Efficiency Costs of Incomplete Markets

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June 18, 2025

Abstract

This paper quantifies misallocation caused by limited risk-sharing and imperfect consumption-smoothing. We measure these losses in terms of how much of society's resources would be left over if financial markets were complete and each household was compensated to maintain its status-quo welfare. Using exact formulas and approximate sufficient statistics, we analyze standard incomplete-market environments—ranging from closed-economy Bewley-Aiyagari models to multi-country settings with input-output linkages. We find that incomplete insurance against idiosyncratic risk is very costly — on the order of 20% of aggregate consumption, using both structural models and sufficient statistics formulas applied to household consumption panel data. We estimate the costs of imperfect international financial markets (abstracting from within-country heterogeneity) to be around 5 percent overall. This depends critically on the inclusion of fast-growing countries such as China and India. Unexploited risk-sharing opportunities among countries at similar levels of development, on the other hand, are fairly limited (less than 1%).

*We thank Rodrigo Adao, Andrew Atkeson, Natalie Bau, Joao Guerreiro, Valentin Haddad, Oleg Itskhoki, Pablo Fajgelbaum, Gianluca Violante, and Pierre-Olivier Weill for their comments. We thank Lautaro Adamovsky for exceptional research assistance.

1 Introduction

If households can neither perfectly share risk across different states of nature nor smooth consumption efficiently over time, the resulting misallocation of resources can be substantial. This paper measures that misallocation by asking a counterfactual question: if financial markets were complete and we compensated every household so that no one was worse off than under the status quo, how much of the available resources would be left over? This is measure of inefficiency introduced in Baqaee and Burstein (2025b). We develop a framework to characterize this measure of misallocation in a range of workhorse models with incomplete financial markets — both in closed-economy (domestic) and open-economy (international) settings.

We provide exact formulas for these welfare losses and show how to approximate them using sufficient statistics derived from observed consumption allocations. In a calibrated Bewley-Aiyagari economy, our quantitative exercises reveal that misallocation is sizeable, around 20 percent — substantially larger than gains from eliminating aggregate fluctuations in complete markets (e.g. Lucas, 1987), yet smaller than measures that ignore households' ex-post heterogeneity (e.g., "behind-the-veil" welfare metrics). We validate this number by applying our sufficient statistics approach to the Panel Study of Income Dynamics (PSID), again finding losses on the order of 20 percent.

Turning to the international economics, we study a setup with multiple countries, each with a representative agent, and industries interconnected by input-output linkages. Here, we assume there are complete markets within countries but not across them. We estimate the costs of imperfect international risk sharing to be around 5 percent overall, but this depends crucially on the inclusion of fast-growing countries such as China and India. If we exclude these high-growth countries from the sample, then misallocation losses fall to below 1 percent. Together, these results underscore the potentially large welfare gains of more complete risk sharing, especially in domestic settings or between emerging and developed economies. Risk-sharing opportunities among countries at similar levels of development, on the other hand, are fairly limited.

Our approach exploits the fact that we need not explicitly model the specific imperfections in financial markets to quantify the resource misallocation they generate. Both our exact and approximate formulas for misallocation losses can be derived solely from the observed (status-quo) consumption allocation and from knowledge of the Pareto efficient frontier. As a result, once the equilibrium consumption process is taken as given, the fine-grained details of which frictions or market failures produced this consumption pattern become irrelevant. This greatly simplifies measurement and eliminates the need to make assumptions about or to calibrate the primitive financial imperfections.

A second key advantage is that our approximate formulas obviate the need to model the underlying productivity process or to disentangle which portion of consumption fluctuations stem from efficient (productivity-driven) shocks versus inefficient (distortion-induced) fluctuations. This distinction is especially pertinent in international settings, where some observed consumption swings are efficient responses to changes in productivity, while others reflect barriers to risk-sharing. By avoiding the necessity of identifying and separating these different sources of fluctuation, our sufficient statistics methodology substantially reduces data and modeling assumptions.

The outline of the paper is as follows. In Section 2 we set up the preferences, technologies, and resource constraints. In Section 3, we define misallocation, and characterize it exactly and approximately. The exact characterization shows how to convert the problem of solving for misallocation into a utility-maximization problem for an as-if representative agent with homothetic preferences. We show that the solution to this particular representative-agent's utility maximization problem yields the degree of misallocation in the primitive economy. This then enables the use of tools and techniques developed for studying single-agent economies to measure misallocation in this multi-agent setting. This result, which is a consequence of Theorem 1 from Baqaee and Burstein (2025b), shows that the as-if representative agent has Leontief preferences over the growth in certainty-equivalent utilities of the households in the primitive model.

Our approximate characterization recasts misallocation in terms of deadweight loss triangles. We show that each triangle depends only on the observed consumption allocation, the input-output network (if there is one), and elasticities of substitution in consumption and production. In particular, we do not need to model the productivity process or to separate consumption fluctuations due to wedges and due to productivities.

In Section 4, we specialize these results by focusing on the case were households have common preferences. This is relevant in a domestic or closed-economy setting, where all agents can access the same common consumption bundle. In this case, we can simplify both our exact and approximate characterization results. In this simplified setting, and in the absence of labor-leisure choice and capital accumulation, our measure is simply the certainty-equivalent of the aggregate endowment, which is uniquely defined since all agents have the same preferences, divided by the sum of certainty-equivalents of all households. In this special case, our measure is similar to the efficiency measure proposed by Benabou (2002). However, we show that with either labor-leisure choice, capital accumulation, or preference heterogeneity, our measure differs. In fact, we show that this with these additional ingredients, the Benabou (2002) measure has some counterintuitive properties — for example, it can prefer more inequality across households if there is labor-leisure choice.

Our deadweight loss triangles formula takes an especially simple form in the setting with a common consumption good. The height of each triangle is the log change in each household's share of the aggregate consumption basket over time divided by the intertemporal elasticity of substitution. The base of each triangle is the gap between each household's share of consumption and the conditional expectation of the net present value of its consumption share.

In Section 5, we apply our theoretical results to some quantitative and empirical examples. We begin using a calibrated Bewley (1972) model. We show that misallocation losses are approximately 20%.[1] Although this is a very high number, it is almost half as big as the losses according to a behind-the-veil of ignorance measure that ignores ex-post heterogeneity. We also conduct some sensitivity analysis showing how the distance to the frontier changes as a function of the size of the idiosyncratic shocks, the amount of public debt, and the size of the borrowing constraint. We show that our second-approximation formula performs well in this model. We also consider how allowing for capital accumulation alters our results. Capital accumulation introduces a new margin for distortions: excessive savings due to the precautionary motive.

We then apply our second-order approximation to consumption panel data from the PSID. In our benchmark specification, misallocation losses from incomplete risk-sharing are also around 20%. Losses are greater, the lower is the intertemporal elasticity of substitution and the impatience parameter. To apply our deadweight loss triangles formula, we must estimate the conditional expectation of the net present value of each household's share of aggregate consumption. To do this, we use a regression that conditions on household observables — for example, income, education, wealth, etc. We show that, as we drop covariates from this regression, estimated losses from misallocation rise. Intuitively, this is because with a less informative regression, we attribute some systematic differences between households to lack of risk-sharing rather than to differences in observable household characteristics.

We end Section 5 by applying our second-order approximation formula to analyze losses from the lack of international risk-sharing. We calibrate an Armington model of world trade with 32 countries, 54 industries in each country, and input-output linkages. We calibrate elasticities of substitution and apply our formula to a sample from 1970 to

[1] A related question studied by e.g. Lucas (1987), Atkeson and Phelan (1994), and Alvarez and Jermann (2004), is the gains from eliminating fluctuations in aggregate consumption. Our counterfactual question is different because we do not modify the process of aggregate consumption when financial markets are completed.

2019. We find misallocation losses roughly in the 5% range. That is, if financial markets were complete and every household was compensated to be indifferent to the initial allocation, then there would be 5% of every consumption good left.

This result crucially depends on the fact that some countries in our sample, principally China but some extent also India, experienced very rapid growth during the sample compared to other countries. This means that there is significant unexploited gains from intertemporal trade between countries. If we exclude the fast-growing countries, like China and India, from our sample, then the misallocation losses drop to around 1%. We consider sensitivity analysis and show, once again, that losses are higher if the intertemporal elasticity of substitution is lower. We also show that losses are larger if the Armington trade elasticity is higher. Intuitively, there are more unexploited opportunities to share risk and smooth consumption, conditional on the data, if the foreign and domestic goods are more substitutable.

Relation to companion papers. Although this paper is self-contained, it has two companions. Baqaee and Burstein (2025b) provides a general framework for studying aggregate efficiency with heterogeneous agents. Many of the results in this paper are therefore applications of the general approach in that paper. In the other companion paper, Baqaee and Burstein (2025a), we apply the the same framework to study changes in aggregate efficiency in spatial economies with discrete choice and heterogeneous consumer tastes.

Additional related literature. This chapter is related to papers that analyze efficiency properties of models with incomplete financial markets. There are two main branches of this literature. The first branch is concerned with domestic risk-sharing of idiosyncratic household-level risks in closed-economy settings. The second branch analyzes efficiency of risk-sharing in an international context with nontraded goods. We discuss these two branches of the literature in sequence.

The first branch derives from Bewley (1972), and its extensions including Imrohoroğlu (1989), Huggett (1993), and Aiyagari (1994). To evaluate aggregate welfare in this class of models, there are two common approaches in the literature. The first is to use a social welfare function, typically by appealing to behind-the-veil of ignorance logic of Harsanyi (1955).[2] It is understood that social welfare functions, including the utilitarian behind-the-veil one, embed some distributional judgement and require interpersonal comparisons. These measures are typically averse to efficient but unequal allocations across

[2] Some examples include Heathcote et al. (2008), Conesa et al. (2009), Dávila et al. (2012), Krueger et al. (2016), and Boar and Midrigan (2022).

households. The second approach, following Benabou (2002) and then Floden (2001), aims to separate Pareto-efficiency considerations from redistributional ones. Instead of aggregating individual consumptions or utilities, these measures sum up consumption certainty-equivalents. Both of these approaches exploit the fact that all goods are tradeable and that all households have the same preferences.

In the special cases where we impose both of these assumptions, our approach is more closely related to the one taken by Benabou (2002). Specifically, we show that our definition of aggregate efficiency collapses to a measure similar to his as long as (1) there is no capital accumulation, and (2) there is no labor-leisure choice. The typical approach in this branch of the literature to dealing with these complications is to hold them fixed in the calculation of certainty equivalents. We complement this prior work by adopting a definition that accounts for inefficient capital accumulation and labor-leisure choice due to imperfect risk-sharing.

However, even abstracting from both capital accumulation and labor-leisure choice, our paper complements Benabou (2002) and the literature that followed it by providing a second-order approximation of the efficiency losses from imperfect risk-sharing. Our approximation formula, which is a Harberger (1954) triangles formula, requires only estimates of the EIS, the risk-free interest rate, and second moments of the household consumption allocations. This allows us to approach the problem of quantifying misallocation with significantly weaker structural assumptions — not taking a stance for example on the nature of financial market imperfections or households' idiosyncratic income processes.

Our focus is on the distance of the allocation from the Pareto frontier. This means that, when financial markets completed, we allow for lump-sum transfers between households to ensure everyone is compensated. Some papers in this literature, including Benabou (2002), consider constrained efficiency and second best policies (with imperfect redistribution). Although our framework can be applied to study such questions, we do not pursue them in this version of the paper.[3]

The second branch of the risk-sharing literature focuses on the international dimension of the problem — taking seriously the fact that goods are non-tradeable. Some examples include Van Wincoop (1994), Gourinchas and Jeanne (2006), Fitzgerald (2012), Heath-

[3]Extending our approach to second best scenarios means that we would ask the same counterfactual question to define aggregate efficiency, but use the restricted possibility set taking into account feasible (potentially imperfect) transfers. This is related to Farhi and Werning (2012), who quantify the aggregate welfare gains from capital taxation in an incomplete market model with private information, using the resources saved when implementing the inverse Euler equation while holding labor decisions and utilities unchanged. This is also related to the approach in Schulz et al. (2023) and Aguiar et al. (2024), who use a Pareto improvement criterion rather than social welfare functions, to evaluate second-best policies.

cote and Perri (2014), Fitzgerald (2024), Corsetti et al. (2024). In this literature, due to non-tradeability, neither the behind-the-veil or sum of certainty-equivalents approaches are justifiable, since the goods and preferences of different households are different. The approach to quantifying inefficiency in this literature is more eclectic. Some papers assume ex-ante symmetry, so that the aggregate gain from completing financial markets is also symmetric. Some papers eschew aggregate comparisons and report country-by-country results only. Lastly, some papers use Bergson (1954)-Samuelson (1983) social welfare functions, typically a so-called utilitarian function. Of course, there are also papers that analyze the efficiency properties of the decentralized aequilibrium, without quantifying inefficiency per se, for example Cole and Obstfeld (1991) and Backus and Smith (1993).

Our paper also contributes to this literature. Given the way we measure aggregate efficiency, we do not need to impose that countries be ex-ante symmetrical or to use a social welfare function. Accordingly, our measure relaxes the unrealistic assumption of symmetry while still eschewing interpersonal comparisons of utility and without taking a stance on distributional questions. On a methodological front, we show that our second-order approximation can be applied to the data without requiring that we model the productivity shocks that hit the economy. Instead, our second-order approximation requires only data on input-output tables at one point in time, an estimate of the risk-free rate, time series information on consumption and real exchange rates for each country, and estimates of elasticities of substitution in consumption and production.

Our paper is also related to recent work that provides approximate decompositions of changes in aggregate welfare using social welfare functions, for example Bhandari et al. (2021) and Dávila and Schaab (2022, 2023). Our paper is different because the measure of aggregate efficiency we use is distinct from the measures used in these papers in two ways. First, we do not specify a social welfare function. Second we define aggregate efficiency exactly and not as part of an approximate decomposition of aggregate welfare. This means that our measure can be integrated, allowing us to study the effect of large changes and, that generically, it does not coincide with what is referred to as efficiency in these papers.

Of course, our paper is also related to a different literature that studies the efficiency consequences of misallocation, following Harberger (1954), and more recently, Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). From a methodological and conceptual point of view, our paper is very closely related to this literature, though we study a very different type of misallocation. Whereas this literature typically emphasizes static cross-sectional misallocation in production, we study dynamic stochastic misallocation in consumption. Notwithstanding this difference, our methodological approach is similar. We analyze the distance to the Pareto-efficient frontier, we use reduced-form wedges to capture the frictions in the decentralized equilibrium, and we repurpose the triangles formulas developed by Baqaee and Farhi (2020) to study a very different class of problems.

2 Preferences, Technologies, and Resource Constraints

Below we introduce the preferences and the feasible set of allocations.

Households. Consider an economy populated by households indexed by $h \in \{1, ..., H\}$. Household h has intertemporal preferences over state-contingent consumption streams $\boldsymbol{c}_h$ represented by the utility function

$$u_h(\boldsymbol{c}_h) = \frac{1}{1 - 1/\eta} \sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}(s)^{1-\frac{1}{\eta}}.$$

Here, $c_{ht}(s)$ denotes consumption of household h at time t in state s (which may be an h-specific homothetic bundle of many goods), the discount factor is $\beta < 1$, and $\eta > 0$ is the elasticity of intertemporal substitution (EIS). Since the consumption bundle may vary across households, we are not imposing common preferences across households.[4] The probability of state s is denoted by $\pi(s)$, where each state s indexes a sample path of shocks (productivities and wedges). We denote the time-zero expectation, using probabilities $\pi(s)$ for each sample path s using the operator $\mathbb{E}_0$. In our baseline model, we abstract from labor-leisure choice and assume that labor is inelastically supplied. We extend our results to allow for labor-leisure choice later.

Technologies. In every period, t, of every state s, there is a set F of primary factor endowments and N of goods. The factors are inelastically supplied and owned by households, and used by producers in the same period (i.e. labor from t cannot be used by producers in $t+1$). Producer $i \in N$ has a CES production function that uses intermediate inputs and primary factor endowments with elasticity of substitution θ_i. Hence, the

[4] As we discuss below, our measure of aggregate efficiency does not depend on how utility functions are cardinalized.

production function of i is

$$y_{it}(s) = z_{it}(s) \left(\sum_{j \in N} \alpha_{ij} \left(y_{ijt}(s) \right)^{\frac{\theta_i - 1}{\theta_i}} + \sum_{f \in F} \alpha_{if} \left(l_{ift}(s) \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

where z_i is a Hicks-neutral productivity shifter, and $y_{ijt}(s)$ and $l_{ift}(s)$ are intermediate input j and factor input f. The scalars α_{ij} and α_{if} are share parameters that affect expenditures shares across inputs for each i.

Note that this structure is general enough to accommodate any pattern of nested-CES producers. This model also accommodates any Armington-style model of trade, and the productivity shifters, $z_{it}(s)$, for specialized intermediaries of imports and exports represent iceberg costs of trade.[5] Without loss of generality, we treat the consumption bundle of each household $c_{ht}(s)$ as-if it is produced by one of the goods producers and order the consumption goods first among the commodities in N.

The production structure above rules out capital accumulation by imposing that there be no intertemporal intermediate inputs. Hence, for the time being, we assume that output is non-storable and production functions in each period t rely only on intermediates and primary factors from the same period. We consider capital accumulation only as an extension.

Resource constraints. The resource constraints of the economy are as follows: consumption of good h equals its production,

$$y_{ht}(s) = c_{ht}(s), \qquad (h \in H)$$

use of intermediate input i equals its production,

$$\sum_{j \in N} y_{jit}(s) = y_{it}(s), \qquad (i \in N, i \notin H)$$

use of factor f equals its endowment

$$\sum_{j \in N} l_{jft}(s) = z_{ft}(s), \qquad (f \in F).$$

[5]This production structure is general enough to capture any nested-CES structure through re-labeling. This is because we allow each producer to have a different elasticity of substitution across its inputs. See Baqaee and Farhi (2019) and their discussion of the "stand-form" representation of nested-CES economies for more details.

Given these technologies and resource constraints, denote the dynamic consumption possibility set of the economy by $\mathcal{C}(\boldsymbol{z})$, where $\boldsymbol{z}$ is the vector of all state-contingent technology processes. That is, each element of $\mathcal{C}(\boldsymbol{z})$ is a vector of state-contingent consumptions streams for every household. By the second welfare theorem, $\mathcal{C}(\boldsymbol{z})$ is the set of perfectly competitive equilibria, with complete markets, given unrestricted lump-sum transfers.

3 Quantifying and Characterization Misallocation

In this section, we define the measure of misallocation, and we provide exact and approximate characterizations of it.

3.1 Definition of Misallocation

We specialize Definition 1 from Baqaee and Burstein (2025b) to this environment. Our definition of aggregate efficiency here coincides with the one proposed by Debreu (1951, 1954). Let $\boldsymbol{c}^0$ be the state- and date-contingent consumption allocation that is the equilibrium of some model with inefficient risk-sharing and financial frictions. We measure misallocation due to imperfect consumption-smoothing across time and states in the following way.

Definition 1. Misallocation in $\boldsymbol{c}^0$ is

$$A(\boldsymbol{c}^0, \mathcal{C}) \equiv \max \left\{ \phi \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \phi^{-1}\mathcal{C} \text{ and } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h \right\}.$$

We define the change in aggregate efficiency relative to the status-quo as

$$\Delta \log A(\boldsymbol{c}^0, \mathcal{C}) = \log A(\boldsymbol{c}^0, \mathcal{C}) - \log A(\boldsymbol{c}^0, \boldsymbol{c}^0) = \log A(\boldsymbol{c}^0, \mathcal{C}).$$

In words, misallocation is measured by the maximum contraction of the consumption possibility set such that it is possible to keep every agent at least in different. The scalar $\Delta \log A$ measures how much of every consumption good (in every period and state) is left over after every agent has been made indifferent. This is a measure of the total waste of resources caused by market incompleteness and frictions. Importantly, we do not take a stance on which agents would or should receive these extra resources if one were to complete markets. That is, this aggregate efficiency measure is silent on redistributions. If the initial consumption allocation is Pareto efficient, then $\Delta \log A$ is zero. Note that conditional on the status-quo allocation, $\boldsymbol{c}^0$, the nature of the specific financial frictions that led to it are irrelevant for calculating $\Delta \log A$ (e.g. borrowing constraints, limited commitment, incomplete assets, portfolio-choice frictions, etc.)

3.2 Exact Characterization

We use Theorem 1 from Baqaee and Burstein (2025b) to characterize $\Delta \log A$ as the solution to a representative-agent planning problem (which, by the first welfare theorem, can also be viewed as the competitive equilibrium allocation of that fictitious representative-agent economy).

Proposition 1 (Calculating misallocation via a planning problem)**.** *Misallocation in $\boldsymbol{c}^0$ is the solution to the planning problem*

$$A(\boldsymbol{c}^0, \mathcal{C}(z)) = \max_{\boldsymbol{c} \in \mathcal{C}(z)} \min_{h \in H} \left\{ \tilde{u}_h(\boldsymbol{c}_h) \right\}, \tag{1}$$

where $\tilde{u}_h(\boldsymbol{c}_h) = \left[u_h(\boldsymbol{c}_h) / u_h(\boldsymbol{c}_h^0) \right]^{\frac{\eta}{\eta-1}}$.

This proposition converts the problem of calculating misallocation into one of maximizing utility for a fictional representative agent. This fictional representative agent has Leontief preferences over growth in certainty-equivalent utilities of the households in the real economy relative to status-quo. To see, this define the consumption-equivalent of a consumption process $\boldsymbol{c}_h = \{c_{ht}(s)\}_{t,s}$ to be the function $CE_h(\boldsymbol{c}_h)$ that solves

$$u_h(\mathbf{1}CE) = u_h(\boldsymbol{c}_h),$$

where $\mathbf{1}$ is constant process equal to one. In words, CE is the constant deterministic consumption path that gives the same utility as $\boldsymbol{c}_h$. Then $\tilde{u}_h(\boldsymbol{c}_h) = CE_h(\boldsymbol{c}_h)/CE_h(\boldsymbol{c}_h^0)$ is the growth in the certainty-equivalent consumption of household h.[6] Solving the utility maximization in Proposition 1 is relatively straightforward, since it is a efficient representative agent problem with homothetic CES preferences.[7]

The function $\min_{h \in H} \{\tilde{u}_h(\boldsymbol{c}_h)\}$ in Proposition 1 is not a Rawlsian social welfare function. First, this function depends on the minimum change in certainty-equivalent utility,

[6] Proposition 1 is an application of Theorem 1 from Baqaee and Burstein (2025b). In Baqaee and Burstein (2025b), the functions $\tilde{u}_h$ are defined using the distance function, not certainty-equivalents. However, given the additional assumptions in this chapter, $\tilde{u}_h$ turns out to coincide with the ratio of certainty-equivalents.

[7] Proposition 1 applies without change even if we allowed households to have different discount factors, intertemporal elasticities of substitution, removed the CES assumption on production functions, and allowed for capital accumulation.

relative to the status-quo, whereas a Rawlsian social welfare function depends on the minimum level of utility. Second, whereas the allocation that maximizes a social welfare function is the optimal allocation, the allocation that maximizes (1) has no such interpretation. Rather, it is simply an analytical device for measuring aggregate efficiency $\Delta \log A$. Indeed, we do not ever define *the* optimal allocation on the Pareto frontier.

3.3 Approximate Characterization

The exact characterization in Proposition 1 requires fully specifying the Pareto frontier, including, for example, the productivity processes. In this subsection, we provide a (second-order) approximation for misallocation that does not require as much information to implement. To derive this approximation, we introduce the concept of an equilibrium with wedges. We decentralize the status-quo allocation using household-specific state- and date-contingent consumption taxes, and then apply Proposition 7 from Baqaee and Burstein (2025b).

Denote the *wedge*, which is an implicit tax, on the consumption of household h at time t in state s by $\mu_{ht}(s)$. The intertemporal budget constraint for household h, in the decentralization with wedges, is

$$\sum_s \sum_t q_t(s)\mu_{ht}(s)p_{ht}(s)c_{ht}(s) \leq I_h,$$

where $q_t(s)$ is the price of an Arrow security and $p_{ht}(s)$ is the price of the consumption good h in state s at time t not including the wedge, and I_h is initial wealth (including factor endowments and revenues from the consumption tax wedges).

We now define general equilibrium with wedges. Since we are focusing only on misallocation from incomplete markets for households, we abstract from other possible distortions for now and assume that firms set prices equal to marginal cost.

Definition 2 (Equilibrium with Wedges)**.** A general equilibrium with wedges is the collection of prices and quantities such that: (1) the price of each good i equals its marginal cost of production; (2) each producer takes prices as given and chooses quantities to maximize profits; (3) each household chooses consumption quantities to maximize utility taking prices, consumption tax wedges, and income as given; (4) household h earns income from primary factors and tax revenues; (5) all resource constraints are satisfied.

Since there are no intertemporal production linkages (i.e. capital accumulation) and firms set prices equal to marginal cost, the allocation in each period is statically Pareto-efficient. That is, holding fixed consumption allocations in every other period and state, and focusing only a single period and state, it is not possible to make one agent better off without making someone worse off. However, the equilibrium allocation is not dynamically Pareto-efficient since the state-contingent consumption taxes distort intertemporal trade and insurance between agents.

The following proposition shows that any feasible consumption allocation that is the equilibrium of a model with incomplete financial markets (and no other distortions) can be decentralized using some pattern of household-state-date-specific consumption taxes.[8]

Proposition 2 (Decentralization with Wedges)**.** *Consider some feasible state-contingent consumption allocation $\boldsymbol{c}^0$. Assume that for each period t and state s, the consumption vector $\{c^0_{ht}(s)\}_{h\in H}$ is statically efficient. Then, setting*

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \omega_{ht}(s) - \log \omega_{h0}(s)\right] + \frac{1-\eta}{\eta}\left[\log p_{ht}(s) - \log p_{h0}\right] \tag{2}$$

implies that $\boldsymbol{c}^0$ is a general equilibrium with those wedges, where $p_{ht}(s)$ is the consumption price and $\omega_{ht}(s) = \frac{p_{ht}(s)c^0_{ht}(s)}{\sum_{h'} p_{h't}(s)c^0_{h't}(s)}$ is the expenditure share of household h in period t and state s. This equilibrium is supported by some lump-sum transfers across households.

We do not need to specify the lump-sum transfers explicitly. For our purposes, all that matters is that there exist an equilibrium with the wedges in (2), with appropriate transfers, that can support $\boldsymbol{c}^0$ as an equilibrium allocation.[9]

Consider the relative wedge in a given date-state between households. Proposition 2 implies that

$$\log \frac{\mu_{ht}(s)}{\mu_{h't}(s)} = -\frac{1}{\eta}\left[\log \frac{\omega_{ht}(s)}{\omega_{h0}(s)} - \log \frac{\omega_{h't}(s)}{\omega_{h'0}(s)}\right] + \frac{1-\eta}{\eta}\left[\log \frac{p_{ht}(s)}{p_{h0}} - \log \frac{p_{h't}(s)}{p_{h'0}}\right].$$

[8]The wedges in Proposition 2 are distinct from the wedges in Berger et al. (2023). They consider preference shifters that, in a representative agent economy, replicate the path of aggregate outcomes (e.g. aggregate consumption, hours, etc.) from a heterogeneous agent New Keynesian model. They show that deviations from perfect risk-sharing map onto discount factor shocks in the representative agent model. They then consider the reduction in output volatility in the absence of these as-if discount factor shocks. In contrast, the wedges in Proposition 2 replicate a microeconomic, rather than just aggregate, allocation in a heterogeneous agent general equilibrium with wedges. We use these wedges to construct a Harberger-triangle formula for incomplete market models.

[9]The wedges in (2) are not the only ones that can decentralize $\boldsymbol{c}^0$. For example, if wedges $\mu_{ht}(s)$ are all raised by the same proportion for every household h in a given t and s, this can still decentralize $\boldsymbol{c}^0$. That is, Proposition 2 is a particular normalization of wedges that can decentralize the status-quo. In contrast, the relative prices $p_{ht}(s)/p_{h't}(s)$ in a given period and state across households are generically pinned down by $\boldsymbol{c}^0_{ht}(s)$ across households in that period and state. This is because production is statically efficient, and so relative prices within a state-date are pinned down by marginal costs of producing each consumption good efficiently (given that there are not intertemporal production linkages in the form of capital accumulation).

If allocations are dynamically efficient, so that wedges are all equal to one, then households whose consumption prices are relatively high, should be spending relatively more if $\eta < 1$. Indeed, setting wedges equal to one and rearranging yields the Backus and Smith (1993) condition for efficient risk-sharing. In our international application, we refer to $\log \mu_{ht}(s)$ as the Backus and Smith (1993) wedges.

There are two salient special cases: (1) if there is a common consumption good, then $p_{ht}(s) = p_{h't}(s)$, and the wedge depends only on fluctuations in consumption shares over time. In this case, efficient risk-sharing requires that all households maintain a constant share of aggregate consumption in every date and state. If one household's share of consumption rises, then it must be receiving an inefficient "subsidy" in that date and state. (2) If $\eta = 1$, then once again, the efficient allocation features constant consumption expenditure shares over time. This is related to the observation by Cole and Obstfeld (1991) that an economy with $\eta = 1$ and constant expenditure shares in equilibrium delivers efficient risk-sharing even if there is financial autarky.

Given these wedges, we can now apply a version Proposition 7 from Baqaee and Burstein (2025b) to this environment. To do so, denote the deviations of productivity shifters from some constant values by $\Delta \log z$. That is, for producer i at time t in state s,

$$\Delta \log z_{it}(s) = \log \frac{z_{it}(s)}{\bar{z}_i},$$

where $\bar{z}_i$ is some constant (over time and states) level of productivity for producer i.

The following proposition approximates misallocation losses in terms of Harberger deadweight loss triangles.

Proposition 3 (Harberger triangles with incomplete markets)**.** *Misallocation comparing the status-quo allocation $\boldsymbol{c}^0$ to the Pareto-frontier $\mathcal{C}(\boldsymbol{z})$ is approximately*

$$\Delta \log A \approx \frac{1}{2}\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{r}{(1+r)^{t+1}} \sum_{h \in H} \omega_h \log \mu_{ht}(s) \sum_{h' \in H} A_{hh'} [\log \mu_{h't}(s) - \log \bar{\mu}_h] \right],$$

where $\log \mu_{ht}(s)$ is given by Proposition 2, $\log \bar{\mu}_h = \mathbb{E}_0[\sum_{t=0}^{\infty} \frac{r}{(1+r)^{t+1}} \log \mu_{ht}(s) | h]$ is the expected discounted consumption wedge for household h, ω_h is the expenditure share of household h at any date or state, r is the riskfree rate at any date or state, and $A_{hh'}$ depends only on the static input-output matrix and elasticities of substitution (including η). The approximation error is order $\log \boldsymbol{\mu}^3$ and $\log \boldsymbol{\mu}^2 \Delta \log \boldsymbol{z}$.

The explicit formula for the $H \times H$ matrix A in terms of the input-output table and elasticities of substitution is given in the appendix. The key is that the matrix A does not depend on either the date or the state. This simplicity comes from the fact that the production possibility set within each date and state is not a function of past choices because there is no capital accumulation.[10]

Proposition 3 is a sufficient statistics formula: misallocation can be approximated conditional on knowledge of the (static) input-output table at some date, elasticities of substitution (include η), the discount factor β, and wedges, $\log \mu_{ht}(s)$, which are recoverable from Proposition 2. Importantly, one does not need know the process driving productivity shocks $\Delta \log \boldsymbol{z}$.

4 Risk-Sharing with Common Consumption Good

We now apply our exact and approximate characterizations, Propositions 1 and 3, to a salient special case. We focus our attention on the case where all households consume the same consumption good. This is the typical assumption in closed-economy settings, and making it allows us to provide sharper and more intuitive versions of Propositions 1 and 3.

The fact that there is only one consumption good in each date and state means all household's have the same preferences. This assumption nests economies that have a single consumption good in every period, like Bewley (1972) and Huggett (1993), but it also accommodates multi-sector versions of these models with input-output linkages, as long as every household's static consumption aggregator is the same.

We begin this section by considering the baseline case without either labor-leisure choice or capital accumulation, and we specialize the exact and approximate characterizations above to this setting.

4.1 Baseline without Capital Accumulation or Labor-Leisure Choice

We start by specializing our exact characterization of misallocation due to market incompleteness to this setting. We then contrast our measure with the popular behind-the-veil-of-ignorance social welfare function (sometimes called "the utilitarian" social welfare function). We end by providing a second-order approximation of misallocation stated in terms of some observable sufficient statistics.

[10] The derivation of Proposition 3 uses Proposition 7 from Baqaee and Burstein (2025b) and shows that the term $\sum_{h' \in H} A_{hh'} [\log \mu_{h't}(s) - \log \bar{\mu}_h]$ is the first-order change in $\log c_{ht}(s)$ due to wedges in the compensated equilibrium with wedges. There is one other subtlety relative to Proposition 7, which is that we assume productivity shifters are close to some constant baseline values through time — i.e. the approximation error is order $\Delta \log \boldsymbol{z} \log \mu^2$.

Exact characterization. Given the assumption that all households consume the same static bundle, we can simplify Proposition 1 further. To do so, recall that $CE(\boldsymbol{c}_h)$ is the certainty-equivalent of the consumption process $\boldsymbol{c}^h$.[11]

Proposition 4 (Misallocation in economies with common consumption good)**.** *Consider the special case where there is a common consumption good in every period and state. In this case, misallocation is*

$$\Delta \log A = \log \frac{CE(\boldsymbol{C}^0)}{\sum_h CE(\boldsymbol{c}_h^0)}. \tag{3}$$

where $C_t^0(s) = \sum_h c_{ht}^0(s)$ is the status-quo aggregate quantity of the consumption good in period t and state s.

In words, when households have common preferences, $\Delta \log A$ is the certainty equivalent of the status-quo aggregate consumption process (evaluated using households' common preferences) relative to the sum of the consumption equivalents of each households' consumption process.[12]

To derive Proposition 4, we use the observation that all consumption allocations on the Pareto frontier satisfy $c_{ht}(s) = \lambda_h C_t(s)$ for some household-specific λ_h with $\sum_h \lambda_h = 1$. This follows from homotheticity of preferences and the fact that aggregate consumption quantity $C_t(s)$ is efficient. Substituting this into (1), we get

$$\Delta \log A = \max_{\boldsymbol{c} \in \mathcal{C}(\boldsymbol{z})} \min_{h \in H} \{\tilde{u}_h(\boldsymbol{c}_h)\} = \max_{\boldsymbol{c} \in \mathcal{C}(\boldsymbol{z})} \min_{h \in H} \left\{ \frac{\lambda_h CE(\boldsymbol{C}^0)}{CE(\boldsymbol{c}_h^0)} \right\}.$$

We know that utility-maximizing choice above must satisfy $\frac{\lambda_h CE(\boldsymbol{C}^0)}{CE(\boldsymbol{c}_h^0)} = \frac{\lambda_{h'} CE(\boldsymbol{C}^0)}{CE(\boldsymbol{c}_{h'}^0)}$. Hence,

$$\lambda_h = \frac{CE(\boldsymbol{c}_h^0)}{\sum_{h'} CE(\boldsymbol{c}_{h'}^0)}.$$

Substituting this back in gives the desired result. This proposition is no longer true if either (1) there is preference heterogeneity, (2) labor-leisure choice, or (3) capital accumulation, as we discuss below.

By inspection, under these assumptions, $\Delta \log A$, also coincides with Kaldor-Hicks efficiency, defined as the ratio of aggregate income in first best relative to the sum of

[11]Using the functional form for utility and solving through, we can write $CE(\boldsymbol{c}_h) = \left(u(\boldsymbol{c}_h)(1-\beta)(1-\frac{1}{\eta})\right)^{\frac{\eta}{\eta-1}}$.

[12]If there are static production inefficiencies, then the numerator of (3) should be replaced by $CE(\boldsymbol{C}^*)$, where $\boldsymbol{C}^*$ is the aggregate consumption process in the undistorted equilibrium (which is unique, because all households have the same homothetic preferences).

compensating variations of the status-quo (the amount of money each household must be given under complete markets to ensure indifferent to their status quo allocation). As we discussed in Baqaee and Burstein (2025b), this equivalence between $\Delta \log A$ and Kaldor-Hicks efficiency breaks down in the presence of preference heterogeneity or non-homotheticity.

Proposition 4 shows that $\Delta \log A$ is related to the measures of inefficiency proposed by Benabou (2002) and Floden (2001). Relative to those papers, Proposition 4 allows for aggregate shocks and intermediate inputs. More generally, once we depart from the common preferences assumption or allow for consumption-leisure choice or capital accumulation, then $\Delta \log A$ no longer coincides with the metrics discussed in those papers.

Contrast to the veil-of-ignorance. A popular measure in the literature is the veil-of-ignorance social welfare function. When all households have common preferences, as we are assuming in this section, this measure is unambiguous to define.[13] Whereas our measure is designed to ignore inequality, a primary motivation for the veil-of-ignorance measure is to capture inequality-aversion using risk-preferences. Define the veil-of-ignorance consumption-equivalent of a consumption allocation $\boldsymbol{c}$ to be:

$$u(\mathbf{1}CE^{VOI}) = \sum_{h \in H} \frac{1}{H} u(\boldsymbol{c}_h^0).$$

In words, CE^{VOI} is the consumption-equivalent of a population-weighted lottery of the consumption allocation of each agent in the equilibrium. Using this as a social welfare function, we can calculate the difference between the value of the first-best allocation, according to this social welfare function, and the value of the status-quo allocation:

$$\Delta \log A^{VOI} = \log \frac{CE(\boldsymbol{C}^0/H)}{CE^{VOI}} = \log \frac{CE(\boldsymbol{C}^0)}{\left(\sum_h \left(CE(\boldsymbol{c}_h^0)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}, \tag{4}$$

where the numerator uses the fact that the first-best allocation with this social welfare function would split aggregate consumption uniformly across all agents. Comparing (3) to (4) clarifies some of the differences between $\Delta \log A$ and $\Delta \log A^{VOI}$. The veil-of-ignorance measure, $\Delta \log A^{VOI}$, uses risk-preferences to discipline inequality-aversion,

[13] See Eden (2020) for a detailed discussion of the veil-of-ignorance approach to quantifying social welfare, and how it must be adapted in the presence of heterogeneous preferences.

whereas $\Delta \log A$ does not feature this effect.[14,15]

The simplest way to see the difference between $\Delta \log A$ and $\Delta \log A^{VOI}$ is to consider a Pareto efficient, but unequal, consumption allocation. Suppose that in the status-quo, each household consumes a constant fraction λ_h of aggregate consumption: $c^0_{ht}(s) = \lambda_h C^0_t(s)$. In this case, the consumption-equivalent is a fraction λ_h of the consumption-equivalent for aggregate consumption: $CE(\boldsymbol{c}_h) = \lambda_h CE(\boldsymbol{C}^0)$. Using Proposition 4, it follows that misallocation is zero. However, as there is inequality in the status-quo (λ_h varies across households), $\Delta \log A^{VOI}$ is positive (unless $\eta = \infty$ and households are risk-neutral).

Approximate characterization of $\Delta \log A$. We now provide a second-order approximation of misallocation. This second-order approximation serves several purposes. First, it provides some useful intuition about how parameters affect misallocation. Second, and more importantly, it identifies some approximate sufficient statistics that can be taken to the data without assuming complete knowledge of the entire distribution of consumption allocations and productivity shifters.

The following proposition specializes Proposition 3 to the special case where there is a common per-period consumption good.

Proposition 5 (Harberger triangles with common consumption good)**.** *Consider the special case where there is a common consumption good in every period and state. Misallocation is approximately equal to*

$$\Delta \log A(\boldsymbol{c}^0, \mathcal{C}(z)) \approx -\frac{1}{2}\frac{1}{\eta}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\sum_h \omega_h\left(\log\omega_{ht}(s) - \log\omega_{h0}\right)\left(\log\omega_{ht}(s) - \log\bar{\omega}_h\right)\right],$$

where $\log \bar{\omega}_h = r\mathbb{E}_0\left[\sum_{t=0}^{\infty}(1+r)^{-(t+1)}\log\omega_{ht}(s')|h\right]$ *is the conditional expected discounted consumption share of household h. The approximation error is order* $\log\mu^3$ *and* $\Delta \log z \log \mu^2$*.*

[14]In other words, the veil-of-ignorance measures sets the Atkinson (1970) parameter for inequality-aversion equal to the coefficient of relative risk aversion.

[15]In this example, the veil of ignorance can also be thought of as a utilitarian social welfare function (sum of utilities) with a particular cardinalization of the utility function. However, with other cardinalizations of the same preferences, the sum of utilities will have different implications. This is because "the" utilitarian welfare function is not well-defined as it depends on how each utility function is cardinalized. For example, if instead of using the functional-form of u, defined above, we cardinalize the same preferences using the monotone nonlinear transformation $(u(1-1/\eta))^{\frac{\eta}{\eta-1}} = \left(\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t c_{ht}^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$, then the utilitarian social welfare function would have zero inequality aversion. We note that our measure of aggregate efficiency, $\Delta \log A$, does not depend on how utility functions are cardinalized.

This approximation is accurate as long as the wedges ($\log \mu$) and aggregate productivity shocks ($\Delta \log z$) are not too big. The intuition for the expression above is exactly the same as the traditional deadweight-loss triangle logic. The height of the triangle is measured by the wedge, $\log \mu_{ht}(s) = -\frac{1}{\eta}[\log \omega_{ht}(s) - \log \omega_{h0}]$ from Proposition 2. The smaller is the EIS, the larger is the implied wedge necessary to reach the same distorted quantity allocation. The base of the triangle is the gap between household h's share of consumption in state s and time t, $\omega_{ht}(s)$, and its expected share of consumption in net-present value terms. The area of the triangle divides the product of the base and the height, and the summation over t, h, and the expectation operator, sum over all deadweight loss triangles using net-present-value Domar weights.

An important difference between Proposition 5 and the classic deadweight loss triangle formulas, like the ones in Baqaee and Farhi (2020), is that here there is no representative agent. However, Proposition 1 shows that there is an as-if representative agent, whose utility values coincide with $\Delta \log A$. It is the allocation that this as-if representative agent selects on the Pareto frontier that is then used for the construction of the Harberger triangles. It is important to stress that in our context there is no single "first-best" allocation. That is, there is no implication that in the absence of wedges, *the* first-best allocation is the one that sets $\omega_{ht}(s) = \log \bar{\omega}_h$. In the absence of wedges, there are many different allocations that are all Pareto dominant to the status-quo, and our measure takes no stance on which one is socially desirable. Instead, because of Proposition 1, $\omega_{ht}(s) = \log \bar{\omega}_h$ is the allocation relative to which the necessary Harberger triangles must be computed if one wishes to recover an approximation of $\Delta \log A$.

4.2 Extension with Labor-Leisure Choice

We now briefly consider an extension with labor-leisure choice. Suppose households have preferences over consumption and leisure

$$u(\boldsymbol{c}_h, \boldsymbol{l}_h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v(c_{ht}(s), l_{ht}(s)).$$

Each household has a unit endowment of time, which they devote either to leisure, $l_{ht}(s)$, or to work, $1 - l_{ht}(s)$. For simplicity, in this extension, we abstract from input-output linkages and assume that labor can be directly converted into the common consumption good using a linear technology. Hence, the resource constraint for consumption at date t in state s is

$$\sum_h c_{ht}(s) = \sum_h z_{ht}(s) \left(1 - l_{ht}(s)\right), \tag{5}$$

where $z_{ht}(s)$ is the idiosyncratic productivity of household h in date t and state s.

The consumption possibility set $\mathcal{C}(z)$ now consists of all consumption and leisure processes that are consistent with the resource constraint above. This is because leisure is an additional consumption good in preferences that must be accounted for. However, unlike the consumption good, leisure is not tradeable across households. We use the same definition of aggregate efficiency as in Definition 1. In words, misallocation is measured by the maximum contraction of the consumption possibility set such that it is possible to keep every agent at least in different. The scalar $\Delta \log A$ measures how much of every consumption good, including leisure, in every state and date is left over after every agent has been made indifferent.[16] This is a measure of the total waste of resources caused by market incompleteness and frictions.

Following Definition 2 from Baqaee and Burstein (2025b), define the homothetized utility function $\tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h)$, implicity by the equation

$$u(\boldsymbol{c}_h^0, \boldsymbol{l}_h^0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v\left(\frac{c_{ht}(s)}{\tilde{u}}, \frac{l_{ht}(s)}{\tilde{u}}\right). \tag{6}$$

Note that when applied to the preferences without leisure, the definition of $\tilde{u}_h$ above coincides with the one in Proposition 1.

Proposition 6 (Misallocation with leisure choice)**.** *Proposition 1 holds with labor-leisure choice using the definition of $\tilde{u}_h$ in Equation* (6)*. Specifically, this means*

$$\Delta \log A = \max_{\boldsymbol{c},\boldsymbol{l}} \tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h),$$

subject to (5) *and $\tilde{u}_h = \tilde{u}_{h'}$ for every $h' \in H$.*

Proposition 6 is a restatement of Theorem 1 from Baqaee and Burstein (2025b). Proposition 6 gives a method for calculating aggregate efficiency by converting the problem into a standard utility maximization problem with a representative agent. The as-if representative agent's preferences are Leontief over $\tilde{u}_h$. We provide some examples of $\tilde{u}_h$ for popular functional forms below.

Example 1 (Homothetic preferences). If the intratemporal utility function, v, is homoge-

[16] Reducing leisure here does not mean we increase work — a proportional reduction in consumption and leisure could instead be achieved by reducing every household's time endowment by the same fraction.

neous of degree $1-1/\eta$, then $\tilde{u}_h$ has a simple explicit form:

$$\tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h) = \left[\frac{u(\boldsymbol{c}_h, \boldsymbol{l}_h)}{u(\boldsymbol{c}_h^0, \boldsymbol{l}_h^0)}\right]^{\frac{1}{1-1/\eta}}.$$

A salient example is when the within period utility function takes the form:

$$v(c_h, l_h) = \frac{1}{1-1/\eta}\left[c_h^{\gamma} l_h^{1-\gamma}\right]^{1-\frac{1}{\eta}}. \tag{7}$$

In many macro applications with labor-leisure choice, the within-period utility function, v, is not homogeneous of degree one in leisure and consumption. In this case, an explicit representation of $\tilde{u}_h$ may not be possible.

Example 2 (MaCurdy preferences). A very popular class of preferences uses

$$v(c_h, l_h) = \frac{1}{1-1/\eta} c_h^{1-\frac{1}{\eta}} + \phi_0 l_h^{1-1/\phi}. \tag{8}$$

In this case, $\tilde{u}$ is implicitly defined by the equation

$$\tilde{u}_h = \left(\frac{1}{u_h^0}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{c_{ht}(s)^{1-\frac{1}{\eta}}}{1-1/\eta} + \tilde{u}_h^{1/\phi-1/\eta}\phi_0 l_{ht}(s)^{1-1/\phi}\right)\right]\right)^{\frac{\eta}{\eta-1}}.$$

We can solve for $\tilde{u}_h$ explicitly only if $\phi = \eta$.

Given $\tilde{u}_h$, solving for $\Delta \log A$ is a matter of solving a standard representative agent utility maximization problem.

Comparison with Benabou (2002) measure of aggregate efficiency. As mentioned earlier, our definition of aggregate efficiency is different to a frequently used alternative in this literature, like Benabou (2002), Floden (2001) and Boar and Midrigan (2022), if households choose between labor and leisure. To compare two allocations, the literature following Benabou (2002) defines the consumption-equivalent of a consumption process $\boldsymbol{c}_h$ and a leisure process $\boldsymbol{l}_h$ to be the function $CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$ that solves

$$u(\mathbf{1}CE, \mathbf{1}\bar{l}) = u(\boldsymbol{c}_h, \boldsymbol{l}_h),$$

where $\bar{l}$ is some fixed level of leisure (e.g. average leisure). This efficiency of allocation is defined by $\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$. Unlike our measure, this is a social welfare function (since it is a monotone increasing function of the underlying preferences of the households).

To understand how this measure differs from $\Delta \log A$ consider the value it assigns to different allocations on the Pareto frontier in the following simple example. Suppose that preferences take the functional form in Example 1. In this case, this measure of efficiency can be written as

$$\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \left(u(\boldsymbol{c}_h^1, \boldsymbol{l}_h^1) \right)^{\frac{\eta}{(\eta-1)\gamma}}.$$

Consider the simple case where labor productivity is equal to one in every date and state for every agent. Then we can show that for every allocation $(\boldsymbol{c}, \boldsymbol{l})$ on the Pareto frontier, there exist a set of numbers on the unit simplex, $\{\boldsymbol{\alpha} \geq 0 : \sum_h \alpha_h = 1\}$, such that

$$\sum_h CE_h(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \alpha_h^{\frac{1}{\gamma}}. \tag{9}$$

In this expression, $\boldsymbol{\alpha}$ are coordinates of the Pareto frontier — i.e. they can be interpreted like Pareto weights — the higher is α_h for household h, the higher is the utility of that agent. Clearly, unless $\gamma = 1$, and there is no labor-leisure choice, (9) assigns different values to different points on the Pareto-frontier. Surprisingly, this measure assigns (weakly) higher values to more unequal Pareto weights since $\gamma \leq 1$. Indeed, by continuity, this shows that there are Pareto-inefficient allocations that receive a higher value according to this measure than alternative Pareto efficient allocations with less inequality. Therefore, once we have labor-leisure choice, this measure is not neutral with respect to pure redistributions (and indeed, prefers inequality). In contrast, our measure of aggregate efficiency, assigns a single value to the whole Pareto frontier by construction.

4.3 Extension with Capital Accumulation

We now consider an extension with capital accumulation along the lines of Aiyagari (1994). For simplicity, in this extension, we abstract from input-output linkages and assume that there is an aggregate output good in each period and labor is inelastically supplied. Aggregate output in each period and state is

$$y_t(s) = z_t(s) k_t(s)^{\alpha}, \tag{10}$$

where we imposed the requirement that the aggregate endowment of labor is equal to one. Capital accumulation satisfies

$$k_{t+1}(s) = (1-\delta)k_t(s) + x_t(s), \tag{11}$$

where $x_t(s)$ is investment. Denote the initial capital stock by k_0. The aggregate resource constraint for output is

$$y_t(s) = c_t(s) + x_t(s) = \sum_{h\in H} c_{ht}(s) + x_t(s). \tag{12}$$

Proposition 1 continues to hold with capital accumulation. Specifically, this means

$$\Delta \log A = \max_{\boldsymbol{c}} \left(\frac{u(\boldsymbol{c}_h)}{u(\boldsymbol{c}_h^0)} \right)^{\frac{\eta}{\eta-1}},$$

subject to (10), (11), (12), and $\frac{u(\boldsymbol{c}_h)}{u(\boldsymbol{c}_h^0)} = \frac{u(\boldsymbol{c}_{h'})}{u(\boldsymbol{c}_{h'}^0)}$ for every $h' \in H$ and some initial capital stock k_0. This implies the following.

Proposition 7 (Misallocation with capital accumulation)**.** *Let $C_t^*(s)$ be the optimal (aggregate) consumption choice of a representative agent in the neoclassical growth model with initial capital stock k_0. Then*

$$\Delta \log A = \log \frac{CE(\boldsymbol{C}^*)}{\sum_h CE(\boldsymbol{c}_h^0)}. \tag{13}$$

That is, computing $\Delta \log A$ requires knowing the consumption-equivalent welfare of each agent in the status-quo as well as the certainty-equivalent of a representative agent given the initial aggregate capital stock. Compared to Proposition 4, this means calculating aggregate efficiency, $\Delta \log A$, above has one more step: solving for the transition dynamics in a standard neoclassical growth model given k_0.

Comparison with Benabou (2002) measure of aggregate efficiency. As with labor-leisure choice, once we account for capital accumulation, our measure of efficiency differs from the approach in Benabou (2002). That measure holds the aggregate capital stock fixed when calculating consumption equivalents for each household, and values allocations by summing up consumption equivalents. However, when there is idiosyncratic risk, households collectively accumulate too many assets relative to the Pareto frontier. Our measure accounts for this, hence, the distance to the Pareto efficient frontier is larger according to $\Delta \log A$ than is implied by the sum of consumption-equivalents in Benabou (2002).

5 Quantitative Applications

In this section, we apply the results in the previous section to some examples. In Section 5.1, we quantify misallocation exactly in calibrated versions of the Bewley (1972)-Aiyagari (1994) model and test the performance of our second-order approximation in finite sample and with non-infinitesimal shocks. In Section 5.2, we apply our second-order approximation sufficient statistics formula to household consumption panel survey data for the United States to estimate misallocation from the absence of complete financial markets. In Section 5.3, we use our second-order approximation to quantify misallocation in the global economy due to incomplete international financial markets.

5.1 Misallocation in Bewley (1972)-Style Models

In this application, we quantify misallocation in a standard calibration of Bewley (1972). We use this example to show how the extent misallocation, relative to the status-quo steady-state equilibrium, changes as a function of parameters like idiosyncratic risk, borrowing constraints, and public debt. We also use this example to test the performance of our second-order approximation and its finite sample properties. We end this application by considering an extension with capital accumulation as in Aiyagari (1994).

Model. There is a unit mass of households, indexed by $h \in [0,1]$, with preferences

$$u_h(\boldsymbol{c}_h) = \frac{1}{1-1/\eta}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t c_{ht}(s)^{1-\frac{1}{\eta}}\right].$$

that maximize utility subject to a per-period budget constraint

$$c_{ht} + a_{ht+1} = (1-\tau)e_{ht} + (1+r_t)a_{ht},$$

where a_{ht} is the quantity of a risk free bond held by h, e_{ht} is labor income, and τ is the tax rate. Each household faces a borrowing constraint

$$a \geq -\underline{a}.$$

Labor is converted into the single consumption good one-for-one and the real wage is equal to one. Income evolves according to

$$\log e_{ht} = \rho \log e_{ht-1} + \sigma\epsilon_{ht},$$

where ϵ_{ht} is an idiosyncratic Gaussian disturbance. The government has issued B risk free bonds and runs a balanced budget every period using labor income taxes, so that

$$rB = \tau.$$

Market clearing condition for goods and bonds is

$$\int_0^1 c_{ht} dh = 1, \qquad \text{and} \qquad \int_0^1 a_{ht} dh = B.$$

Calibration. We use a quarterly calibration. We set quarterly persistence of log income to be $\rho = 0.975$ with standard deviation 0.1555 to match estimates of the quarterly persistence and the cross-sectional standard deviation of the persistent component of log income in the United States.[17] We set the borrowing limit to be -5, so households can borrow at most 5 times their quarterly income. We set the annual risk-free $r = 3\%$ (so the quarterly rate is 0.75%). We set the EIS $\eta = 0.5$. Finally, we set $B = 5.6$ — so that total bonds outstanding relative to quarterly output is 560% (or 140% of annual GDP).

Results. Figure 1 plots the extent of misallocation, calculated using Proposition 4, and compares it to the second-order approximation from Proposition 5, using the steady-state invariant distribution for the status-quo allocation. The approximation performs well and, as expected, becomes exact as $\sigma \to 0$. The benchmark values are indicated by the dashed black line, where misallocation is approximately 0.22. This means that if agents perfectly insure each other and everyone is kept indifferent to their status-quo allocation, then there is 22 log points (or 24% percentage points) of output left over to be split across agents as desired. For comparison, we also compare the status-quo allocation to the first-best allocation under the veil-of-ignorance criteria, using equation (4). The veil-of-ignorance measure, which penalizes inequality across agents analogously to uncertainty for each agent, assigns roughly double the losses to the status-quo. That is, behind the veil, households would be prepared to give up 38 log points of aggregate consumption if they could equalize consumption across dates, states, and the cross-sectional population.

Figure 2 plots the quality of the second-order approximation against the exact misallocation losses as the sample length used in the approximation increases. The second-order approximation stabilizes after about 100 quarters (25 years), but suffers from some small sample bias when the number quarters is significantly shorter than that. The second-

[17] We target a cross-sectional standard deviation of log income equal to 0.7, which means that the standard deviation to the innovations must be $\sigma = 0.7 \times \sqrt{1 - 0.975^2}$.

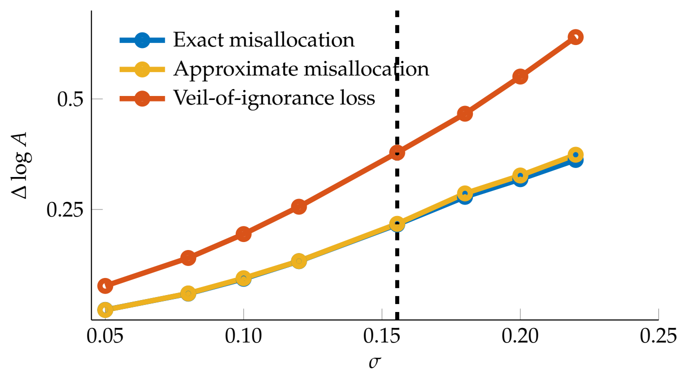


Figure 1: Losses as a function of idiosyncratic income risk. Dashed line is benchmark.

order approximation systematically underestimates the extent of misallocation because the Harberger triangles in the first few periods are, by construction, equal to zero.[18]

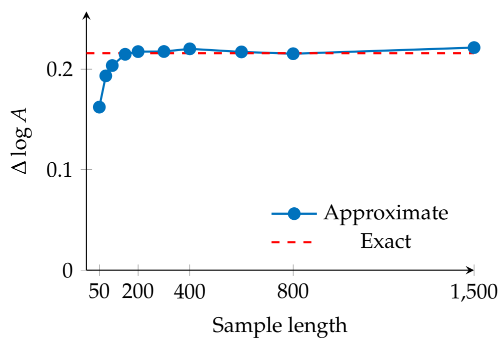
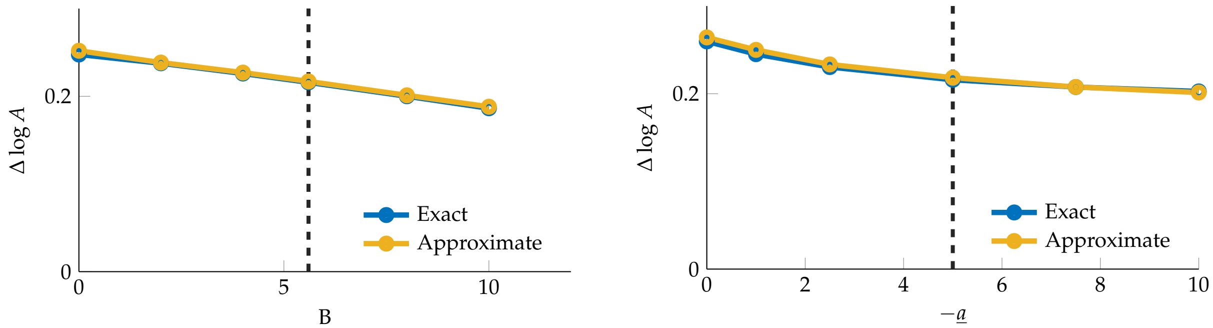


Figure 2: Quality of approximation for benchmark calibration

Figure 3 plots misallocation, relative to the status-quo in the invariant distribution, as a function of the aggregate supply of bonds and the borrowing limit. In both cases, misallocation falls mildly as the borrowing limit is increased and as the supply of bonds rises. The approximation continues to perform well. It is important to note that, as we change parameters, the invariant distribution changes — hence, in plotting these curves, we are not holding fixed the status-quo allocation. So, for example, if misallocation falls from 0.21 at benchmark to 0.19 when aggregate bond supply is doubled, this does not imply that aggregate efficiency rises by $0.22 - 0.19 \approx 0.03$ when bond supply is doubled. To answer this question, using Definition 1, we would have to specify the distributive tools available to society, which would give rise to a consumption possibility frontier $\mathcal{C}(B)$, hold fixed the status-quo allocation at $B = 5.6$, and solve the problem in Proposition 1.

[18] The second-order approximation is much less sensitive to the number of households in the sample.



(a) Bond supply

(b) Borrowing limit

Figure 3: Misallocation as a function of parameters (dashed line is benchmark).

Extension with labor-leisure choice. We use the results in Section 4.2 to extend the model to include labor-leisure. We assume preferences take the form in (8). We calibrate the EIS and the Frisch elasticity of labor supply to equal $\eta = \phi = 0.5$, and set ϕ_0 so that leisure is, on average, equal to 40% of the time endowment. We re-calibrate the productivity process so that the standard deviation and persistence of the equilibrium income process with endogenous leisure coincides with that in the model with no leisure. The remaining parameters the same as in the benchmark calibration. The losses are shown in Figure 4 as a function of idiosyncratic risk σ. Even as σ goes to zero, the losses are non-zero since there is a tax on labor. However, this effect is small because the baseline tax rate is small. Misallocation for the baseline parameters is 15 log points, which is smaller than in the model without leisure because goods consumption only accounts for a fraction of total spending on consumptions and leisure.

As before, we also compare the losses to those implied by a money-metric veil-of-ignorance measure.[19] As expected, the veil-of-ignorance measure assigns a bigger degree of misallocation compared to first-best as compared to our measure.

[19] Concretely, the veil-of-ignorance loss is

$$\Delta \log \left(\frac{\sum_h u_h^1}{\sum_h u_h^0} \right)^{\frac{\eta}{\eta - 1}},$$

where u_h^1 and u_h^0 are the first-best (behind the veil) and status-quo intertemporal utilities of agent h. In words, this is the (log change) in wealth required to go from the indifference curve $\sum_h u_h^0$ to $\sum_h u_h^1$. Since preferences are homothetic, this is also the proportional increase in a given commodity bundle (consumption and leisure bundle) required to go from one indifference curve to the other as in Lucas (1987). In this sense, its magnitude is comparable to $\Delta \log A$, which is also stated in terms of proportional changes in the set of feasible commodity bundles.

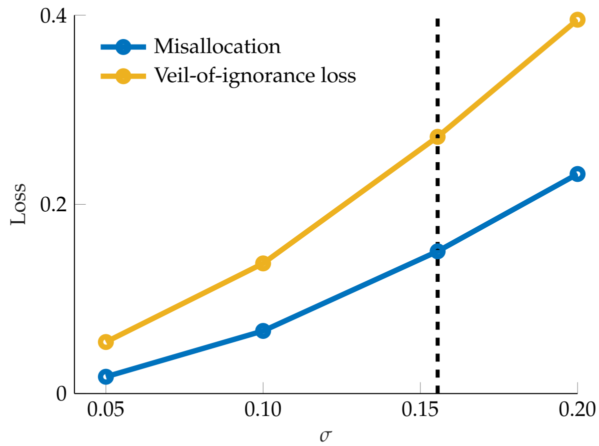


Figure 4: Losses with labor-leisure choice as a function of idiosyncratic income risk. Dashed line is σ in original calibration.

Extension with capital. With endogenous capital accumulation, incomplete risk-sharing results in excessive saving relative to first best. This means that the steady-state status-quo allocation has an inefficiently high stock of capital (the steady-state capital stock in every allocation on the Pareto frontier is lower than the steady-state capital stock in status-quo). To study the importance of this effect, we alter the baseline model along the lines of Section 4.3. Below, we describe the equilibrium that determines the status-quo.

Household preferences are the same as before, but the per-period budget constraint is now

$$c_{ht} + x_{ht} = e_{ht} + R_t k_{ht},$$

where x_{ht} is investment by household h and R_t is the rental price of capital. Each household faces a borrowing constraint, so $k_{ht} \geq 0$. The labor income process is the same as before. Each household's capital stock follows

$$k_{ht+1} = (1-\delta)k_{ht} + x_{ht}$$

The aggregate resource constraints are as in (10)-(12). Aggregate output is produced by a perfectly competitive representative firm that hires labor and capital on competitive spot markets. The rental rate of capital clears the capital market: $\int_0^1 k_{ht} dh = k_t$.

We calibrate capital's share of GDP to be $\alpha = 0.35$ and calibrate δ to match a capital output ratio of 14. We keep ρ, σ, and β the same as in the benchmark Bewley calibration, which means that the annual interest rate is 4.4% in the benchmark calibration.

We calculate distance to the frontier, $\Delta \log A$, using (13). The results are plotted in Figure 5 as a function of idiosyncratic risk. Misallocation at the benchmark values is 15.7

log points. This number should not be compared to the one in Figure 1 since we did not recalibrate the model to keep the volatility of consumption the same across the two models.

As is well-known, in this Aiyagari (1994) environment, households overinvest in capital relative to the first-best allocation. When financial markets are complete, households dissave since they no longer have a precautionary motive. This means that the long-run steady-state capital stock is lower at the Pareto frontier.

To quantify the importance of overinvestment for misallocation, we compare the benchmark distance to the frontier with one where we impose that the capital stock remains constant — that is, we use equation (3) rather than (13). As expected, the distance to the frontier holding the capital stock constant is smaller than the distance to the frontier allowing the capital stock to adjust — and the gap grows as idiosyncratic risk, and the strength of the precautionary motive, rise. At the benchmark values however, this effect is relatively mild, raising misallocation losses from 14.5 log points to 15.7 log points.

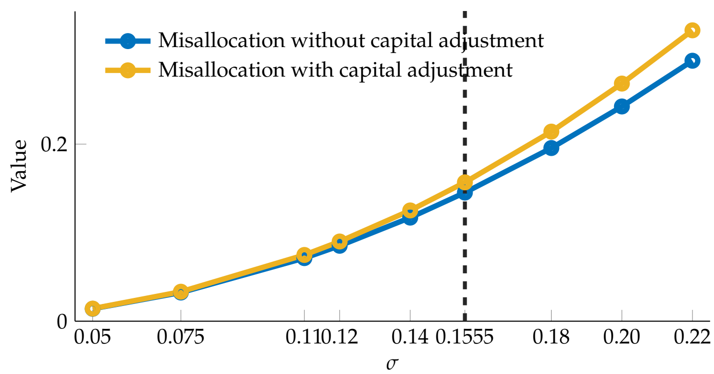


Figure 5: Losses as a function of idiosyncratic income risk. Dashed line is σ in original calibration.

5.2 Misallocation in PSID

In this application, we quantify misallocation from the lack of complete domestic insurance markets in the United States. We study how much consumption (in every date and state) is left over if domestic insurance markets work perfectly and every household is kept indifferent relative to their status-quo allocation. The larger is this number, the greater is the extent of misallocation from incomplete risk-sharing in the status-quo. Recall that the status-quo allocation is not consumption in the first period, but it is the state- and date-contingent equilibrium consumption processes for each household starting in the first period.

Approach. We use Proposition 5 to examine the extent of misallocation. In particular, we assume the data arise from an economy that meets the assumptions laid out in Section 4 — that is, every household has the same static consumption aggregator, and production is efficient in a static sense, but consumption allocations may be Pareto inefficient over time or across different states of nature.[20]

The key benefit of using the second-order approximation in Proposition 5, instead of specifying a fully detailed structural model and applying the exact outcome in Proposition 1, is that it imposes far lower informational requirements. Proposition 5 can be applied without needing to specify any particular stochastic process for either the wedges or the productivity shifts.

Description of data. We use the Panel Study of Income Dynamics (PSID) which is a longitudinal panel survey of American households. We use a balanced panel of households from 1999 to 2021 with $2,096$ households. We use household consumption expenditures across six consumption categories collected once every two years. These categories are food (at home and away), child care, healthcare, education, transportation, and housing. We leave out other expenditure categories (like clothing and electronics) which are not collected in every wave. Housing expenditures do not measure owner-occupied rental value for home owners, so we use the methodology in Baqaee et al. (2024). That is, we regress rent on observables for non-owners, and then use the estimates to predict rents for home-owners.

Mapping data to terms in Proposition 5. To apply Proposition 5, specify the EIS $\eta = 0.5$ and the risk-free rate $r = 0.03$. We use household h's share of total consumption expenditures in 1999 to calibrate ω_h.[21] We perform sensitivity analysis with respect to these choices when we present our results.

To estimate $\log \bar{\omega}_h = r\mathbb{E}_0\left[\sum_{t=0}^{\infty}(1+r)^{-(t+1)} \log \omega_{ht}(s^t)|h\right]$, the expected discounted consumption share of household h, we run a regression of household h's consumption share in period t on its initial consumption share (in 1999), and a vector of household-

[20]In particular, this means that we abstract from labor-leisure choice at the individual, and net capital accumulation at the aggregate level.

[21]We experimented with using contemporaneous shares ω_{ht} every period instead of freezing them in 1999 and the results are very similar.

level covariates.[22]

$$\log \omega_{ht} = \gamma_t \log(\omega_{h0}) + \boldsymbol{\psi_t X_{h0}} + \epsilon_{ht}. \tag{14}$$

The estimated regression equation is the best linear predictor of household h's consumption share at t conditional on observables at date 0 (1999). We use this linear predictor in place of the conditional expectation in the formulas (i.e. in place of the best nonlinear predictor). Hence, to calculate $\log \bar{\omega}_h = r\mathbb{E}_0 \left[\sum_{t=0}^{\infty}(1+r)^{-(t+1)} \log \omega_{ht}(s')|h\right]$ we predict household h's consumption share at each horizon t and sum them up discounted using $r = 0.03$. Because we do not observe all terms in the infinite sums for present value calculations, we treat unobserved terms as equal to the last observed value. As shown in Figure 2, this imputation does reasonably well in small sample settings.

Results. Figure 6 plots estimated misallocation losses in the PSID as a function of the annual interest rate. Our benchmark interest rate of 3% implies that misallocation is 23 log points (roughly 20%). This is in the same ballpark as the calibrated Bewley (1972) model in Section 5.1. Estimated misallocation decreases as the risk-free rate, or degree of impatience, rises. This is because deadweight loss triangles in the future are more heavily discounted. In the limit, as $r \to \infty$, households are infinitely impatient, there is no possibility to share risk, and misallocation is zero.

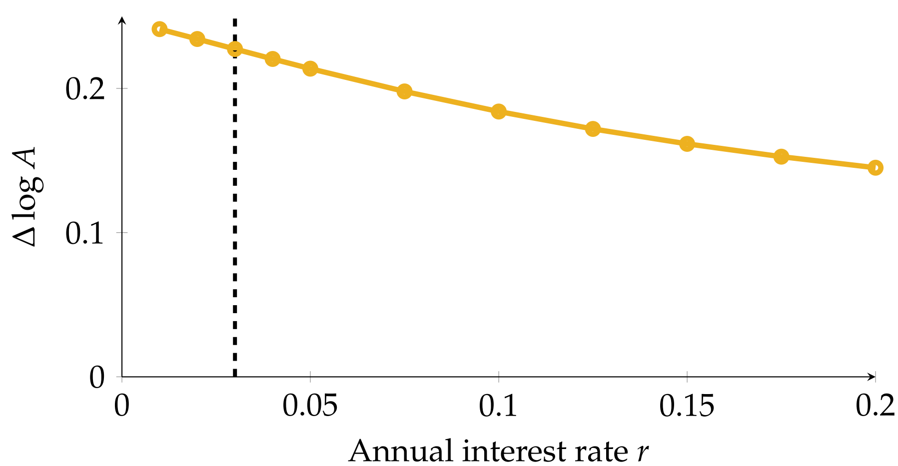


Figure 6: Estimated misallocation in PSID. Dashed line is benchmark.

Recall, from the discussion following Proposition 5, that the base of each deadweight loss triangle is the gap between household h's share of consumption in state s and time

[22]The covariates are household wealth without home equity, state of residence, household size, home ownership status (0 or 1), household head's age, race, ethnicity, and college degree status, business assets of the head and spouse, household head's labor income, and spouse's labor income.

t, $\omega_{ht}(s)$, and its expected share of consumption in net-present value terms. To estimate this conditional expectation for each household, we rely on the regression in (14).

Table 1 shows how estimates of misallocation change as we drop covariates from this regression. As the regression becomes less informative, estimated misallocation rises. Intuitively, this is because with a less informative regression, we attribute some systematic differences between households to lack of risk-sharing rather than to differences in observable household characteristics. In other words, once the regression model stops controlling for certain variables, previously explained variation in consumption shares now appears as evidence of incomplete insurance, artificially inflating the measured misallocation. This upward bias in estimated misallocation can be large: including no additional controls besides initial consumption, causes the estimate to rise from 0.23 to 0.28.

Eliminated variable	**Estimated misallocation**
None (Baseline)	0.227
Spouse labor income	0.239
Household head labor income	0.240
Business assets (household & spouse)	0.239
Household head college degree	0.247
Household head race and ethnicity	0.252
Household head age	0.260
Renter status	0.262
Household size	0.267
State of residence	0.279
Wealth	0.279

Table 1: Estimated misallocation under the baseline calibration with annual interest rate $r = 0.03$. The top row includes all covariates; each subsequent row eliminates one additional covariate (e.g., the third row excludes both spouse and head labor income).

5.3 Misallocation in Open-Economy Production Networks

In our final application, we quantify misallocation from the lack of complete international financial markets relative to the status-quo. More precisely, we calculate how much of every consumption good (in the world) is left over if financial markets are completed and agents in every country are kept indifferent relative to the status-quo. The larger is this number, the greater is the extent of misallocation from incomplete risk-sharing.

We use Proposition 3 to study the extent of misallocation. Specifically, we assume that the data is generated by an economy satisfying the assumptions in Section 2 — allocations are efficient from a static perspective, but potentially inefficient over time and states of nature. The advantage of using the second-order approximation in Proposition 3, versus writing a fully-specified structural model and applying the exact result in Proposition 1, is that the informational requirements are much weaker. Specifically, we can apply Proposition 3 without taking a stance on the stochastic process driving either the wedges or the productivity shifters (and indeed, there may be productivity changes that we did not explicitly model, like changes in iceberg costs at the industry-country pair level, and they would not alter the validity of the second approximation, as long as these shocks are small).

Structural model. Following standard practice in the international macro literature, we assume that each country has a representative agent, but in contrast to the previouse quantitative applications, we allow each country to have a different static consumption aggregator.

We specialize the technologies introduced in Section 2 as follows. There are H countries (households), S industries in each country, and one primary factor endowment per country (i.e. equipped labor). The static preferences of household h are an h-specific CES aggregator across different industries with elasticity of substitution θ_S. Consumption by h from industry i is an h, i-specific Armington CES aggregator over different origin countries, with elasticity of substitution θ_T.

The production function of industry i in country h is a CES aggregator, with elasticity θ_Y, of the local primary factor and an h, i-specific bundle of intermediate inputs from other industries. This intermediate bundle is also an h, i-specific CES aggregator across different industries with elasticity θ_S. The industry-j input used by producers in country h is an h, j-specific Armington aggregator with elasticity θ_T across different origin countries.[23]

To build some intuition for the upcoming quantitative results, we present a simple symmetric two country example of this model and apply Proposition 3.

Example 3 (Symmetric country example). Consider two symmetric countries, $h \in \{1, 2\}$, and suppose that each country produces one good using a linear technology from the local factor endowment (i.e. there is one industry in each country and no intermediate inputs). Let α denote the import share in both countries at the point we calibrate to.

Then given a path of consumption expenditure shares and prices in each country, de-

[23]Intuitively, this means we assume the same country-composition of the intermediate input bundle by industry. For example, mining & quarrying and the manufacture of basic metals in Australia, have the same expenditure shares on rubber and plastic products from China relative to India.

fine

$$\log \tau_t(s) = \eta \log \frac{\mu_{1t}(s)}{\mu_{2t}(s)} = \log \frac{(\omega_{1t}(s)/\omega_{10}) \,/\, (\omega_{2t}(s)/\omega_{20})}{(p_{1t}(s)/p_{10})^{1-\eta} \,/\, (p_{2t}(s)/p_{20})^{1-\eta}}$$

to be the log ratio of the wedge for country 1 relative to country 2 multiplied by the EIS. Then, applying Proposition 3 and rearranging yields

$$\Delta \log A \approx \left[\frac{\frac{1}{2}(1-\alpha)\alpha}{\eta 4\alpha(1-\alpha)\left[1-\frac{\eta}{\theta_T}\right]+\frac{\eta^2}{\theta_T}}\right] \sum_{t=0}^{\infty} \frac{r}{(1+r)^{t+1}} \mathbb{E}_0 \left[\log \tau_t(s) \left(\sum_{t'=0}^{\infty} \frac{r}{(1+r)^{t'+1}} \log \frac{\tau_t(s)}{\tau_{t'}(s)}\right)\right].$$

This example illustrates several important lessons. First, conditional on a given set of observed Backus-Smith wedges, misallocation goes to zero if either the import share of consumption, α, approaches zero or one. Intuitively, if the two countries are consuming completely unrelated goods, then there is no insurance possible between them. Second, as the EIS, η, tends to zero misallocation goes to infinity. In this case, consumption fluctuations are very costly, and so to justify the fact that consumption fluctuates in the data, we require very large consumption wedges. Finally, as the Armington elasticity, θ_T, rises to infinity misallocation rises because there is more scope for international risk sharing when foreign and domestic goods are more substitutable. These three lessons are borne out in our quantitative model below.[24]

Calibration. We calibrate the expenditure shares in Proposition 3 using the 2014 release of the world input-output database (Timmer et al., 2015).[25] That is, we calibrate the consumption share of each country ω_h using that country's share of total consumption, investment, and government expenditures. We calibrate the input-output matrix required for $A_{hh'}$ using the transaction flows in 2014. We calibrate each industry-country's expenditures on intermediate inputs from other industries and value-added from the WIOD. The model has 32 countries and 54 industries covering all sectors of the economy, including primary sectors, manufacturing, and services.[26]

[24] Note that, as we vary the elasticities θ_T and η, we keep the Backus-Smith wedge process, $\log \mu_{ht}(s)$, and import share, α, unchanged. If, instead, one were to hold some other primitives constant, then the effects of changing these parameters may be very different if the implied equilibrium Backus-Smith wedge process or import share change. Our approach is to keep the data constant as we vary parameters.

[25] To calibrate Proposition 3, we take advantage of the fact that, since allocations are statically efficient by assumption, the observed relative prices of goods within each period and state, not including the consumption wedges, in the decentralized equilibrium with wedges are equal to the relative marginal costs of production. Hence, if, in the data, relative prices within periods and states reflect marginal costs, we can calibrate the expenditures shares in the model directly to those in the data.

[26] We drop the activities of private households as employers industry and the activities of extraterritorial organizations and bodies industry from the sample. The list of countries is Australia, Austria, Belgium,

To measure wedges, we apply Proposition 2, which states that

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \omega_{ht}(s) - \log \omega_{h0}(s)\right] + \frac{1-\eta}{\eta}\left[\log p_{ht}(s) - \log p_{h0}\right].$$

These are the traditional Backus and Smith (1993) wedges. We apply this formula at annual frequency from 1970 to 2019 using the nominal consumption and CPI-based real exchange rates from the Global Macro Database from Müller et al. (2025). This means that we treat the equilibrium starting in 1970 as part of the (date- and state-contingent) status-quo allocation. Hence, we treat the observed path of wedges as one realization (i.e. sample path) of the wedges from the decentralized equilibrium with wedges in status-quo. The wedges are different to zero if changes in log relative consumption and real exchange rates between countries do not comove perfectly. For example, in our data, the correlation between annual changes in real exchange rates and real consumption between the US and each country is 0.17 for the median country, whereas perfect risk sharing implies that this correlation should be -1. We quantify the extent of misallocation that results from these wedges, abstracting from other possible distortions in the economy.

We assume an annual riskfree rate $r = 0.05$. We set the EIS $\eta = 0.5$, and the Armington trade elasticity $\theta_T = 2$. We assume the other elasticities of substitution, θ_S and θ_Y, are equal to one. We vary these parameters in sensitivity analyses.

To estimate the time-zero expectations in Proposition 3, we treat the wedges from 1970 to 2019 as one sample path from the distribution generating the data. Since we only have one realization of the sample path, we estimate the expectation using this single observation. Because we do not observe all terms in the infinite sums for present value calculations, we treat unobserved terms as equal to an average of the observations in the last five years of the data (2015 to 2019).[27]

Results. Misallocation in our baseline calibration is 5.2% — that is, with complete insurance markets, every country can be made indifferent to the status-quo allocation with 5.2% of every good left over. Recall that the status-quo allocation here is the date- and state-contingent consumption processes in the observed equilibrium. The extent of misallocation depends strongly on whether countries with rapid growth rates are included in the sample. For example, if we exclude just China, then the extent of misallocation falls to 1.9% instead. If we drop China, India, Korea, and Indonesia as well, misallocation falls to

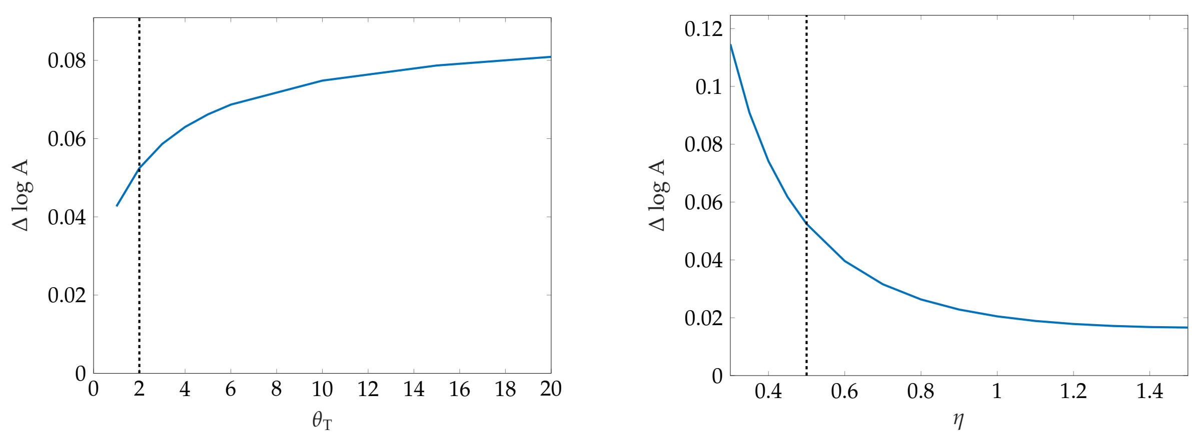
Brazil, Canada, Switzerland, China, Cyprus, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Hungary, Indonesia, India, Ireland, Italy, Japan, South Korea, Luxembourg, Mexico, Malta, Netherlands, Norway, Poland, Portugal, Sweden, Turkey, and the US.

[27] Results are very similar if we set unobserved terms equal to the last observed year of these terms.

only 1.0%. After this, the results are quite stable to dropping more countries. This shows that if we include large countries with very different growth rates in the sample, then the extent of misallocation from lack of international financial markets becomes larger. In the rest of this section, we report results including all 32 countries.

We experimented with varying the start date, for example, if we start in 1980 instead of 1970, then misallocation is slightly larger at 6.3%. If we start in 1993, then we can increase the number of countries in the sample by 10 by including additional countries that belonged to the Eastern Bloc. This raises misallocation to 6.6%.

We also vary the WIOD release we calibrate to. If we use an earlier release date, say 2006 instead of 2014, then misallocation is smaller, around 3.6% instead. This is because the world economy is less open in 2006 compared to 2014, so there is less scope for international risk-sharing, as discussed in Example 3.



(a) Armington elasticity (b) EIS

Figure 7: Misallocation varying parameters. Dashed line is benchmark value.

Figure 7 shows how our estimates of misallocation change as a function of the Armington trade elasticity and the EIS. As expected from Example 3, misallocation is larger the higher is the Armingon elasticity, since more substitutablity between domestic and foreign varieties facilitate more risk sharing; and misallocation is larger the lower is the EIS, since observed fluctuations in consumption are most costly for lower values of the EIS. Whereas our estimates for misallocation are fairly insensitive to the Armington elasticity, within the range the literature considers empirically plausible (e.g. from 1 to 5). However, our estimates are sensitive to lower values of the EIS. For example, if the EIS is 0.25, misallocation is around 10% — and these losses will go to infinity as η approaches zero. We do not present graphs for how estimated misallocation varies as a function of the elasticity of substitution between industries, θ_S, or between intermediates and value-added, θ_Y. Estimated losses are slightly increasing in these elasticities.

6 Conclusion

We characterize misallocation costs of financial market incompleteness for households in both open and closed economies. We find that misallocation costs due to market incompleteness are substantial within countries, especially if the elasticity of substitution across time and states of nature is low. Misallocation losses from imperfect consumption-smooth across countries, assuming a representative agent in each country, are much smaller. This is particularly true among developed economies, who do not have very different growth rates, and if trade elasticities are relatively low, so that imports are poor substitutes for domestic goods. A promising area for future research is to extend our characterizations to study misallocation relative to the constrained efficient Pareto frontier, accounting for the imperfections of policy.

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Appendix A Proofs

The results in the paper build on Theorem 1 and Theorem 2 from Baqaee and Burstein (2025b). To apply these results we recall the following definitions from that paper.

Define the fictitious Hicksian representative agent as follows.

Definition 3. The *Hicksian representative agent* is an agent whose preferences are represented by

$$U(\boldsymbol{c}) = \min_h \{\tilde{u}_h(\boldsymbol{c}_h)\},$$

where $\tilde{u}_h(\boldsymbol{c}_h) = \left[u_h(\boldsymbol{c}_h)/u_h(\boldsymbol{c}_h^0)\right]^{\frac{\eta}{\eta-1}}$.

Define the compensated equilibrium as follows.

Definition 4 (Compensated Equilibrium)**.** A *compensated equilibrium* is the general equilibrium of an economy with the same technologies, resource constraints, and wedges as the original economy but where there is a representative agent with preferences as in Definition 3. For any equilibrium variable $X(t)$, denote the same variable in the compensated equilibrium by $X^{\text{comp}}(t)$.

Theorem 2 from Baqaee and Burstein (2025b) implies that aggregate efficiency can be calculated via the utility of the Hicksian representative agent in the compensated equilibrium.

Proof of Proposition 2. We now derive equation (2). Consider first a decentralized Arrow-Debreu economy with wedges. Household h solves

$$\max_{c_{ht}(s)} \frac{1}{1-\frac{1}{\eta}} \sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}$$

subject to

$$\sum_s \sum_t q_t(s) \mu_{ht}(s) p_{ht}(s) c_{ht}(s) \leq I_h$$

where $q_t(s)$ denotes Arrow-Debreu prices. Given Lagrange multiplier λ_h, the FOC is

$$\beta^t \pi(s) c_{ht}^{-\frac{1}{\eta}}(s) = \lambda_h q_t(s) \mu_{ht}(s) p_{ht}(s)$$

Taking ratios between household h and H and between t and 0,

$$\frac{c_{ht}(s)/c_{h0}}{c_{Ht}(s)/c_{H0}} = \left(\frac{p_{ht}(s)/p_{h0}}{p_{Ht}(s)/p_{H0}}\right)^{-\eta} \left(\frac{\mu_{ht}(s)}{\mu_{Ht}(s)} / \frac{\mu_{h0}}{\mu_{H0}}\right)^{-\eta}$$

or

$$\frac{p_{ht}(s)\, c_{ht}(s) / p_{h0} c_{h0}}{p_{Ht}(s)\, c_{Ht}(s) / p_{H0} c_{H0}} = \left(\frac{p_{ht}(s) / p_{h0}}{p_{Ht}(s) / p_{H0}}\right)^{1-\eta} \left(\frac{\mu_{ht}(s)}{\mu_{Ht}(s)} / \frac{\mu_{h0}}{\mu_{H0}}\right)^{-\eta}.$$

Solving for $p_{ht}(s)\, c_{ht}(s)$,

$$p_{ht}(s)\, c_{ht}(s) = \frac{p_{Ht}(s)\, c_{Ht}(s)}{p_{H0} c_{H0}} p_{h0} c_{h0} \left(\frac{p_{ht}(s) / p_{h0}}{p_{Ht}(s) / p_{H0}}\right)^{1-\eta} \left(\frac{\mu_{ht}(s)}{\mu_{Ht}(s)} / \frac{\mu_{h0}}{\mu_{H0}}\right)^{-\eta}$$

so

$$\sum_h p_{ht}(s)\, c_{ht}(s) = \frac{p_{Ht}(s)\, c_{Ht}(s)}{p_{H0} c_{H0}} \sum_h p_{h0} c_{h0} \left(\frac{p_{ht}(s) / p_{h0}}{p_{Ht}(s) / p_{H0}}\right)^{1-\eta} \left(\frac{\mu_{ht}(s)}{\mu_{Ht}(s)} / \frac{\mu_{h0}}{\mu_{H0}}\right)^{-\eta}. \quad (15)$$

Define

$$\omega_{ht}(s) \equiv \frac{p_{ht}(s)\, c_{ht}(s)}{\sum_{h'} p_{h't}(s)\, c_{h't}(s)}.$$

Using (15),

$$\begin{aligned}
\omega_{ht}(s) &= \frac{p_{h0} c_{h0} \left(\frac{p_{ht}(s)}{p_{h0}}\right)^{1-\eta} \left(\frac{\mu_{ht}(s)}{\mu_{h0}}\right)^{-\eta}}{\sum_{h'} p_{h'0} c_{h'0} \left(\frac{p_{h't}(s)}{p_{h'0}}\right)^{1-\eta} \left(\frac{\mu_{h't}(s)}{\mu_{h'0}}\right)^{-\eta}} \\
&= \frac{\omega_{h0} \left(\frac{p_{ht}(s)}{p_{h0}}\right)^{1-\eta} \left(\frac{\mu_{ht}}{\mu_{h0}}\right)^{-\eta}}{\sum_{h'} \omega_{h'0} \left(\frac{p_{h't}(s)}{P_{h'0}}\right)^{1-\eta} \left(\frac{\mu_{h't}(s)}{\mu_{h'0}}\right)^{-\eta}}.
\end{aligned}$$

Set in every t, s

$$\sum_{h'} \omega_{h'0} \left(\frac{p_{h't}(s)}{p_{h'0}}\right)^{1-\eta} \left(\frac{\mu_{h't}}{\mu_{h'0}}\right)^{-\eta} = 1.$$

This is without loss of generality, as $q_t(s)$ will absorb different normalizations of wedges in different states. Then,

$$\frac{\omega_{ht}(s)}{\omega_{h0}} = \left(\frac{p_{ht}(s)}{p_{h0}}\right)^{1-\eta} \left(\frac{\mu_{ht}(s)}{\mu_{h0}}\right)^{-\eta}$$

so

$$\frac{\mu_{ht}(s)}{\mu_{h0}} = \left(\frac{p_{ht}(s)}{p_{h0}}\right)^{\frac{1-\eta}{\eta}} \left(\frac{\omega_{ht}(s)}{\omega_{h0}}\right)^{-\frac{1}{\eta}}$$

Without loss of generality, we can set $\mu_{h0} = 1$ for all h (since different household-level wedges can be absorbed in I_h). Hence,

$$\log \mu_{ht}(s) = \frac{1-\eta}{\eta} \log \frac{p_{ht}(s)}{p_{h0}} - \frac{1}{\eta} \log \frac{\omega_{ht}(s)}{\omega_{h0}}, \tag{16}$$

which corresponds to (2) in Proposition 2.

We show now that (16) also holds in the compensated equilibrium that replicates the status-quo allocation. In the compensated equilibrium, the representative household maximizes

$$\min_h \{\tilde{u}_h(\boldsymbol{c}_h)\}$$

with

$$\tilde{u}_h(\boldsymbol{c}_h) = \left[\frac{\sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}}{\sum \beta^t \pi(s) c^0_{ht}(s)^{1-\frac{1}{\eta}}}\right]^{\frac{\eta}{\eta-1}},$$

subject to a single budget constraint

$$\sum_h \sum_{s,t} q_t(s) \mu_{ht}(s) p_{ht}(s) c_{ht}(s) \leq I.$$

The solution to this problem can be solved as a two-step budgeting problem. The Hicksian representative agent distributes income across h,

$$\sum I_h = I,$$

and for each h maximizes $\tilde{u}_h(\boldsymbol{c}_h)$ subject to

$$\sum_{s,t} q_t(s) \mu_{ht}(s) p_{ht}(s) c_{ht}(s) = I_h.$$

The choice of $\{I_h\}_h$ must be such that

$$\tilde{u}_h(\boldsymbol{c}_h) = \tilde{u}_{h'}(\boldsymbol{c}_h).$$

Setting $I_h = I^0_h$ for all h, where I^0_h is the income level of household h in the status-quo of the primitive AD economy with wedges, gives the status-quo allocation $\boldsymbol{c}^{\mathbf{0}}$, which satisfies $\tilde{u}_h(\boldsymbol{c}^0_h) = \tilde{u}_{h'}(\boldsymbol{c}^0_h) = 1$. Hence, there is an equilibrium in a AD economy with a Hicksian representative agent that coincides with the primitive AD equilibrium in the status-quo. $\square$

Proof of Proposition 3. We apply Proposition 7 from Baqaee and Burstein (2025b) and evaluate the derivative at $(\boldsymbol{z}+\Delta\boldsymbol{z},\log\boldsymbol{\mu})$

$$\Delta\log A\approx-\frac{1}{2}\sum_i\lambda_i(\mu,z+\Delta z)\log\mu_i\sum_j\frac{\partial\log y_i^{comp}(\mu,z+\Delta z)}{\partial\log\mu_j}\log\mu_j$$

where Δz is chosen to turn-off the productivity shocks and discrepancy between derivatives at $(\boldsymbol{z},\log\boldsymbol{\mu})$ and $(\boldsymbol{z}+\Delta\boldsymbol{z},\log\boldsymbol{\mu})$ is higher order. Since there are only household-level wedges, we must only calculate

$$\sum_{h'}\frac{\partial\log c_h^{comp}(\mu,z+\Delta z)}{\partial\log\mu_{h'}}\log\mu_{h'}.$$

We omit in the remainder of the proof the superscript *comp*. Recall that the Hicksian representative agent maximizes $\tilde{u}_h(\boldsymbol{c}_h)$ subject to $\sum_{s,t}q_t(s)\,\mu_{ht}(s)\,p_{ht}(s)\,c_{ht}(s)=I_h$ and constant $\tilde{u}_h/\tilde{u}_H$. The first order condition of the maximization problem is

$$\left[\frac{\sum\beta^t\pi(s)c_{ht}(s)^{1-\frac{1}{\eta}}}{\sum\beta^t\pi(s)c^0_{ht}(s)^{1-\frac{1}{\eta}}}\right]^{\frac{\eta}{\eta-1}}\frac{\beta^t\pi(s)c_{ht}(s)^{-\frac{1}{\eta}}}{\sum\beta^t\pi(s)c_{ht}(s)^{1-\frac{1}{\eta}}}=\lambda_h q_t(s)\,\mu_{ht}(s)\,p_{ht}(s)$$

or

$$\tilde{u}_h(\boldsymbol{c}_h)\,\beta^t\pi(s)c_{ht}(s)^{-\frac{1}{\eta}}=\underbrace{\lambda_h\sum\beta^t\pi(s)c_{ht}(s)^{1-\frac{1}{\eta}}}_{\tilde{\lambda}_h}q_t(s)\,\mu_{ht}(s)\,p_{ht}(s)$$

Taking the ratio between h and H,

$$\frac{c_{ht}(s)}{c_{Ht}(s)}=\left(\frac{\mu_{ht}(s)\,\tilde{\lambda}_h p_{ht}(s)}{\mu_{Ht}(s)\,\tilde{\lambda}_H p_{Ht}(s)}\right)^{-\eta}\left(\frac{\tilde{u}_h}{\tilde{u}_H}\right)^{\eta}$$

Consider a change in wedges, keeping productivities fixed. Differentiating the equation above and using the fact that the Hicksian RA keeps $\tilde{u}_h/\tilde{u}_H$ constant, and denoting static expenditure shares by $\chi_{ht}(s)=p_{ht}(s)\,c_{ht}(s)\,/\sum p_{h't}(s)\,c_{h't}(s)$, we have

$$d\log\chi_{ht}(s)-d\log\chi_{Ht}(s)=(1-\eta)\,(d\log p_{ht}(s)-d\log p_{Ht}(s))-\eta\left(d\log\frac{\mu_{ht}(s)}{\tilde{\lambda}_h}-d\log\frac{\mu_{Ht}(s)}{\tilde{\lambda}_H}\right).\tag{17}$$

Our goal is to solve for (compensated) $d\log c_{ht}(s)=d\log\chi_{ht}(s)-d\log p_{ht}(s)$ in response to changes in wedges, to a first order. We consider neoclassical economies without intertemporal production links and with household-time level wedges. The following equations hold to a first-order in the compensated equilibrium:

$$d\log p_t(s) = \sum_f \Psi_{(:,f)} d\log w_{ft}(s)$$

$$d\log w_{ft}(s) = d\log \tilde{\lambda}_{ft}(s)$$

$$d\Omega_t(s) = \Theta d\log w_t(s)$$

$$d\tilde{\lambda}'_t(s) = \chi' \Psi d\Omega_t(s)\Psi + d\chi'_t(s)\Psi$$

where $\lambda'_t(s) = \chi'_t(s)\Psi_t(s)$ denotes static Domar weights and $\Psi = (I - \Omega)^{-1}$ is the static inverse Leontieff matrix. In combination with (17), we can solve for changes in quantities and prices, given $\{d\log\mu_{ht}(s) - d\log\tilde{\lambda}_h\}$. We consider the approximation around the allocation with no wedges and no productivity changes.

Inspecting the above linearized system, we can express $d\log c_{ht}(s)$ as a function of $\{d\log\mu_{ht}(s) - d\log\tilde{\lambda}_h\}$:

$$d\log c_{ht} = A^h\left(d\log\boldsymbol{\mu}_t - d\log\tilde{\boldsymbol{\lambda}}\right),$$

where $d\log\tilde{\boldsymbol{\lambda}} = \{d\log\tilde{\lambda}_h\}$, $d\log\boldsymbol{\mu}_t = [d\log\mu_{1t}, ..., d\log\mu_{ht}]$, and A_h are vectors for each h with coefficients that depend on parameters and shares in the allocations without shocks. We now solve for $d\log\tilde{\lambda}_h$. The condition $d\log\tilde{u}_h = d\log\tilde{u}_H$ can be expressed as

$$\frac{\sum_{s,t}\pi(s)\beta^t\bar{c}_{ht}(s)^{1-\frac{1}{\eta}}}{\sum_{s,t'}\pi(s)\beta^{t'}\bar{c}_{ht'}(s)^{1-\frac{1}{\eta}}}d\log c_{ht}(s) = \frac{\sum_t\pi(s)\beta^t\bar{c}_{Ht}(s)^{1-\frac{1}{\eta}}}{\sum_{t'}\pi(s)\beta^t\bar{c}_{Ht'}(s)^{1-\frac{1}{\eta}}}d\log c_{Ht}(s)$$

around the point of approximation $\bar{c}_{ht}(s) = \bar{c}_h$. So, we have

$$\frac{\sum\pi(s)\beta^t d\log c_{ht}(s)}{\sum\pi(s)\beta^t} = \frac{\sum\pi(s)\beta^t d\log c_{Ht}(s)}{\sum\pi(s)\sum\beta^t}$$

or substituting

$$\frac{\sum\pi(s)\beta^t A^h d\log\boldsymbol{\mu}_t}{\sum\pi(s)\beta^t} - \frac{\sum\pi(s)\beta^t A^h d\log\tilde{\boldsymbol{\lambda}}_{\boldsymbol{h}}}{\sum\pi(s)\beta^t} = \frac{\sum\pi(s)\beta^t A^H d\log\boldsymbol{\mu}_t}{\sum\pi(s)\beta^t} - \frac{\sum\pi(s)\beta^t A^H \boldsymbol{d\log\tilde{\lambda}_h}}{\sum\pi(s)\beta^t}$$

or

$$\sum_{s,t} \pi(s) \beta^t \sum_{h'} \left[A^h_{h'} - A^H_{h'} \right] d\log \mu_{h't}(s) = \sum_{s,t} \pi(s) \beta^t \sum_{h'} \left[A^h_{h'} - A^H_{h'} \right] d\log \tilde{\lambda}_{h'}$$
$$\sum_{h'} \left[A^h_{h'} - A^H_{h'} \right] \frac{\sum_{s,t} \pi(s) \beta^t d\log \mu_{h't}(s)}{\sum_{s,t} \pi(s) \beta^t} = \sum_{h'} \left[A^h_{h'} - A^H_{h'} \right] d\log \tilde{\lambda}_{h'}$$

Therefore, a solution is to set

$$d\log \tilde{\lambda}_{h'} = \frac{\sum_{s,t} \pi(s) \beta^t d\log \mu_{h't}(s)}{\sum_{s,t} \pi(s) \beta^t}.$$

Therefore, to solve for changes in consumption by household in the compensated equilibrium, we use the static system above plus

$$\begin{aligned} d\log \chi_{ht}(s) - d\log \chi_{Ht}(s) = {} & (1-\eta)\left(d\log p_{ht}(s) - d\log p_{Ht}(s)\right) \\ & - \eta \Bigg(d\log \mu_{ht}(s) - \frac{\sum_t \pi(s) \beta^t d\log \mu_{ht}(s)}{\sum \pi(s) \beta^t} \\ & - d\log \mu_{Ht}(s) + \frac{\sum_{s,t} \pi(s) \beta^t d\log \mu_{Ht}(s)}{\sum \pi(s) \beta^t} \Bigg) \end{aligned} \tag{18}$$

The input to this system is the static input-output matrix, Ψ, and static expenditure shares χ in the status-quo (both defined in detail below) and

$$d\log \mu_{ht}(s) - \frac{\sum_{t,s} \pi(s) \beta^t d\log \mu_{ht}(s)}{\sum \pi(s) \beta^t},$$

which can be calculated from the status-quo wedges.

We now provide an expression for $A_{hh'}$. Define the within-period (static) $(H + N +$

$F) \times (H+N+F)$ input-output matrix:

$$\Omega = \left[\begin{array}{ccc|ccc|ccc} 0 & \cdots & 0 & b_{11} & \cdots & b_{1N} & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & & \cdots & & & \cdots & \\ 0 & \cdots & 0 & b_{H1} & \cdots & b_{HN} & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & \Omega_{11} & \cdots & \Omega_{1N} & \Omega_{1N+1} & \cdots & \Omega_{1N+F} \\ \vdots & \cdots & \vdots & & \ddots & & & & \\ 0 & \cdots & 0 & \Omega_{N1} & & \Omega_{NN} & \Omega_{NN+1} & \cdots & \Omega_{NN+F} \\ \hline 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{array}\right]$$

The first H rows correspond to the households consumption baskets. The next N rows correspond to the expenditure shares of each producer on every other producer and factor. The last F rows correspond to the expenditure shares of the primary factors. The Leontief inverse matrix is the $(H+N+F) \times (H+N+F)$ matrix defined as $\Psi \equiv (I-\Omega)^{-1}$. The within-period Domar weights are[28]

$$\lambda' = \chi' \Psi,$$

where χ_h denotes the share of expenditures of household h in total expenditures in a given period. Define the $F \times F$ matrix B with element (f,f) given by

$$b_{f,f'} = (1-\eta)\, Cov_{\chi'}(\Psi_{(:,f)}, \Psi_{(:,f')}) + \sum_{f' \in F} \sum_{j \in H \cup N} \lambda_j (1-\theta_j) Cov_{\Omega_{(j,:)}}(\Psi_{(:,f)}, \Psi_{(:,f')}),$$

and the $F \times H$ matrix D with element (f,h) given by

$$d_{f,h} = Cov_{\chi'}(e_h, \Psi_{(:,f)})$$

where e_h is the h-th basis vector column vector (with hth element equal to 1). Define the $H \times H$ matrix.

$$\mathcal{F} = -\eta \Psi_{HF} \left(diag(\lambda_F) - B\right)^{-1} D.$$

where Ψ_{HF} is the $H \times F$ block of Ψ and λ_F is the $F \times 1$ vector of static factor shares (the

[28]Within-period Domar weights are sales in a period divided by total consumption expenditures in that period. We refer to these as within-period Domar weights to contrast them with Arrow-Debreu Domar weights which are net present value sales divided by net present value of total consumption using Arrow securities.

last F elements of λ). Finally, define the $H \times H$ matrix,

$$A = -\eta\left(F + I\right) - (1 - \eta)\mathbf{1}\chi' F + \mathbf{1}\eta\chi'.$$

The element (h, h') of A is $A_{hh'}$ in Proposition 3. Note that A depends on the input-output matrix Ω, expenditure shares χ, and elasticities of substitution in production and consumption, θ_j for $j \in H \cup N$, and the EIS η. $\square$

Other proofs to be added.