Long-Run Comparative Statics

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March 7, 2025

Question

- What are the long-run effects of permanent changes to the economy:
  - e.g. industry productivity shocks? trade costs and tariffs? within and cross-sector distortions?
- In the long-run, capital stock will adjust to changes.
- Characterize comparative statics for large class of models in terms of primitives.

Key Takeaway

- Long-run analysis can be done using a static economy with as-if wedge distortions.
- "Wedge" reflects deviation from Golden Rule, given by ratio of capital income & investment.
- Long-run outcomes obey logic of the theory of the second best (even in efficient economies!)
- That is, long-run effects of changes depend on interaction with initial "distortions."
- If capital income exceeds investment, reallocation towards capital boosts long-run consumption.

Quantitative Findings

- Changes to distortions
  - Large long-run consumption effects even from small markups.
  - Large long-run consumption effects even from small tariffs.
  - Large long-run consumption effects even from small distortions to formal firms.
  - Static economy or fully depreciated capital $\rightarrow$ no first-order effect due to envelope theorem
- Changes to productivities
  - Long-run consumption effects $>>$ sales shares for industries upstream of investment
  - Effects amplified if capital-labor substitutability $\sigma_{KL} > 1$, and mitigated if $\sigma_{KL} < 1$
  - Static economy or fully depreciated capital $\rightarrow$ effects $=$ sales share indep. of elasticities

Selected Related literature

- Sufficient statistics for production networks in trade and macro:
  Hulten (1978), Gabaix (2011), Costinot & Rodriguez-Clare (2014), Caliendo & Parro (2015), Baqaee & Farhi (2020), Bigio & La'O (2020), Liu (2020), Barro (2022), Baqaee & Farhi (2024), Andersen et al. (2024).

- Development accounting:
  Klenow & Rodriguez-Clare (1997), Hall & Jones (1999), Caselli (2005), Hsieh & Klenow (2007), Hsieh & Klenow (2010).

- Trade and capital:
  B. Ravikumar et al. (2019), Lyon & Waugh (2019), Dix-Carneiro et al. (2023), Antras (2023), Kleinman et al. (2024), Sposi et al. (2024).

- Investment networks in trade & macro:
  Vom Lehn & Winberry (2022), Foerester et al. (2022), Ding (2024).

- Intertemporal approach to current account:
  Obstfeld & Rogoff (1995), Caballero et al. (2008), Mendoza et al. (2009), Angeletos & Panousi (2011).

Agenda

1. Setup
2. Comparative Statics
3. Quantitative Model
4. Conclusion

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Assumptions

- Class of models defined by the following balanced growth equations

Producer behavior:

$$Y_i = A_i F_i \left[\{L_{if}\}_{f \in \mathcal{F}}, \{Y_{ij}\}_{j \in \mathcal{N}}, \{K_{ij}\}_{j \in \mathcal{N}} \right]$$

$$Y_i, L_{if}, Y_{ij}, K_{ij} \text{ maximizes } \pi_i = p_i Y_i - \sum_f w_f L_{if} - \sum_j p_j Y_{ij} - \sum_j (r_j + \delta_j) p_j K_{ij}$$

Household behavior:

$$\{C_i\} \text{ maximizes } U(C_1, \ldots, C_N) \text{ given } \sum_i p_i C_i \leq \sum_f w_f L_f + \sum_j (r_j - g) B_j$$

Resource constraints and asset market clearing:

$$Y_i = C_i + X_i + \sum_j Y_{ji}, \quad X_i = (g + \delta_i) K_i, \quad \sum_i L_{if} \leq L_f, \quad \sum_j K_{ij} \leq K_i$$

$$B_j = p_j K_j$$

Proposition: Isomorphism of BGP and Static Economy

Consider BGP with returns $\{r_i\}$ — its prices & quantities are also eqm. of a static economy where:

1. The production functions and preferences are the same as in the dynamic economy.
2. Capital goods are intermediates produced with technology $K_i = A_{K_i} X_i$ where $A_{K_i} = \frac{1}{g+\delta_i}$.
3. Capital goods are sold at a markup $\mu_i = \frac{r_i+\delta_i}{g+\delta_i}$ with profits distributed to households.

For long-run outcomes, capital subject to *as-if* markup unless $r_i = g$ or $\delta_i = \infty$ for every i.

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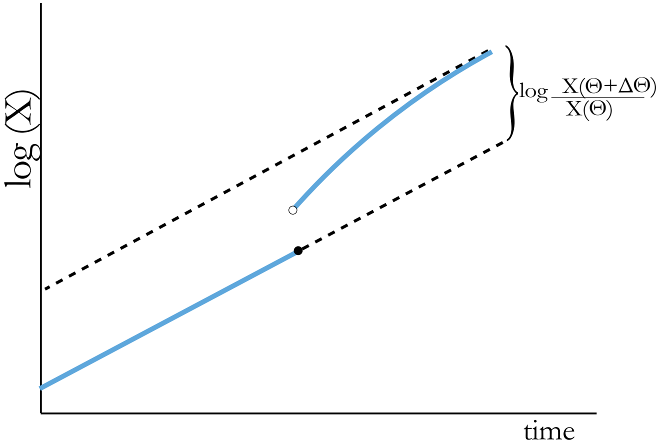
Extensions: Open economy and Initial Distortions

- Open-Economy
  - Modify BGPs equations to feature households in each country
  - Index BGPs by $\{r_i\}$ and the distribution of net capital income $\{\pi_c\}$ across countries
  - Proposition carries through if profits are distributed according to π_c in the static economy
- Initial Distortions
  - Modify BGP equations to have linear taxes, with revenues distributed lump-sum to households
  - Proposition carries through with taxes as additional wedges in the static economy
- Non-physical capital
  - E.g., entry costs as in Hopenhayn can also be accommodated

Agenda

Illustration of BGP comparative static



Comparative Statics Using Isomorhpism

- Suppose $X^{BGP}(\Theta)$ is a smooth function of parameter Θ.

- Proposition implies $X^{BGP}(\Theta) = X^{static}(\Theta, \mu(\Theta))$.

$$\frac{dX^{BGP}}{d\Theta} = \frac{\partial X^{static}}{\partial \Theta} + \sum_i \frac{\partial X^{static}}{\partial \mu_i} \frac{d\mu_i}{d\Theta}$$

- First term: effect of Θ shock in static economy holding as-if markup fixed.

- Second term: effect of markup shock in static economy $\times$ change in markup.

- The endogenous change in as-if markup depends on changes in rates of return:

$$\mu_i = \frac{r_i + \delta_i}{g + \delta_i} \quad \Longrightarrow \quad \frac{d\mu_i}{d\Theta} = \frac{1}{g + \delta_i} \frac{dr_i}{d\Theta}.$$

Capital Supply (Asset Demand)

- To derive returns, we use capital/asset market clearing
- We use a model's *capital supply* correspondence $\mathcal{A}[\mathbf{r}(\Theta)] = \frac{B_i}{\sum_f w_f L_f}(\boldsymbol{r})$
  - Necessary conditions on household asset holdings on a balanced growth path
  - Different models give rise to different asset demand functions, e.g. neoclassical growth model

$$\mathcal{A}(r) = \begin{cases} -\frac{1}{r-g} & \text{if } r < \rho + \sigma g \\ \left[-\frac{1}{r-g}, \infty\right) & \text{if } r = \rho + \sigma g \\ \varnothing & \text{if } r > \rho + \sigma g \end{cases}, \tag{1}$$

  - Other accumulation problems: Blanchard-Yaari, Aiyagari

Capital Market Clearing

- On a BGP, define capital demand to be $\mathcal{K}_i^{static}[\Theta, \mu(\boldsymbol{r})] = \frac{p_i K_i}{\sum_f w_f L_f}(\Theta, \mu(\boldsymbol{r}))$
- Note: capital demand is purely a function of the static model solution
- Equilibrium condition

$$\mathcal{K}_i^{static}[\Theta, \mu(\boldsymbol{r})] = \mathcal{A}[\mathbf{r}(\Theta)]$$

- Total differentiation yields $d\mathbf{r}$ as a function of $d\Theta$.
- Let $\epsilon_r^s = \frac{d \log \mathcal{A}}{d\mathbf{r}}$ and $\epsilon_r^d = -\frac{\partial \log \mathcal{K}^{static}}{d\boldsymbol{r}}$.

Long-Run Comparative Statics with Endogenous Returns

Proposition

Assume $X^{BGP}(\Theta)$, $X^{static}(\Theta, \mu)$, and $\mathcal{A}(\mathbf{r})$ are diffable, then:

$$\frac{dX^{BGP}}{d\Theta} = \frac{\partial X^{static}}{\partial \Theta} + \sum_i \frac{\partial X^{static}}{\partial \mu_i} \frac{d\mu_i}{d\Theta},$$

where, changes in as-if markups are

$$\frac{d\boldsymbol{\mu}}{d\Theta} = \frac{1}{g + \boldsymbol{\delta}} \frac{d\boldsymbol{r}}{d\Theta} = \frac{1}{g + \boldsymbol{\delta}} \left(\epsilon_r^d + \epsilon_r^s\right)^{-1} \frac{\partial \log \mathcal{K}^{static}}{\partial \Theta}.$$

- Partial derivatives of X^{static} and $\mathcal{K}^{static}$ and ϵ_r^d known functions of input-output table and elasticities of substitution (see Baqaee & Farhi, 2024).
- Full comparative statics pinned down given semi-elasticity of savings to returns, ϵ_r^s.

Agenda

Quantification

- Production as in standard quantitative trade models (Costinot & Rodriguez-Clare, 2014) with global input-output structure.
- Cobb-Douglas across sectors, CES within sectors with elasticity $\theta = 5$.
- Closed-form asset demand from OLG + industry-specific idiosyncratic capital risk.
  - Endogenous risk premia clears physical capital markets by country.
  - Risk-free rate pinned down by world bond market.
- Data sources:
  - World Input-Output Database (Timmer et al. 2015) + investment flows (Ding 2022).
  - Initial NFA positions from External Wealth of Nations (Milesi-Ferretti 2022).
- Initial as-if markup for each capital good: $\mu_i = \frac{\text{Gross operating surplus}_i}{\text{Investment}_i}$.

Calibration Results

Parameter	**Description**	**USA**	**CHN**	**EU**	**JPN**	**GBR**
r	Risk-free rate	0.025	0.025	0.025	0.025	0.025
g	Growth rate	0.024	0.024	0.024	0.024	0.024
$\bar{r}_c$	Average return on capital	0.124	0.052	0.156	0.119	0.093
$\bar{\mu}_c$	Harmonic average wedge on capital	2.372	1.799	2.443	2.175	2.100

- As-if markup on capital is high, since capital compensation roughly double investment.

Quantitative Experiments

- Distortions: revisiting classic questions in different literatures:
  - Macro-IO: Uniform increase in markups across industries;
  - Trade: Uniform increase in tariffs across countries;
- Productivity: revisiting importance of different industries for long-run consumption
  - Uniform increase in an industry's productivity in all countries

Breakdown of Long-Run Consumption Response

- In all distortion experiments, country consumption changes satisfy:

$$d \log C_c = \underbrace{\sum_{i \in \mathcal{K}_c} \left[\frac{GOS_i - INV_i}{P_c C_c} \right] d \log K_i}_{\Delta \text{Harberger ``misallocation''}} + \underbrace{\sum_{i \in \mathcal{N}} \frac{NX_{ci}}{P_c C_c} d \log p_i}_{\Delta \text{terms of trade}} + \underbrace{\frac{1}{P_c C_c} d \left[(r - g) B_c \right]}_{\Delta \text{net factor payments}} .$$

- Last two terms are zero sum, implying that world consumption satisfies

$$d \log C = \sum_{i \in \mathcal{K}} \left[\frac{GOS_i - INV_i}{P_c C_c} \right] d \log K_i$$

- In particular, no long-run effect on global C if gross operating surpluses equal investment

Elasticity of Consumption to Universal Increase in Markups

Scenario	**Description**	**Global Consumption**
Benchmark	Baseline calibration	-0.770
Rep. agent	Baseline calibration holding returns and current accounts constant	-1.293
Static	Investment treated as a final expenditure and capital treated as an endowment	0.000
$\sigma_{KL} = 1.2$	Higher elasticity of substitution between capital and labor	-0.878
$\sigma_{KL} = 0.6$	Lower elasticity of substitution between capital and labor	-0.425
$\theta = 1$	Benchmark calibration, but trade elasticities equal to zero ($\theta - 1 = 0$)	-0.763
$\delta = \infty$	All depreciation rates set to infinity. Implies that all as-if markups equal 1, and that capital is treated as an intermediate	0.000

- Markups very costly for long-run consumption even if no initial markups.
- Higher markups increase prices of investment goods relative to labor, depressing capital stock.
- No first-order effect in a static economy or when capital fully depreciates

Elasticity of Consumption to Universal Increase in Tariffs

Country	$d \log C_c$	**Harberger**	**Terms of trade**	**Δ Current account**
United States	-0.106	-0.098	-0.007	-0.001
Canada	-0.314	-0.271	-0.041	-0.002
China	-0.105	-0.120	0.011	0.004
United Kingdom	-0.229	-0.185	-0.044	-0.000
India	-0.216	-0.212	-0.002	-0.002
Japan	-0.111	-0.110	-0.005	0.003
Mexico	-0.669	-0.672	0.010	-0.008
European Union	-0.083	-0.087	0.006	-0.001
Rest of the World	-0.185	-0.215	0.027	0.003
Global	-0.137	-0.137	0.000	-0.000

- Most of the effect driven by the “misallocation” term.

Understanding Mechanisms

Selected regions	**Benchmark**	**Rep. agent**	**Static**	$\sigma_{KL} = 0.6$	$\sigma_{KL} = 1.2$	$\theta = 1$	$\delta = \infty$
United States	-0.106	-0.127	0.006	-0.049	-0.124	-0.136	-0.023
European Union	-0.083	-0.107	0.006	-0.030	-0.101	-0.116	-0.005
China	-0.105	-0.119	-0.022	-0.042	-0.130	-0.127	-0.000
Japan	-0.111	-0.124	0.008	-0.050	-0.132	-0.151	-0.018
Canada	-0.314	-0.397	-0.074	-0.198	-0.348	-0.143	-0.077
Global	-0.137	-0.190	0.000	-0.073	-0.158	-0.128	0.000

- For world long-run consumption:
  - Trade elasticity not very important.
  - Inelastic asset demand mitigates effects (consistent with misallocation logic)
  - Elasticity of substitution between capital and labor important.
  - No effect in static economy or when capital fully depreciates

Effect of increasing industry productivities

- Benchmark results for undistorted economies:
  - Static economy w/ exogenous capital: output elasticity $= \frac{\text{Sales}}{\text{GDP}}$ (Hulten, 1978)
  - If Golden Rule wedge absent: long-run consumption elasticity $= \frac{\text{Sales}}{\text{Consumption}} > \frac{\text{Sales}}{\text{GDP}}$
- Result for distorted economies (Baqaee and Farhi, 2020)

$$\frac{\partial \log C}{\partial \log A_i} = \tilde{\lambda}_i - \sum_{f \in F} \tilde{\lambda}_f \frac{\partial \log \lambda_f}{\partial \log A_i}$$

  - First term: technological effect, given by *cost-based* Domar weight $\tilde{\lambda}_i$
    - Exceed sales share for sectors with big markups/taxes between them and final consumption
    - Possible to calcualte from input-output table plus estimated capital wedges μ_i
  - Second term: reallocation effect, positive if labor shares tend to fall

Elasticity of Long-Run Consumption to Productivity Shocks

			Sales weights		Cost weights	
Sector	$\frac{\partial \log C}{\partial \log A_i}$	$\frac{\text{Sales}_i}{\text{GDP}}$	$\frac{\text{Sales}_i}{\text{C}} (= \lambda_i)$	$\frac{\left(\frac{\partial \log C}{\partial \log A_i}\right)}{\lambda_i}$	$\tilde{\lambda}_i$	$\frac{\left(\frac{\partial \log C}{\partial \log A_i}\right)}{\tilde{\lambda}_i}$
Agriculture	0.111	0.080	0.099	1.117	0.111	1.000
Basic and Feb Metals	0.200	0.075	0.094	2.131	0.197	1.014
Machinery	0.135	0.044	0.054	2.484	0.134	1.007
Electrical Eqmt	0.188	0.070	0.087	2.150	0.185	1.012
Transport Eqmt	0.183	0.070	0.087	2.115	0.180	1.015
Construction	**0.406**	**0.124**	**0.154**	**2.633**	**0.399**	**1.018**
Hotel and Restaurants	0.077	0.056	0.070	1.108	0.077	1.000
Prof. Services	0.291	0.146	0.181	1.604	0.292	0.997
Health	**0.109**	**0.087**	**0.108**	**1.009**	**0.109**	**1.000**

- Labor share roughly constant in model $\Rightarrow$ cost-based BGP Domar weight very close to correct.
- If industry upstream from capital $\Rightarrow$ cost-based Domar weight much larger than sales/consumption.

Effect of capital-labor substitutability: reallocation

Panel A: Health Sector				
	$\tilde{\lambda}$	Benchmark	Rep. agent	$\delta = \infty$
$\sigma_{KL} = 0.6$	0.109	0.108	0.108	0.109
$\sigma_{KL} = 1.0$	0.109	0.109	0.109	0.109
$\sigma_{KL} = 1.2$	0.109	0.109	0.109	0.109
Panel B: Machinery Sector				
	$\tilde{\lambda}$	Benchmark	Rep. agent	$\delta = \infty$
$\sigma_{KL} = 0.6$	0.135	0.091	0.078	0.135
$\sigma_{KL} = 1.0$	0.135	0.135	0.136	0.135
$\sigma_{KL} = 1.2$	0.135	0.149	0.165	0.135

- Reallocations further boost effects if labor/capital share responds.

Agenda

Conclusion

- What are the long-run effects of permanent shocks?
- Finding: possible to use static analysis with as-if distortions.
- Long-run outcomes obey logic of the theory of the second best (even in efficient economies!)
- Distortions: big first-order long-run consumption effects if distortions reduce capital
- Productivities: long-run consumption effects $>$ sales share for sectors upstream of investment