International Income Comparisons by Matching Households Across Countries

David Baqaee Ariel Burstein Javier Cravino

UCLA and University of Michigan

October 1 2024.

How do standards of living differ across countries?

- Focus on specific question:

  What income makes households in country A indifferent to facing country B's prices?

  e.g., What is the equivalent French income for a Polish household? How does this compare with French households' income? And with German's equivalent income?

- Adjust Polish income by price differences, weighted by compensated budget shares.

- But compensated budget shares often based on strong assumptions.

What we do

- Calculate equivalent income across countries using non-parametric method.
  - Recover compensated shares w/o imposing homotheticity or estimating demand system.
  - Requires uncompensated budget shares across income distribution in base country and budget shares for income of interest in other countries.
- Apply method for average income in ICP countries using France as base country.
  - Average HH in poor countries less poor than implied by using aggregate shares.

    Driven by income effects in housing spending.
- Apply method across income distribution in Eurostat countries (requires more data).
  - In Eurostat countries, cross-country price adjustment larger for poor.

Selected Literature

- International income comparisons:

  Summers & Heston (‘88, ‘91), Neary (‘04), Feenstra, Ma & Rao (‘09), Deaton and Dupriez (‘11), Feenstra-Inklaar-Timmer (‘15)...

- Economic approach to index number theory:

  Diewert (‘76, ‘99, ‘08...), Caves, Christensen & Diewert (‘82), Balk (‘96), Hill (‘99)...

- Non-parametric methods to estimate cost of living over time:

  Blundell et al (‘03), Jaravel & Lashkari (‘24), Baqaee, Burstein & Koike-Mori (‘24)...

- Spatial price indices using micro data:

  Jaravel & Beck (‘21), Argente Hsieh & Lee (‘22), Cavallo Feenstra-Inklaar (‘23), Diamond-Moretti (‘24), ...

- Paper: Infer compensated budget shares w/out estimating demand and use it for international income comparisons.

Agenda

Agenda

Setup

- Common preferences $\succeq$ over $\boldsymbol{c}$ in $\mathbb{R}^N$ represented by $\mathscr{U}(\boldsymbol{c})$.
- Indirect utility function $v(\boldsymbol{p}, \mathrm{I}) = \max_{\boldsymbol{c}}\{\mathscr{U}(\boldsymbol{c}) : \boldsymbol{p} \cdot \boldsymbol{c} \leq \mathrm{I}\}$.
- Expenditure function $e(\boldsymbol{p}, u) = \min_c\{\boldsymbol{p} \cdot \boldsymbol{c} : \mathscr{U}(\boldsymbol{c}) \geq u\}$.
- Index countries by τ, with price vector $\boldsymbol{p}_\tau$.

Definition: Equivalent Income (money metric)

$$e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))$$

Converts income I in country τ to equivalent income in country τ_0.

Setup

- Common preferences $\succeq$ over $\boldsymbol{c}$ in $\mathbb{R}^N$ represented by $\mathscr{U}(\boldsymbol{c})$.
- Indirect utility function $v(\boldsymbol{p}, \mathrm{I}) = \max_{\boldsymbol{c}}\{\mathscr{U}(\boldsymbol{c}) : \boldsymbol{p} \cdot \boldsymbol{c} \leq \mathrm{I}\}$.
- Expenditure function $e(\boldsymbol{p}, u) = \min_{c}\{\boldsymbol{p} \cdot \boldsymbol{c} : \mathscr{U}(\boldsymbol{c}) \geq u\}$.
- Index countries by τ, with price vector $\boldsymbol{p}_\tau$.

Definition: Equivalent Income (money metric)

$$e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))$$

Converts income I in country τ to equivalent income in country τ_0.

- **Goal:** measure $e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))$ to compare incomes across countries given data on prices $\boldsymbol{p}_\tau$ and budget shares $\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I})$ for $\mathrm{I} \in [\underline{\mathrm{I}}_\tau, \bar{\mathrm{I}}_\tau]$.

Calculating equivalent income

- Equivalent income is income deflated by the True Cost of Living (Könus) Index:

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \log \frac{e(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I}))}{e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))}$$

Calculating equivalent income

- Equivalent income is income deflated by the True Cost of Living (Könus) Index:

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \log \frac{e(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I}))}{e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))}$$

- To a second-order approximation in price changes (Diewert 1976):

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \approx \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{b}^H(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I})) + \boldsymbol{b}^H(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))\right] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

Calculating equivalent income

- Equivalent income is income deflated by the True Cost of Living (Könus) Index:

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \log \frac{e(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I}))}{e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))}$$

- To a second-order approximation in price changes (Diewert 1976):

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \approx \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{b}^H(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I})) + \boldsymbol{b}^H(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))\right] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

$$= \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{b}^H(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))\right] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

Observed shares for income I in τ

Compensated shares not observed

Calculating equivalent income

- Equivalent income is income deflated by the True Cost of Living (Könus) Index:

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \log \frac{e(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I}))}{e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))}$$

- If **homothetic**, use aggregate shares (‘**Aggregate Deflator**’):

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \approx \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{b}^H(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))\right] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

$$= \log \mathrm{I} - \frac{1}{2}[\underbrace{\boldsymbol{B}(\boldsymbol{p}_\tau)}_{\text{Observed shares in } \tau} + \underbrace{\boldsymbol{B}(\boldsymbol{p}_{\tau_0})}_{\text{Observed shares in } \tau_0}] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

Calculating equivalent income

- Equivalent income is income deflated by the True Cost of Living (Könus) Index:

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \log \frac{e(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I}))}{e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))}$$

- **General case**: compensated equals share of ‘matched’ HH (**‘Matched Deflator’**).

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \approx \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{b}^H(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))\right] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

$$= \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{B}(\boldsymbol{p}_{\tau_0}, e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})))\right] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

Observed shares for income I in τ Observed shares for ‘matched’ income in τ_0

Calculating equivalent income

- Equivalent income is income deflated by the True Cost of Living (Könus) Index:

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \log \frac{e(\boldsymbol{p}_\tau, v(\boldsymbol{p}_\tau, \mathrm{I}))}{e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))}$$

- $e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))$ is unique solution to a fixed point problem:

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \approx \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{B}(\boldsymbol{p}_{\tau_0}, e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})))\right] \cdot [\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}]$$

Without extrapolating $B(\boldsymbol{p}_{\tau_0}, \cdot)$, match exists if $e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \in [\underline{\mathrm{I}}_{\tau_0}, \bar{\mathrm{I}}_{\tau_0}]$.

Iterative algorithm

- Guess $e_0(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))$, e.g. $e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \mathrm{I}$
- Solve for $e_k(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}))$ and iterate until convergence:

$$\log e_k(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \approx \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{B}(\boldsymbol{p}_{\tau_0}, e_{k-1}(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})))\right] \cdot \left[\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}\right]$$

- Similar to Deaton and Dupriez (‘11) approach to international poverty comparisons.

Exact results

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) \approx \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{B}(\boldsymbol{p}_{\tau_0}, e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})))\right] \cdot \left[\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}\right]$$

- Different ways to approximate Könus index, as in homothetic case.

Exact results

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \frac{1}{2}\left[\boldsymbol{B}(\boldsymbol{p}_\tau, \mathrm{I}) + \boldsymbol{B}(\boldsymbol{p}_{\tau_0}, e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})))\right] \cdot \left[\log \boldsymbol{p}_\tau - \log \boldsymbol{p}_{\tau_0}\right]$$

- Exact if $\log e(\boldsymbol{p}, u)$ is quadratic in $\log \boldsymbol{p}$ with u-specific params (translog).

$$\log e(\boldsymbol{p}, u) = \alpha(u) + \sum_{i=1}^{I} \alpha_i(u) \log p_i + \frac{1}{2} \sum_{j=1}^{I} \sum_{i=1}^{I} \gamma_{ij}(u) \log p_i \log p_j, \quad \text{with} \quad \gamma_{ij}(u) = \gamma_{ji}(u)$$

Exact results

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \sum_{i=1}^{I} \frac{B_i(\boldsymbol{p}_\tau, \mathrm{I}) - B_i(\boldsymbol{p}_{\tau_0}, e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})))}{\log B_i(\boldsymbol{p}_\tau, \mathrm{I}) - \log B_i(\boldsymbol{p}_{\tau_0}, e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})))} [\log p_{i\tau} - \log p_{i\tau_0}]$$

- Exact with Sato-Vartia weights if $e(\boldsymbol{p}, u)$ is generalized non-homothetic CES.

$$e(\boldsymbol{p}, u) = \left[\sum_{i=1}^{I} \alpha_i(u) p_i^{1-\sigma(u)} \right]^{\frac{1}{1-\sigma(u)}}.$$

Exact results

- Exact with any preferences if continuously chained (Divisia Index).

$$\log e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I})) = \log \mathrm{I} - \int_{\tau_0}^{\tau} \sum_i \left[B_i(\boldsymbol{p}_s, e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_s, \mathrm{I})))\right] \frac{d \log p_{is}}{ds} ds$$

Agenda

Data

- Need prices $\boldsymbol{p}_\tau$ and $\boldsymbol{p}_{\tau_0}$.
- Need budget shares $B(\boldsymbol{p}_\tau, \mathrm{I})$ for I and $B(\boldsymbol{p}_{\tau_0}, \mathrm{I}')$ for $\mathrm{I}' \in [\underline{\mathrm{I}}_{\tau_0}, \bar{\mathrm{I}}_{\tau_0}]$.

- Price level indices across 12 COICOP Divisions from ICP 2011.
- Country-level aggregate expenditures data from ICP 2011.
  - Individual Consumption Expenditures by Households by COICOP Division.
  - Calculate aggregate budget shares and expenditures per adult-equivalent.
  - Calculate adult-equivalent from World Development Indicators.
- Household-level expenditures for 25 countries from Eurostat HBS 2015.
  - Household level expenditures across 12 COICOP Divisions.
  - Total expenditures per adult-equivalent by household.
  - Impute owner-occupied-housing expenditures using data for similar HHs that rent.

Agenda

Equivalent income for average household in each country

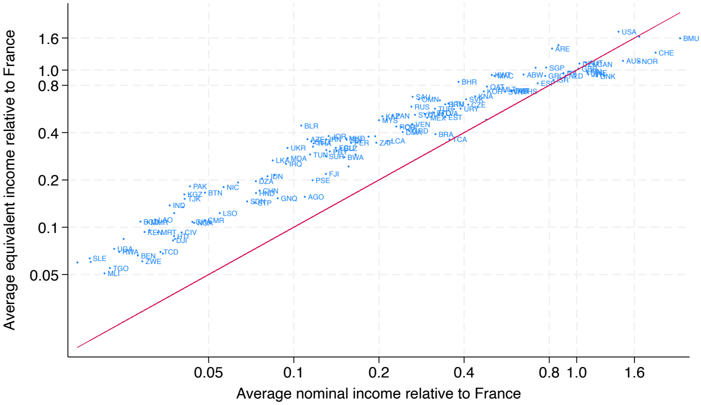
- Calculate equivalent French income for 'average-household' in each ICP country.

- Define average-household in each country τ as:
  - Income I_{τ}^{av} equals consumption expenditures per adult-equivalent (ICP).
  - Assume budget shares of average household equal aggregate budget shares.
  - This assumption is very good in countries where we can check it.

- For base country $\tau_0 = $ France:
  - Budget shares $B(\boldsymbol{p}_{\tau_0}, \mathrm{I})$ for $\mathrm{I} \in [\underline{\mathrm{I}}_{\tau_0}, \bar{\mathrm{I}}_{\tau_0}]$ (Eurostat HBS).
  - Given discrete data, smooth $B(\boldsymbol{p}_{\tau_0}, \mathrm{I})$ flexibly within observed income support.
  - Aggregate budget shares in Eurostat HBS similar to ICP.

- Report:
  1. Equivalent income in τ relative to average income in France: $e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_{\tau}, \mathrm{I}_{\tau}^{av}))/\mathrm{I}_{\tau_0}^{av}$.
  2. Compare Matched deflator with Aggregate deflator (homothetic).

Equivalent vs. nominal income for average household



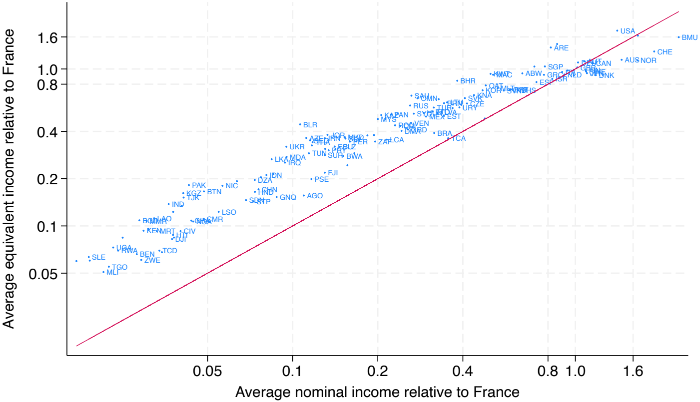
- Report results for countries where $e(\boldsymbol{p}_{\tau_0}, v(\boldsymbol{p}_\tau, \mathrm{I}_\tau^{av}))/\mathrm{I}_{\tau_0}^{av} > 0.05$.
- Market-exchange rates comparisons overstate equivalent income differences.

Matched vs. Aggregate deflator for average household

- Aggregate deflator makes poor countries look expensive.
- Most poor countries less poor than what is implied by aggregate deflator.

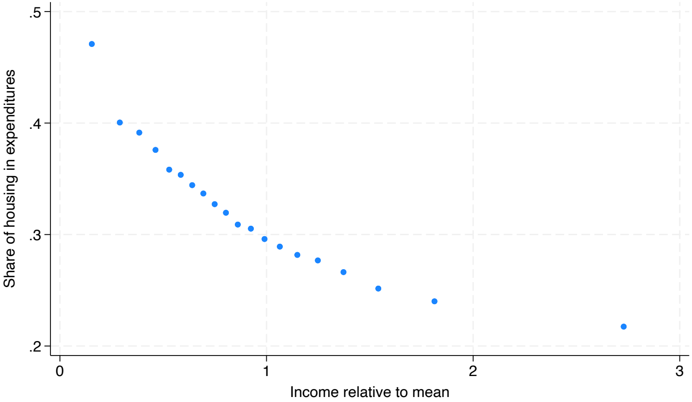
Why is Matched deflator lower than Aggregate deflator?

1. For most countries, matched French household is poorer than average French.



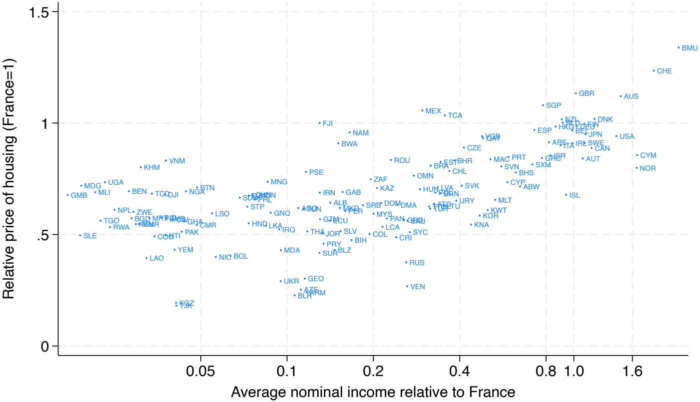
Why is Matched deflator lower than Aggregate deflator?

1. For most countries, matched French household is poorer than average French.
2. Within France, poor spend more on housing.

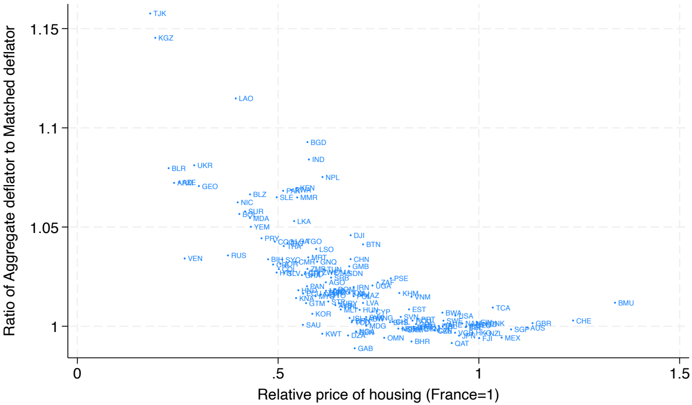


Why is Matched deflator lower than Aggregate deflator?

1. For most countries, matched French household is poorer than average French.
2. Within France, poor spend more on housing.
3. Housing is relatively cheap in poor countries.



Matched deflator is relatively lower where housing is cheap

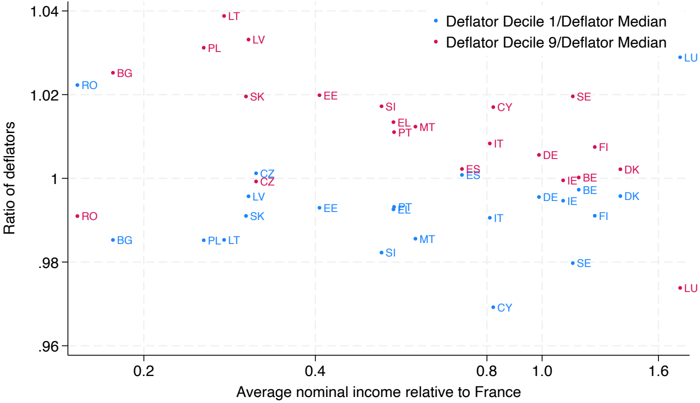


Agenda

Equivalent income across income distribution in Eurostat countries

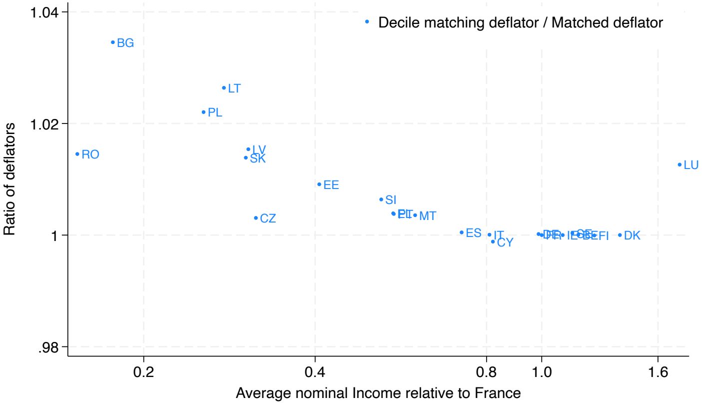
- Calculate equivalent French income across income distribution by country.
- For base and comparison countries τ_0 and τ:
  - Smoothed budget shares $B(\boldsymbol{p}_\tau, \mathrm{I})$ for $\mathrm{I} \in [\underline{\mathrm{I}}_\tau, \bar{\mathrm{I}}_\tau]$ (HBS).
  - Calculate equivalent income for I for which we can find matched household in France.
- Report:
  1. Matched deflator, by income level.
  2. Compare with Matched and Aggregate deflators across income deciles.
  3. Compare with deflators that match HHs by percentile in income distribution.
- Robustness to different approximations

Matched deflators by income decile

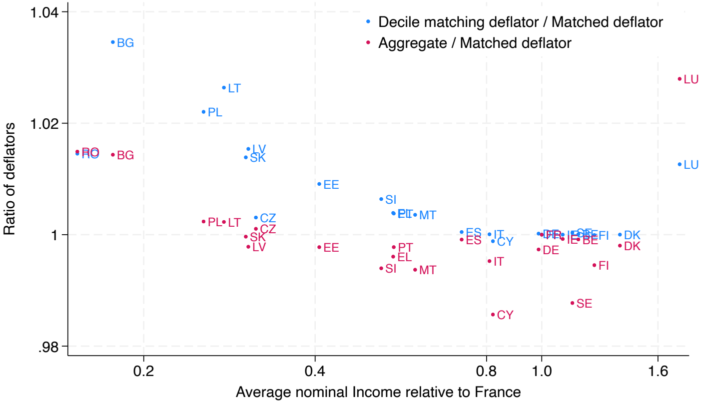


- Small differences, Matched deflator is lower for poorer households.

Matched deflator vs. ‘decile matching’ for ninth decile

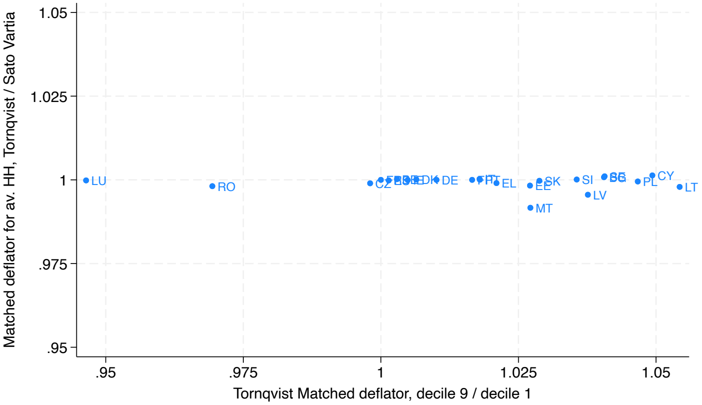


Matched deflator vs. ‘decile matching’ for ninth decile



- Error from decile matching may be larger than error from aggregate deflator.
- Equivalent income of ‘rich’ Poles closer to French average income than to French rich.
- Data requirement of our method is the same as for decile matching.

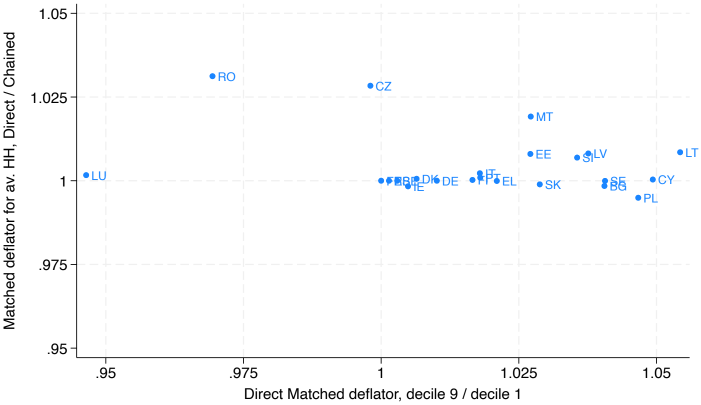
Robustness: Sato-Vartia weights



- Difference across formulas small relative to differences across income deciles.

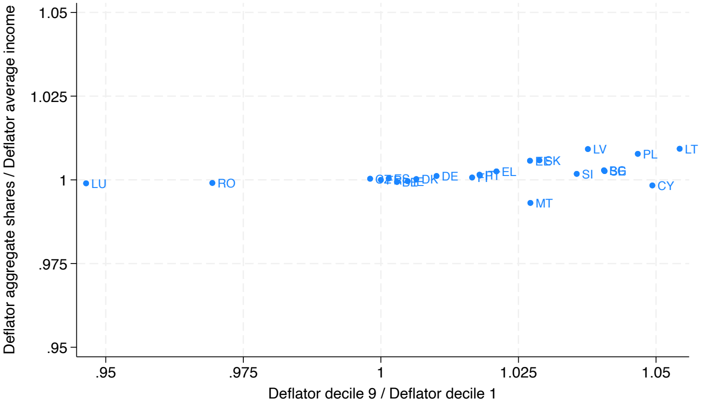
Robustness: Approximate integral by chaining

- Choose a path connecting τ and τ_0 using MST (Hill '99), WRPD 'W3' (Diewert '02).



- For most countries, differences small relative to differences across income deciles.

Deflator for average income vs. Deflator with aggregate shares



- Differences small relative to differences across income deciles.

Conclusion and future work

- Method for cross-country income comparisons allowing for income effects.
- Same requirement as decile-specific deflators.
- Future work:
  - Calculate standard erros.
  - Time series of cross-country income differences.
  - Allow for differences in tastes across groups by matching within groups. e.g. If Poles have different tastes than French, match to Polish immigrants in France.
  - Cross-country wealth comparisons to account for future (Baqaee, Burstein & Koike-Mori).