$ by $V(\mathcal{C}) = \max_{\boldsymbol{c}\in\mathcal{C}} U(\boldsymbol{c})$. Denote the solution to $V(\mathcal{C})$ by $\boldsymbol{c}^{**}$, with $\boldsymbol{c}^{**} \in \mathcal{C}$. Since $U(\cdot)$ is homogeneous of degree 1, it follows that $V((\phi^*)^{-1}\mathcal{C}) = (\phi^*)^{-1}V(\mathcal{C})$, with solution $(\phi^*)^{-1}\boldsymbol{c}^{**}$. Note that $V\left((\phi^*)^{-1}\mathcal{C}\right) \geq 1$ because $\boldsymbol{c}^* \subseteq (\phi^*)^{-1}\mathcal{C}$ and $U(\boldsymbol{c}^*) = 1$. Moreover, $V\left((\phi^*)^{-1}\mathcal{C}\right) = 1$ because, if it were strictly higher than 1, the solution to (1) would be higher than ϕ^*. It thus follows that:

$$V(\mathcal{C}) = \frac{V(\mathcal{C})}{V\left((\phi^*)^{-1}\mathcal{C}\right)} = \phi^*.$$

$\square$

Proof of Theorem 2. Let $C(t) = \mathcal{C}(\boldsymbol{z}(t), \boldsymbol{\mu}(t))$ be the set of consumption allocations that can be attained in general equilibrium with transfers. Define

$$\boldsymbol{c}^*(t) \in \text{argmax}\left\{U(c) : c \in C(t)\right\}.$$

By definition, there exists some $\boldsymbol{p}^*(t)$ and $\boldsymbol{T}^*(t)$ supporting $\boldsymbol{c}^*(t)$ with aggregate income $I(t)$. Consider the rep agent problem

$$\boldsymbol{c}^{comp}(t) \in \text{argmax}_{\boldsymbol{c}}\left\{U(\boldsymbol{c}) : \boldsymbol{p}^*(t) \cdot \boldsymbol{c} \leq I(t)\right\}.$$

Since $\boldsymbol{c}^*(t)$ is affordable to the compensated representative agent, we know that

$$U(\boldsymbol{c}^{comp}(t)) \geq U(\boldsymbol{c}^*(t)).$$

Since $\boldsymbol{c}^{comp}(t) \in C(t)$, we also know that

$$U(\boldsymbol{c}^*(t)) \geq U(\boldsymbol{c}^{comp}(t)).$$

Hence,

$$A(t) = U(\boldsymbol{c}^*(t)) = U(\boldsymbol{c}^{comp}(t)).$$

This proves the first equality. Since the compensated representative agent has a homogeneous of degree 1 utility function, the remaining equalities follow as a consequence of standard results for representative agent economies. $\square$

Proof of Lemma 1. Consider the decentralized equilibrium at the status-quo. We wish to show that the same prices and allocations are a compensated equilibrium. This simply requires showing that given decentralized price vector $\boldsymbol{p}(0)$ and aggregate income $I(0)$, the compensated representative agent makes the same consumption choices. This follows from the fact that (i) $\boldsymbol{c}(0)$ is affordable to the compensated representative agent; (ii) any other consumption choice that dominates $\boldsymbol{c}(0)$ is unaffordable to the compensated representative agent.

(ii) follows from the fact that if $U(\boldsymbol{c}') > U(\boldsymbol{c}(0))$, then it must be that $\boldsymbol{c}'_h \succ_h \boldsymbol{c}_h(0)$ for every h. The latter fact implies that $\boldsymbol{p}(0) \cdot \boldsymbol{c}_h^{'} > \boldsymbol{p}(0) \cdot \boldsymbol{c}_h(0)$ for every h (otherwise $\boldsymbol{c}(0)$ would not be a decentralized equilibrium). Summing this across all h guarantees that $\boldsymbol{c}'$ is unaffordable because $\boldsymbol{p}(0) \cdot \sum_h \boldsymbol{c}_h' > \boldsymbol{p}(0) \cdot \sum_h \boldsymbol{c}_h = I(0)$. □

Proof of Proposition 1. By Theorem 2, we know that

$$A(t) = Y^{comp}(t).$$

If preferences are identical, homothetic, and all households face the same relative prices, then the distribution of spending across households has no effect on equilibrium relative prices. Hence, $\boldsymbol{p}^{comp}(t) = \boldsymbol{p}(t)$. Given these prices, and homotheticity of preferences, we also know that

$$\sum_h \boldsymbol{c}^{comp}(t) = \sum_h \boldsymbol{c}(t).$$

From this, it follows that

$$A(t) = Y^{comp}(t) = Y(t).$$

Since there is a positive representative agent with homothetic preferences, it follows from standard results that

$$Y(t) = A^{RA}(t).$$

Finally, letting $u(\boldsymbol{c})$ be the homogeneous of degree one representation of the utility function of every agent (since all agents have the same preferences), we have that

$$A^{KH}(t) = \frac{\sum_h e_h(\boldsymbol{p}(t), u_h(t))}{\sum_h e_h(\boldsymbol{p}(t), u_h(0))} = \frac{\sum_h u_h(t)}{\sum_h u_h(0)} = \frac{\sum_h u(\boldsymbol{c}_h(t))}{\sum_h u(\boldsymbol{c}_h(0))},$$
$$= \frac{\sum_h u(\boldsymbol{c}(t)\chi_h(t))}{\sum_h u(\boldsymbol{c}(0)\chi_h(0))},$$

where $\chi_h(t)$ is household h's share of aggregate expenditures at t,

$$= \frac{\sum_h \chi_h(t) u(\boldsymbol{c}(t))}{\sum_h \chi_h(0) u(\boldsymbol{c}(0))},$$
$$= \frac{u(\boldsymbol{c}(t))}{u(\boldsymbol{c}(0))} = A^{RA}(t).$$

□

Proof of Proposition 2. This is restatement of Hulten (1978). □

Proof of Proposition 3. This follows from combining Theorem 2 with Proposition 2. □

Proof of Proposition 4. This follows from combining Theorem 2 with Proposition 3 from Baqaee and Farhi (2019b). □

Proof of Proposition 6. Kaldor-Hicks efficiency can be defined using the aggregate consumption-equivalent variation where the set we shift is the aggregate budget set. Specifically,

$$A^{KH}(t) = \max \left\{ \phi \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \phi^{-1}\mathcal{B}(\boldsymbol{p}(t), I(t)) \text{ and } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h \right\},$$

where $\mathcal{B}(\boldsymbol{p}(t), I(t)) = \{\boldsymbol{c} : \boldsymbol{p}(t) \cdot \sum_h \boldsymbol{c_h} \leq I(t)\}$ and $I(t) = \sum_h I_h(t)$. In the absence of distortions ($\boldsymbol{\mu} = 1$), $\boldsymbol{p}(t)$ and $I(t)$ are prices and income in the competitive equilibrium.

If $\mathcal{C}' \subseteq \mathcal{C}$ then $A(\boldsymbol{c}^0, \mathcal{C}') \leq A(\boldsymbol{c}^0, \mathcal{C})$. To see this, let c^* and ϕ^* be the solution to $A(\boldsymbol{c}^0, \mathcal{C}')$. That is, $c^* \in \mathcal{C}'$ and $u_h(\boldsymbol{c}_h^*) \geq u_h(\boldsymbol{c}_h^0)$ for every h. Since $c^* \in \phi^*\mathcal{C}$, the statement above follows. Hence, because the Pareto frontier $\mathcal{C}(\boldsymbol{z}(t))$ is contained in the aggregate budget set $\mathcal{B}(\boldsymbol{p}(t), I(t))$, it follows that $A(t) \leq A^{KH}(t)$. Finally, pure redistributions leave the Pareto frontier unchanged, so $\mathcal{C}(t)$ is unchanged with t and $A(t) = 1$. □

Proof of Proposition 7. This is a consequence of Theorem 2 and Petrin and Levinsohn (2012). □

Proof of Proposition 8. Index wedges by t, and denote the status-quo with wedges by $\boldsymbol{c}^0(t)$. Let $t = 0$ denote the point where $\boldsymbol{\mu}(0) = \mathbf{1}$. The set $\mathcal{C}(0)$ is then the Pareto efficient frontier. From Proposition 7, we have that

$$\log A(\boldsymbol{c}_0(t), C(0)) = -\int_0^t \sum_i \lambda_i^{\text{comp}}(s) \left(1 - \frac{1}{\mu_i(s)}\right) \frac{d \log y_i^{\text{comp}}}{ds} ds.$$

Differentiate the expression above to get

$$\frac{d}{dt}[\log A] = -\sum_i \lambda_i^{\text{comp}}(t) \left(1 - \frac{1}{\mu_i(t)}\right) \frac{d \log y_i^{\text{comp}}}{dt}.$$

Differentiate a second time to get

$$\frac{d^2}{dt^2}[\log A] = -\sum_i d\lambda_i^{\text{comp}}(t) \left(1 - \frac{1}{\mu_i(t)}\right) \frac{d \log y_i^{\text{comp}}}{dt} - \sum_i \lambda_i^{\text{comp}}(t) \frac{1}{\mu_i(t)} \frac{d \log \mu_i}{dt} \frac{d \log y_i^{\text{comp}}}{dt}$$
$$- \sum_i \lambda_i^{\text{comp}}(t) \left(1 - \frac{1}{\mu_i(t)}\right) \frac{d^2 \log y_i^{\text{comp}}}{dt^2}.$$

Evaluate these derivatives at $t = 0$ and write the second-order Taylor approximation:

$$\log A \approx 0 - \frac{1}{2}\sum_i \lambda_i^{\text{comp}}(0)\frac{1}{\mu_i(0)}\frac{d\log\mu_i}{dt}dt\frac{d\log y_i^{\text{comp}}(0)}{dt}dt.$$

To a second-order, this can also be written as

$$\Delta\log A \approx 0 - \frac{1}{2}\sum_i \lambda_i^{\text{comp}}(t)\frac{d\log\mu_i}{dt}dt\frac{d\log y_i^{\text{comp}}(t)}{dt}dt \approx -\frac{1}{2}\sum_i \lambda_i^{\text{comp}}(t)\Delta\log\mu_i\Delta\log y_i^{\text{comp}},$$

since the differences are higher-order. □

Let $\boldsymbol{\tau}^*(t)$ and $\boldsymbol{T}^*(t)$ to be the maximizers of (12). Using $\boldsymbol{\tau}^*(t)$, we provide a slightly more general definition of the compensated equilibrium.

Definition 6 (Compensated Equilibrium)**.** A *compensated equilibrium* is the general equilibrium of an economy with the same technologies, resource constraints, wedges, and linear taxes $\boldsymbol{\tau}^*(t)$ but where there is a representative agent with preferences as in Definition 3. For any equilibrium variable $X(t)$, denote the same variable in the compensated equilibrium by $X^{\text{comp}}(t)$.

The following result, which is a consequence of Theorem 1, generalizes Theorem 2 to allow for limited redistribution.

Theorem 3 (Aggregate Efficiency Using Compensated Equilibrium)**.** *If outcomes are interior, then aggregate efficiency can be calculated using the compensated equilibrium:*

$$A(t) = Y^{comp}(t) = A^{KH,comp}(t) = A^{RA,comp}(t).$$

In words, aggregate efficiency, $A(t)$, can be computed by solving for changes in real output, or welfare of the Hicksian representative agent, in the compensated equilibrium.[36] Once again, this means that tools and results used to calculate welfare in homothetic representative agent economies can be converted into results about aggregate efficiency with heterogeneous and non-homothetic preferences.

The main challenge lies in knowing the necessary taxes $\tau^*(t)$ which the proposition takes as given. However, given these taxes, then the change in every price and quantity in the compensated equilibrium can be calculated as a function of t by applying the results in Baqaee and Farhi (2020).

[36]The fact that Kaldor-Hicks efficiency, $A^{KH,comp}(t)$, in the compensated equilibrium with distortionary taxes τ^* coincides with the rest follows trivially from the fact that the compensated equilibrium has a single agent with homothetic preferences. It is important to note that $A^{KH,comp}(t)$ is not the same as $A^{KH}(t)$.

Proof of Theorem 3. Consider

$$A(\boldsymbol{z},\boldsymbol{\mu}) = \max\left\{U\left(\boldsymbol{c}(\boldsymbol{z},\boldsymbol{\mu},\boldsymbol{\tau},\boldsymbol{T})\right) : (\boldsymbol{\tau},\boldsymbol{T}) \in \mathcal{T}(\boldsymbol{z},\boldsymbol{\mu})\right\}.$$

Index technology and wedges by t and let $\tau^*(t)$ and $T^*(t)$ be optimizers. Let $\boldsymbol{c}^*(t) = \boldsymbol{c}(\boldsymbol{z}(t),\boldsymbol{\mu}(t),\boldsymbol{\tau}^*(t),\boldsymbol{T}^*(t))$ be a consumption allocation that maximizes the problem above. Interior outcomes implies that

$$\tilde{u}_h(\boldsymbol{c}_h^*(t)) = \tilde{u}_{h'}(\boldsymbol{c}_{h'}^*(t)).$$

Let $\boldsymbol{p}^*(t)$ and $I^*(t)$ be the equilibrium price vector and income corresponding to maximizer of problem above. Let $\tilde{e}_h(p)$ be the unit cost function associated with $\tilde{u}$. Hence, we have that spending by household h, denoted by $\chi_h^*(t)$ is

$$\chi_h^*(t) = \tilde{e}_h(t)\tilde{u}_h(t).$$

Consider the compensated representative agent's problem in a decentralized equilibrium, facing the same relative prices,

$$\boldsymbol{c}^{comp}(t) \in \text{argmax}_{\boldsymbol{c}}\left\{U(\boldsymbol{c}) : \boldsymbol{p}^*(t)\cdot\boldsymbol{c} \le I(t)\right\},$$

where by definition $U(\boldsymbol{c}) = \min_{h\in H}\{\tilde{u}_h(\boldsymbol{c}_h)\}$. Hence, by two-stage budgeting, we can rewrite the representative agent's problem as

$$\chi_h^{\text{comp}} \in \text{argmax}_{\chi_h}\left\{\min_{h\in H}\left\{(\chi_h/\tilde{e}_h(\boldsymbol{p}^*(t)))\right\} : \sum_h \chi_h \le I(t)\right\}.$$

This is a convex optimization problem and the first-order conditions are necessary and sufficient for the global optimum. That is, $\chi_h^{\text{comp}}(t)$ is pinned down by that

$$\chi_h^{\text{comp}}(t)/\tilde{e}_h(\boldsymbol{p}^*(t)) = \chi_{h'}^{\text{comp}}(t)/\tilde{e}_{h'}(\boldsymbol{p}^*(t)),$$

and

$$\sum_h \chi_h^{\text{comp}}(t) = I(t).$$

This is to say that

$$\tilde{u}_h(\boldsymbol{c}_h^{\text{comp}}(t)) = \tilde{u}_{h'}(\boldsymbol{c}_h^{\text{comp}}(t)) \qquad \forall h, h'$$

and, from budget balance, we know that

$$\sum_h \tilde{e}_h(\boldsymbol{p}^*(t))\tilde{u}_h(\boldsymbol{c}_h^{\text{comp}}(t)) = I(t).$$

But these conditions are also satisfied by $\boldsymbol{c}^*(t)$. Hence, $\boldsymbol{c}^*(t)$ constitutes a compensated equilibrium. □

Proof of Proposition 9. Combine Theorem 3 with Proposition 4 from Baqaee and Rubbo (2023).

□

Proposition 11 (Equivalence of Kaldor-Hicks and Aggregate Efficiency)**.** *If there is one primary factor, so that relative prices are independent of demand, then* $A(t) = A^{KH}(t)$*.*

Proof. With one primary factor of production and constant-returns technologies, it is well-known that relative prices do not depend on final demand. Hence, the vector of equilibrium prices and aggregate income in the decentralized (multi-agent) economy $\boldsymbol{p}(t)$ and $I(t)$ are also compensated equilibrium prices and aggregate income. Theorem 2 implies that $A(t) = A^{KH,comp}(t)$. Since relative prices and aggregate income are the same, it follows that $A^{KH,comp}(t) = A^{KH}(t)$. □

Appendix D Additional Examples and Derivations

Homothetized non-homothetic CES. Consider a household with non-homothetic CES preferences, as in Comin et al. (2021),

$$u_h(\boldsymbol{c}_h) = \left(\sum_i (c_{hi})^{\frac{\eta-1}{\eta}} (u_h(\boldsymbol{c}_h))^{\xi_i} \right)^{\frac{\eta}{\eta-1}}.$$

where η is the compensated elasticity of substitution and ξ_i controls income effects. Then $\tilde{u}_h(\boldsymbol{c}_h)$ is homothetic CES given by

$$\tilde{u}_h(\boldsymbol{c}_h) = \frac{1}{u_h^0} \left(\sum_i (c_{hi})^{\frac{\eta-1}{\eta}} \left(u_h^0\right)^{\xi_i} \right)^{\frac{\eta}{\eta-1}},$$

where $u_h^0 \equiv u_h(\boldsymbol{c}_h^0)$ is treated as a constant. If ξ_i are the same for every i, then $\tilde{u}_h$ and u_h are both cardinalizations of the same preference rankings.

Derivations in Example 2 The homothetized utility function associated to

$$u_h(\boldsymbol{c}_h) = c_{hg}^{\alpha} c_{hs}^{1-\alpha},$$

is

$$\tilde{u}_h(\boldsymbol{c}_h) = \frac{c_{hg}^{\alpha} c_{hs}^{1-\alpha}}{\left(c_{hg}^0\right)^{\alpha} \left(c_{hs}^0\right)^{1-\alpha}}.$$

The compensated representative agent assuming interior outcomes sets

$$A = \tilde{u}_h = \tilde{u}_{h'}$$

for all h' subject to

$$\sum_h c_{hg} = z_g, \qquad c_{hs} = z_{hs}.$$

Hence, $\{c_{hg}\}_{h=1}^{H}$ solves

$$A = \frac{c_{hg}^{\alpha} z_{hs}^{1-\alpha}}{\left(c_{hg}^0\right)^{\alpha} \left(c_{hs}^0\right)^{1-\alpha}} = \frac{c_{h'g}^{\alpha} z_{h's}^{1-\alpha}}{\left(c_{h'g}^0\right)^{\alpha} \left(c_{h's}^0\right)^{1-\alpha}}, \qquad \text{for all } h'$$

subject to $\sum_h c_{hg} = z_g$. The solution is

$$A = \left(\frac{z_g}{\sum_h z_{hs}^{\frac{\alpha-1}{\alpha}} c_{hg}^0 \left(z_{hs}^0\right)^{\frac{1-\alpha}{\alpha}}} \right)^{\alpha}.$$

In status-quo, $c_{hg}^0 = \chi_h^0 z_g^0$,[37] so

$$A = \left(\frac{z_g / z_g^0}{\sum_h \chi_h^0 \left(z_{hs}/z_{hs}^0\right)^{\frac{\alpha-1}{\alpha}}} \right)^{\alpha}.$$

A second-order approximation yields the expression in the text.

[37] To see that χ_h^0 is also region h's share in total income, note that (under the Cobb-Douglas specification of this example) the first-order condition for c_{hg} and c_{hs} is $p_{hs}^0 z_{hs}^0 = \frac{1-\alpha}{\alpha} \chi_h^0 z_g^0$, where we normalize the price of the tradable good to 1. Therefore, $\chi_h^0 z_g^0 + p_{hs}^0 z_{hs}^0 = \frac{1}{\alpha} \chi_h^0 z_g^0$ and $\frac{\chi_h^0 z_g^0 + p_{hs}^0 z_{hs}^0}{\sum_{h'} \chi_{h'}^0 z_g^0 + p_{h's}^0 z_{h's}^0} = \frac{\chi_h^0}{\sum \chi_{h'}^0} = \chi_h^0$.

Derivations in Example 3 The homothetized utility function associated to

$$u_h(\boldsymbol{c}_h) = \left[(\alpha_h)^{\frac{1}{\theta_h}} \left(u_h(\boldsymbol{c}_h)\right)^{\zeta_h} c_{hd}^{\frac{\theta_h - 1}{\theta_h}} + (1-\alpha_h)^{\frac{1}{\theta_h}} c_{hf}^{\frac{\theta_h - 1}{\theta_h}} \right]^{\frac{\theta_h}{\theta_h - 1}},$$

is

$$\tilde{u}_h(\boldsymbol{c}_h) = \frac{1}{u_h^0} \left[(\alpha_h)^{\frac{1}{\theta_h}} (u_h^0)^{\zeta_h} c_{hd}^{\frac{\theta_h - 1}{\theta_h}} + (1-\alpha_h)^{\frac{1}{\theta_h}} c_{hf}^{\frac{\theta_h - 1}{\theta_h}} \right]^{\frac{\theta_h}{\theta_h - 1}},$$

In autarky, $c_{hf} = 0$, so

$$\tilde{u}_h\left([c_{hd}, 0]\right) = \frac{1}{u_h^0} c_{hd} \left[(\alpha_h)^{\frac{1}{\theta_h}} (u_h^0)^{\zeta_h} \right]^{\frac{\theta_h}{\theta_h - 1}}$$

The domestic expenditure share of household h in the status quo is

$$s_{hd}^0 \equiv \frac{p_d^0 c_{hd}^0}{p_d^0 c_{hd}^0 + p_f^0 c_{fd}^0} = \frac{p_d^0 c_{hd}^0}{p_h^0 c_h^0} = \frac{(\alpha_h)^{\frac{1}{\theta_h}} (u_h^0)^{\zeta_h} \left(c_{hd}^0\right)^{\frac{\theta_h - 1}{\theta_h}}}{\left(u_h^0\right)^{\frac{\theta_h - 1}{\theta_h}}}$$

Solving for $\left[(\alpha_h)^{\frac{1}{\theta_h}} (u_h^0)^{\zeta_h} \right]$ and substituting into the homothetized utility function yields

$$\begin{aligned}
\tilde{u}_h\left([c_{hd}, 0]\right) &= \frac{c_{hd}}{c_{hd}^0} \left(s_{hd}^0\right)^{\frac{\theta_h}{\theta_h - 1}} \\
&= \frac{p_d^0}{p_h^0 c_h^0} \frac{p_h^0 c_h^0}{p_d^0 c_{hd}^0} c_{hd} \left(s_{hd}^0\right)^{\frac{\theta_h}{\theta_h - 1}} \\
&= \frac{p_d^0}{p_h^0 c_h^0} c_{hd} \left(s_{hd}^0\right)^{\frac{1}{\theta_h - 1}} \\
&= \frac{p_d^0 y_d^0}{p_h^0 c_h^0} \frac{c_{hd}}{y_d^0} \left(s_{hd}^0\right)^{\frac{1}{\theta_h - 1}} \\
&= \frac{1}{\chi_h^0} \frac{c_{hd}}{y_d^0} \left(s_{hd}^0\right)^{\frac{1}{\theta_h - 1}},
\end{aligned}$$

where $\chi_h^0 = p_h^0 c_h^0 / p_d^0 y_d^0$ is the share of $h's$ expenditures in total income (assuming balanced trade), and y_d^0 is the aggregate quantity of the home produced good in the status-quo (which is consumed and exported).

The compensated representative agent assuming interior outcomes sets

$$\tilde{u}_h = \tilde{u}_{h'}$$

for all h' subject to

$$\sum c_{hd} = y_d = y_d^0,$$

where y_d is the total output of domestic good in autarky (which is equal to that in status-quo). Combining, we obtain

$$\Delta \log A = \log \tilde{u}_h = \log \frac{1}{\sum \chi_h^0 \left(s_{hd}^0\right)^{-\frac{1}{\theta_h - 1}}},$$

which is the expression in the text.

Example with skill-biased technical change and costly redistribution. We now consider a simple example with skill-biased technical change that raises the real wage of high-skill workers but lowers the real-wage for low-skill workers. We compare how the response of aggregate efficiency changes depending on the redistributive tools available. Suppose that output (and consumption) are a CES aggregate of the output of manufacturing and services:

$$c = y = \left[\gamma_1^{\frac{1}{\rho}} y_m^{\frac{\rho-1}{\rho}} + (1-\gamma_1)^{\frac{1}{\rho}} y_s^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}},$$

where each sector's output is a CES aggregate of low- and high-skill labor

$$y_o = \left[\alpha_o^{\frac{1}{\sigma}} \left(z_{o1} l_{o1}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_o)^{\frac{1}{\sigma}} \left(z_{o2} l_{o2}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where l_{o1} is low- and l_{o2} is high-skill labor. The resource constraints are that

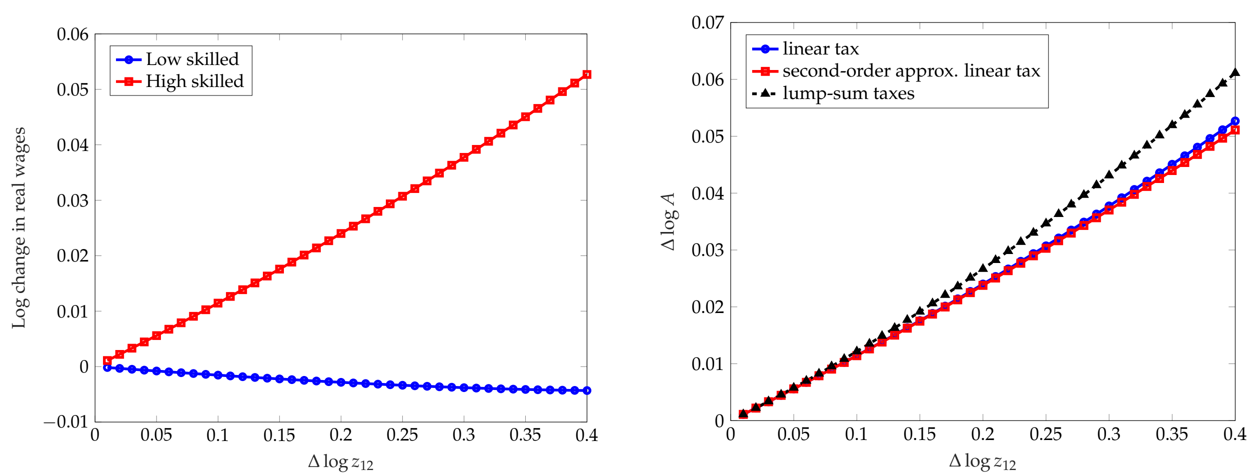
$$\sum_h c_h = c, \qquad \sum l_{o1} = l_1, \qquad \sum l_{o2} = l_2.$$

We assume that workers are much more substitutable than sectors: $\rho \ll \sigma$. We also assume that manufacturing is more intensive in low-skill labor use than services.

Consider an increase in automation or the productivity of capital, which we capture via an increase in the productivity of high-skill labor in manufacturing: $\Delta \log z_{m2} > 0$. This is a reduced-form representation for the idea that high-skill labor in manufacturing is equipped by capital, and hence an increase in the quality of capital makes high-skill more productive.[38]

Again, we contrast two scenarios: (1) lump-sum taxation is available, (2) lump-sum transfers must be non-negative and the government can only levy a linear tax on machine use in manufacturing, which we capture as a linear tax, τ, on manufacturing's use of high-skill labor.

Figure 7 illustrates the results in a numerical example. Panel 7a shows that skill-biased technical change raises the real wage for high-skill workers and lowers them for low-skill workers in the decentralized equilibrium. The fact that low-skill wages decline means that they need to be compensated via transfers financed by either lump-sum or distortionary taxes. Panel 7b shows the increase in efficiency depending on which taxes are used. As expected, the increase in aggregate efficiency is lower if only distortionary redstributive tools are available. Panel 7b also shows that the second-order approximation is very accurate. In the absence of any redistributive tools whatsoever, aggregate efficiency in this example actually declines because the low-skill workers are worst off and there is no feasible way to compensate them.



(a) real wages

(b) aggregate efficiency

Figure 7: A numerical example of skill-biased technical change. The parameter values are $\rho = 1$, $\sigma = 8$, $\gamma = 0.5$, $\alpha_{m1} = 0.9$, and $\alpha_{s1} = 0.5$. We normalize steady-state quantities so that the CES share parameters are equal to expenditure shares in the status-quo.

[38]For example, high-skill labor and capital are combined in a Leontief nest together called equipped labor, and then equipped labor is substitutable with low-skill labor. We can then think of altering the productivity of equipped labor by varying the productivity of capital.

Appendix E Explicit Characterization of Compensated Equilibrium

Theorem 2 and Theorem 3 (in Appendix C) show that calculating changes in aggregate efficiency can be boiled down to solving for the compensated equilibrium. This section provides some formulas for calculating variables in the compensated equilibrium. To do so, we rely on the differential hat algebra approach in Baqaee and Farhi (2020), which characterizes equilibria of representative agent economies with wedges using differential equations. Alternatively, one could also use exact-hat algebra methods, as in Dekle et al. (2008).

For concreteness, assume that all production and utility functions are nested-CES. (Non-CES economies can be analyzed in a similar way following the non-CES extensions in Baqaee and Farhi (2019b)). To make the notation more compact, represent the economy in such a way that each producer, i, is associated with a single elasticity of substitution θ_i (by treating each sub-nest as a separate producer).

E.1 Input-Output Notation

Stack the expenditure shares of the representative household, all producers, and all factors into the $(H+N+F)\times(H+N+F)$ input-output matrix Ω. The first H rows correspond to the households consumption baskets. The next N rows correspond to the expenditure of each producer on every other producer and factor as a share of its sales (where the sales price is always inclusive of the wedge and tax). The last F rows correspond to the expenditure shares of the primary factors (which are all zeros, since primary factors do not require any inputs). With some abuse of notation, the heterogeneous agent input-output matrix can be written as

$$\Omega = \left[\begin{array}{ccc|ccc|ccc} 0 & \cdots & 0 & b_{11} & \cdots & b_{1N} & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & & \cdots & & & \cdots & \\ 0 & \cdots & 0 & b_{H1} & \cdots & b_{HN} & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & \Omega_{11} & \cdots & \Omega_{1N} & \Omega_{1N+1} & \cdots & \Omega_{1N+F} \\ \vdots & \cdots & \vdots & & \ddots & & & & \\ 0 & \cdots & 0 & \Omega_{N1} & & \Omega_{NN} & \Omega_{NN+1} & \cdots & \Omega_{NN+F} \\ \hline 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{array}\right].$$

Note that our convention is that rows (not columns) record costs relative to revenues inclusive of wedges and taxes. If wedges and taxes are greater than one, then the rows of this matrix will generally sum to a number less than one. The Leontief inverse matrix is the $(H+N+F)\times(H+N+F)$ matrix defined as

$$\Psi \equiv (I-\Omega)^{-1} = I + \Omega + \Omega^2 + \dots,$$

where I is the identity matrix. The Leontief inverse matrix $\Psi \geq I$ records the *direct and indirect* exposures through the supply chains in the production network.

Denote the distribution of expenditures by each household by χ, which is an $(H+N+F)\times 1$ vector. The first H elements are equal to each household's share of aggregate consumption expenditures, and the remaining $N+F$ elements are all zeros. As a matter of accounting identities, the vector of Domar weights satisfies:

$$\lambda' = \chi'\Psi.$$

In this equation λ is a $(H+N+F)\times 1$ vector. The first H elements are equal the expenditures of each household relative to aggregate consumption expenditures, χ', the next $N+F$ elements are equal to the sales of each good and factor relative to aggregate consumption expenditures.

Let μ and τ denote the diagonal matrices whose iith element is equal to μ_i and τ_i respectively. Recall that μ are exogenous wedges, whereas τ are linear taxes that can be used for redistribution. Define the cost-based Leontief inverse to be

$$\tilde{\Psi} = (I-(\tau\mu)\Omega)^{-1}.$$

Note that the cost-based Leontief inverse coincides with Ψ in the absence of wedges. Intuitively, $\tilde{\Psi}$ is a version of the Leontief inverse that calculates exposures of i to j in terms of cost shares rather than revenue shares (revenues exceed costs if wedges and taxes are greater than one).

For any non-negative vector a, define

$$Cov_a(b,c) = \mathbb{E}_a[bc] - \mathbb{E}_a[b]\mathbb{E}_a[c] = \sum_i \frac{a_i}{\sum_{i'} a_{i'}} b_i c_i - \sum_i \frac{a_i}{\sum_{i'} a_{i'}} b_i \sum_i \frac{a_i}{\sum_{i'} a_{i'}} c_i,$$

where $\mathbb{E}_a[\cdot]$ denotes averages of vectors weighted by the elements of a. For any matrix X, denote its ith row and column by $X_{(i,:)}$ and $X_{(:,i)}$.

E.2 Differential Hat-Algebra

The next proposition characterizes compensated variables in terms of initial expenditure shares, wedges, and shocks.

Proposition 12 (Differential Equations for Compensated Equlibrium)**.** *Let aggregate spending be the numeraire. Then, assuming interior outcomes, the compensated equilibrium satisfies the following system of differential equations. For each $i \in H + N + F$, the compensated price satisfies*

$$d \log p_i^{comp} = \sum_j \tilde{\Psi}_{ij}^{comp} [d \log \mu_j \tau_j^* - d \log z_j] + \sum_{f \in F} \tilde{\Psi}_{if}^{comp} d \log \lambda_f^{comp}. \tag{14}$$

Compensated Domar weights for goods and factors satisfy

$$d\lambda_l^{comp} = \sum_j \lambda_j^{comp} (1-\theta_j) \mu_j^{-1} Cov_{\Omega_{(j,:)}^{comp}} \left(d \log p^{comp}, \Psi_{(:,l)}^{comp} \right) + Cov_{\chi^{comp}} \left(d \log \chi^{comp}, \Psi_{(:,l)}^{comp} \right) - \sum_j \lambda_j \left(\Psi_{jl} - \mathbf{1}[j = l] \right) d \log \mu_j \tau_j^*. \tag{15}$$

Changes in compensated expenditure shares for household h satisfy

$$d \log \chi_h^{comp} = d \log p_h^{comp}, \tag{16}$$

where $d \log p_h^{comp}$ is the price of the consumption bundle for household h. The compensated input-output matrix satisfies

$$d\Omega_{ij}^{comp} = (1 - \theta_i) \left(d \log p_j^{comp} - \mathbb{E}_{\Omega_{(i,:)}^{comp}} [d \log p^{comp}] \right) - d \log \mu_i. \tag{17}$$

Finally, $d \log y_i^{comp}$ is given by $d \log \lambda_i^{comp} - d \log p_i^{comp}$. The initial conditions are given by Lemma 1 that all prices and expenditures are equal to the ones in the competitive equilibrium for $t = 0$.

Equation (14), (15), and (17) are standard and identical to expressions in Baqaee and Farhi (2020). They are loglinearizations of marginal cost-functions, market clearing conditions, and demand curves respectively. The key equation, which distinguishes the compensated equilibrium from the decentralized equilibrium is (16). Whereas in the decentralized equilibrium changes in household expenditures are determined by changes in the income of each household, in the compensated equilibrium, they are determined by the choices of the compensated agent (who tries to equate homothetized utilities across agents). The term $d\log p_h^{\text{comp}}$, which is pinned down by (14), is the change in the compensated price index of household h.

The taxes $\boldsymbol{\tau}^*(t)$ are given by the maximizers of the problem in (12). If only lump-sum transfers are used for redistribution, as in Sections 4 and 5, then $\boldsymbol{\tau}^*(t) = 0$, and Proposition 12 fully characterizes the compensated equilibrium in terms of exogenous parameters: $\boldsymbol{z}(t)$ and $\boldsymbol{\mu}(T)$. If lump-sum transfers are unavailable, then solving for $\tau^*(t)$ requires specifying more details about the set of available tax instruments. Specifically, we would need to add the log-linearized first-order conditions for the tax instruments from (12) as additional equations in Proposition 12 to pin down how τ^* evolves.

There is one case where this optimization problem can be avoided. If there are only $H-1$ taxes available, and outcomes are interior, then (16) can pin down $\tau^*(t)$. For example, suppose that there are $H-1$ taxes, and the share of revenues from the ith tax sent to household h are given by α_{ih}:

$$T_h(t) = \sum_i \alpha_{ih}\left(1 - \frac{1}{\tau_i^*(t)}\right)\lambda_i(t).$$

Log-differentiating household h's budget constraint gives:

$$d\log\chi_h^{\text{comp}} = \sum_f \frac{\omega_{hf}\lambda_f^{\text{comp}}}{\chi_h^{\text{comp}}} d\log\lambda_f^{\text{comp}} + \frac{dT_h}{\chi_h^{\text{comp}}},$$

differentiating $T_h(t)$ above, and substituting it into the log-linearized budget constraint gives $H-1$ additional equations which, assuming regularity conditions, will pin down $d\log\boldsymbol{\tau}^*$.

Generally, solving the system of linear equations in Proposition 12 requires inverting a system of equations. When there is a single primary factor of production and we evaluate these derivatives at a perfectly competitive point, then the change in efficiency can be solved out easily up to a second-order.

Proposition 13 (Aggregate Efficiency with One Factor)**.** *Consider a competitive economy with a single primary factor of production. The change in aggregate efficiency in response to a vector of productivity shocks, $\Delta\log\boldsymbol{z}$ and changes in wedges $\Delta\log\boldsymbol{\mu}$ is*

$$\Delta\log A \approx \sum_i \lambda_i \Delta\log z_i + \frac{1}{2}\sum_{i\in N+H}\lambda_i(\theta_i - 1)Var_{\Omega(i,:)}\left(\sum_k \Psi_{(:,k)}\log z_k\right) - \frac{1}{2}\sum_{i\in N+H}\lambda_i\theta_i Var_{\Omega(i,:)}\left(\sum_k \Psi_{(:,k)}\Delta\log(\mu_k\tau_k^*)\right).$$

to a second-order approximation in $\Delta \log z$ *and* $\Delta \log \mu$.

There are three summands. The first one is just Hulten's theorem. The second summand is a nonlinear adjustment due to changes in Domar weights. The second summand is also equal to: $1/2 \sum_k \left[\sum_j \partial \lambda_k^{\text{comp}} / \partial \log z_j \Delta \log z_j \right] \Delta \log z_k$. If the compensated Domar weight for k rises due to productivity shocks, then the shock to k is more important. This happens if exposure to k is heterogeneous, captured by the variance term, and if elasticities of substitution, θ_i, are far from unity. The final summand are the Harbeger triangles caused by the taxes and wedges. The triangles are larger the higher are elasticities of substitution, θ_i, and the more heterogeneous are exposures to the taxes and wedges, captured by the variance terms.